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STUDY RAYLEIGH DAMPING IN STRUCTURES; UNCERATINTIES AND TREATMENTS

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ABSTRACT :
Characterization of energy damping sources in structures has been an active area of research for many years. Despite all the improvements in modeling and analysis techniques for capturing the behavior of structural components of building, our understanding about inherent energy dissipating characteristics is far from reality. The current state-of-knowledge for modeling inherent structural damping is to use a linear viscous damping model that assumes the global energy dissipating characteristics of a structural system is proportional to its dynamic degrees of freedom velocities, and does not depend on the level of nonlinear behavior of the structural system. On the other hand a proportional damping model called Rayleigh damping is used to model the energy dissipation characteristics of the structure for decades. This paper studies the consequences of using Rayleigh damping in analysis of inelastic structures. It is shown that using the stiffness proportional part of the damping based on the original damping ambiguous forces will develop which may result to overestimated designs and lack of static equilibrium will be observable. A proposed model is used to control this source of problem in this paper.

KEYWORDS:
DAMPING, RAYLEIGH DAMPING, INELASTIC BEHAVIOR

1. INTRODUCTION

All structures show some degree of energy loss during motion. This energy loss is referred to three main sources in numerical analysis which are nonlinearity of members, energy radiation and inherent damping. Characterization of damping forces in vibrating structures has long been under focus of Earthquake Engineers. Damping capacity is defined as the ratio of the energy dissipated in one cycle of oscillation to the maximum amount of energy accumulated in the structure. There are many mechanisms of damping in a structure the most identified of which are material damping and interfacial damping. The material damping contribution comes from a complex molecular interaction within the material, thus the damping is dependant on type of material, methods of manufacturing and finishing processes (Kareem and Gurley, 1996). Since the material microscopic properties may differ from one sample to the other one, estimation of material damping me be complicated.

The interfacial damping mechanism is Coulomb friction between members and connections of a structural system and non structural components like partitions, facades and other frictional mechanisms which are still unidentified. Experiments have shown that most of the energy dissipation mechanisms through the structure are dependant on displacement amplitude rather than frequency of the structure. Therefore the most appropriate form of formulating damping in structures may be friction damping. The mathematical model for Coulomb (friction) damping is as below:

$$f_d = \mu \frac{\dot{u}}{|\dot{u}|} = \mu \operatorname{sgn}(\dot{u}) \tag{1.1}$$

In which μ is the friction coefficient of contacting surfaces, $\dot{u}$ is the first derivative of displacement and $\operatorname{sgn}(\bullet)$ is the mathematical sign function. This approach needs nonlinear analysis which may consume large amount of time and computer power to analyze the structure. That's why in typical engineering practice viscous damping models are used for the sake of simplicity as they lead to the linear analysis of equation of motion. The mathematical model for viscous damping is :

$$f_d = c\dot{u} \tag{1.2}$$

One can understand that the viscous damping is frequency dependant and not displacement dependant. Since viscous damping is used to linearly model the structural behavior, any source of nonlinearity is relatively unknown and consequences of using this model are ignored. Consider the equation of motion for a linear elastic MDOF system with linear viscous damping as below:

$$\mathbf{M}\ddot{u}(t) + \mathbf{C}\dot{u}(t) + \mathbf{K}u(t) = 0 \tag{1.3}$$

In which **M**, **C** and **K** are mass, damping and stiffness matrices and $u(t)$ is the displacement vector and $(\dot{u})$ and $(\ddot{u})$ represent the first and second order derivatives of time at different degrees of freedom respectively. The damping of structure is assumed to be viscous and frequency dependant for the sake of convenience in analysis. The most popular method is to solve the equation of motion using the modal analysis; in this case damping values are directly assigned to the modes. Damping ratios can be calculated using the Caughey series (Caughey and O'Kelly, 1965), Rayleigh damping (Rayleigh, 1954) is a special case of which. Rayleigh damping known as proportional damping or classical damping model expresses damping as a linear combination of the mass and stiffness matrices, that is,

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \tag{1.4}$$

Where α and β are real scalars with 1/sec and sec units respectively. Modes of classically damped systems preserve the simplicity of the real normal modes. Given the system mass, stiffness and damping matrices, the response of the system can be calculated using the available methods. However the accuracy of response may be questionable due to the fact that this approach is based on two main assumptions, a) The model used for damping of a structure is viscous and b) the approach is formulated for the linear response of the structure which may not be the available situation for all cases (i.e. structures with nonlinearities). Since it is not possible to completely present the consequences of modeling damping in structures resulted from these two reasons in this paper, the effects of usage of damping in structures with nonlinearities will be considered. Further discussion about the consequences of usage of viscous damping in modeling structures behavior will be studied in companion papers which are being prepared by the authors.

The aim of this paper is to depict the uncertainties resulted from usage of Rayleigh damping in analysis of structures and have an overview of the treatments that can improve the results using this model in analysis of structures.

2. RAYLEIGH DAMPING

The structural cases which have entered to their nonlinear behavior range are of interest of this paper. In these cases the equation of motion of the structure should be solved directly and the modal procedure is of no use anymore. To solve the equation of motion the mass, stiffness and stiffness matrices of Eqn. (1.3) should be known. Using the assumption of the linear viscous damping in structures focusing on Rayleigh damping

(Eqn. (1.4)) the damping matrix can be defined as a function of mass and stiffness matrices. The damping ratio for the nth mode of such a system is:

$$\zeta_n = \frac{\alpha}{2}\frac{1}{\omega_n} + \frac{\beta}{2}\omega_n \tag{2.1}$$

The coefficients α and β can be determined from specified damping ratios ζ_i and ζ_j for the *i*th and *j*th modes, respectively (Chopra, 2007).

$$\frac{1}{2}\begin{bmatrix} 1/\omega_i & \omega_i \\ 1/\omega_j & \omega_j \end{bmatrix}\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \begin{Bmatrix} \zeta_i \\ \zeta_j \end{Bmatrix} \tag{2.2}$$

The procedure introduced by Hall (2006) is convenient for determining α and β. Selecting a desired amount of damping and a frequency range of ϖ to $R\varpi$ covering those modes of interest, *R*>1, Δ can be computed;

$$\Delta = \zeta \frac{1+R-2\sqrt{R}}{1+R+2\sqrt{R}} \tag{2.3}$$

Where Δ determines bounds on the damping ratios that are imparted to those modes within the specified frequency range. The modes in the given frequency range will have a damping ration bounded between ξ+Δ and ξ-Δ. α and β can be calculated from:

$$\alpha = 2\zeta\varpi \frac{2R}{1+R+2\sqrt{R}} \qquad \beta = 2\zeta \frac{1}{\varpi}\frac{2}{1+R+2\sqrt{R}} \tag{2.4}$$

And can be used to compute an actual damping value for *n*th mode from Eqn. (2.1) if ω_n is known. Figure 1 shows a comparison between different choices of *R* with a given ξ=%10. In this figure ϖ =1 for all cases.

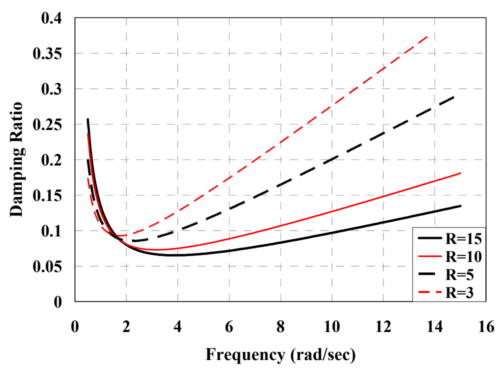

Figure 1: Actual damping ratio as a function of frequency

It can be shown that larger *R* results in larger Δ which is of less desire. This means that although we may like to choose a larger *R* to cover a larger range of frequencies but it results in larger Δ which decreases the accuracy of the damping ratio used as the basis of formation of the range. For *R*=3, although a small range of frequency is covered, the fluctuation between maximum and minimum damping in this range 2Δ=0.01436 which results in $\zeta_{\max-\omega=1,3}$ = 0.1072 and $\zeta_{\min-\omega=\varpi+\sqrt{3}=2.7}$ =0.0928 which are close to ζ=%10, for *R*=15 on the other hand the frequency range covered is a large one but the difference between maximum and minimum damping is 2Δ = 0.06952 which results in $\zeta_{\max-\omega=1,15}$ = 0.1347 and $\zeta_{\min-\omega=\varpi+\sqrt{15}=4.87}$ =0.0670 which have relatively large divergence from ζ =%10. As a result it's better to use to close frequencies to enter Eqn. (2.3).

3. BACKGROUND

3.1 Uncertainties

Some of the problems resulted from the usage of Rayleigh damping in structures are addressed in this section. One of the main problems is when using Rayleigh damping in structures with added dampers or base isolators. This occurs in analysis of base-isolated structures if the Rayleigh damping matrix is constructed using properties of the relatively stiff superstructure alone. The damping ratio imparted to the overall structure consisting of the superstructure and the flexible isolators can be very high owing to the mass-proportional damping term. No non-linearity in the restoring forces is the reason of the unrealistic damping effects; a possible solution to the problem is to use only stiffness-proportional damping (Hall, 2005).

Another type of problem arises if a portion of a structure breaks loose and thereby develops a high velocity; in which case, large mass-proportional damping forces can also develop (Hall, 2005). The stiffness-proportional part of Rayleigh damping corresponds physically to linear viscous dampers that interconnect the degrees of freedom of a structure. In a non-linear analysis where the non-linearity is of the softening type, limits on the restoring forces are imposed by various mechanisms such as yielding, cracking, sliding and buckling. If the initial linear stiffness matrix is employed to construct the stiffness-proportional damping term, then the damping forces in a softening element can reach unrealistically high values compared to the element's restoring. The reason is that although the stiffness is reduced due to nonlinearities, the velocity of the structure increases due to the nonlinearities but because the initial, high stiffness of the structure is used in forming the motion equation, the calculated damping is extravagant. The linearity assumption on the damping forces means they are always proportional to the velocities, no matter how large they become (Hall, 2005). This problem is also addressed by Bernal (1995) and Leger and Dussault (1992).

3.2. Treatments

Several solutions to this problem have been proposed by different researchers which will are summarized in here:

3.2.1. Bernal (1995)

Bernal stated that when using the classical damping in inelastic analysis, the damping matrix is assembled to provide classical damping on the basis of initial inelastic characteristics. If the damping matrix is assumed to remain constant through out the response, proportionality is lost when yielding occurs because the undamped mode shapes for the tangent properties are not the same as those of the original elastic structure. Bernal relates the spurious behavior of damping mechanism to appearance of large damping forces at degrees of freedom having small associated inertias. However, this approach does not preclude spurious damping forces to appear when masses are assigned to rotational degrees of freedom.

3.2.2 Hall (2006)

Hall suggested remedy is to eliminate mass-proportional damping contribution and Bound the stiffness proportional damping contribution.

3.2.3 Charney (2005)

Charney proposed a solution in which the stiffness-proportional component of the damping matrix is based only on the diagonal terms of the initial stiffness, i.e., terms that correspond to the dynamic degrees of freedom, in order to avoid assigning stiffness-proportional damping to degrees of freedom without mass.

Charney also stated that even a better solution would be to eliminate the use of viscous damping altogether, and utilize nonlinear frictional or hysteretic damping. Another solution provided by Charney would be to assemble a damping matrix based on the tangent stiffness of the system, i.e., the damping matrix is updated at each time step but the proportionality coefficients are based on elastic stiffness as shown in Equation 3, in this case a moderate degree of artificial damping may be generated.

$$\mathbf{C}(t) = \alpha\,\mathbf{M} + \beta\,\mathbf{K}_t(t) \tag{3.1}$$

A more efficient solution proposed by Charney is to update the proportionality coefficients with tangent stiffness:

$$\mathbf{C}(t) = \alpha_t\mathbf{M} + \beta_t\mathbf{K}_t(t) \tag{3.2}$$

Where K_t, α_t and β_t are tangent stiffness, mass proportional coefficient and stiffness proportional coefficient updated each step, respectively. However, the application of this approach is questionable once the overall tangent stiffness of the system becomes negative due to second order, P- Δ effects and/or significant material strength and stiffness deterioration. An alternative could be to use only mass-proportional damping:

$$\mathbf{C} = \alpha\,\mathbf{M} \tag{3.3}$$

Although this latter approach will satisfy static equilibrium, displacements responses will exhibit significant higher-frequency responses, which are not present in the response of real structures. This is due to the presence of small damping ratios at the higher modes once this solution is devised. Moreover, several studies such as Otani (1980) demonstrate that stiffness-proportional damping models provide a better correlation with experimental results.

3.2.4 Medina and Krawinkler (2004)

Medina and Krawinkler proposed modeling each beam element with a combination of an elastic beam element and rotational end springs. Plastic hinging occurs in these zero length rotational spring elements with zero damping. As the initial stiffness of the spring is set to be large all the elastic deformations occur in the beam with the given damping. This will result in stiffness proportional damping which will be relevant to the stiffness of the elastic beam and eliminates the effect of ambiguous forces resulted from stiffness proportional part of the damping in nonlinear cases.

3.2.5 Zareian (2006)

Zareian proposed an extension to the solution proposed by Medina and Krawinkler (2004) for proper modeling of viscous damping using the Rayleigh model that can be applied to beam/column elements whose moment gradient can vary with time. This extension is useful in modeling beams and columns in Multi-Degree-Of-Freedom (MDOF) systems. Zareian's solution involves changing the stiffness matrix of the elastic internal beam element explained previously such that the effect of fixed stiffness of the springs at the two ends of the elastic beam element is compensated.

4. PROCEDURE

In order to study the effect of Rayleigh damping on inelastic behavior of the systems, two SDOF frames were developed. The models are based on the model suggested by Krawinkler and Medina (2004) and Zareian (2006). As discussed in previous sections, using the Rayleigh damping with initial stiffness leads to unrealistic damping which can not predict the real seismic response of the structure. To overcome this problem a remedy is to use a lumped inelastic hinge in which all the inelasticity of the structure is concentrated. In these models, the elastic stiffness of members with concentrated plasticity is divided into the elastic stiffness of the rotational springs at the end of the member and the elastic stiffness of the beam-column element. In this step of the study a single-bay single-story frame is modeled and the rotational stiffness of the member can be derived from structural properties of the frame. The rotational stiffness of the member is the combination of beam and spring stiffness in series:

$$\frac{1}{K_{mem}} = \frac{1}{K_s} + \frac{1}{K_{bc}} \Rightarrow K_{mem} = \frac{K_s K_{bc}}{K_s + K_{bc}} \tag{4.1}$$

In which K_{mem}, K_s, and K_{bc} are stiffness of the member after adding rotational springs, stiffness of rotational springs and stiffness of the elastic beam element respectively. After some trials, it is decided to use an elastic spring that its stiffness K_s is "n" times larger than the rotational stiffness of the beam element, K_{bc}:

$$K_s = nK_{bc} \tag{4.2}$$

The stiffness of the sub-element can now be expressed as a function of the total stiffness of the member and the multiplier n:

$$K_{bc} = \left(\frac{n+1}{n}\right) K_{mem} \qquad K_s = (n+1)K_{mem} \qquad K_{mem} = 6EI_i / L \tag{4.3}$$

In which I_i and L are the initial moment of inertia and length of the beam respectively. In this case, plastic hinging is modeled by using nonlinear rotational springs with an initial stiffness several times larger than that of the beam-column element. Zero stiffness proportional damping is assigned to the springs and as a result the β factor is multiplied by the $(1+1/n)$ factor to compensate for the lack of stiffness-proportional damping provided by the rotational springs.

In this study, the frame under consideration has been modeled as an SDOF system. The SDOF system was first selected because when there is only one DOF, the effect of mass and stiffness proportional damping can be considered separately, and all sources of the problem can be identified. The columns have axial and flexural rigidity and the column bases are hinged to eliminate the influence of columns rigidity in analyses. The damping ratio is %10 and the models are elasto- plastic. It is assumed that maximum and minimum yielding moments M_y^+ and M_y^-, for all the cases are 3000kips-in, which are defined as yielding point of the bilinear model. The geometry of frames is shown in Figure 3. There are two different models considered for the beams, one without (Model A) and one with rotational springs (Model B). the two models are dynamically identical except for the treatment of damping.

In model A, the beam element can plastify at its yield surfaces in both ends and can perform a bilinear elasto-plastic behavior (Figure 2-a). In model B the beam is flexible in bending and is linear while the rotational spring element has large initial stiffness with bilinear elasto-plastic behavior. The stiffness-proportional damping is then assigned to the beam and no damping is assigned to the spring (Figure 2-b). Since the model is an SDOF, the effect of mass and stiffness proportional damping are considered separately. Figure 3 to 5 depict the roof displacement, moments at the end of the columns, and the base shear of both models compared to each other.

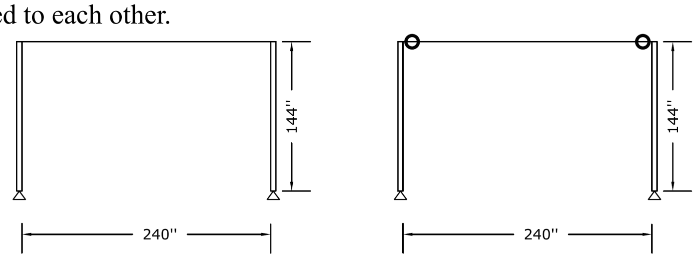

Figure 2: (a) Model without rotational springs, (b) Proposed model with rotational springs

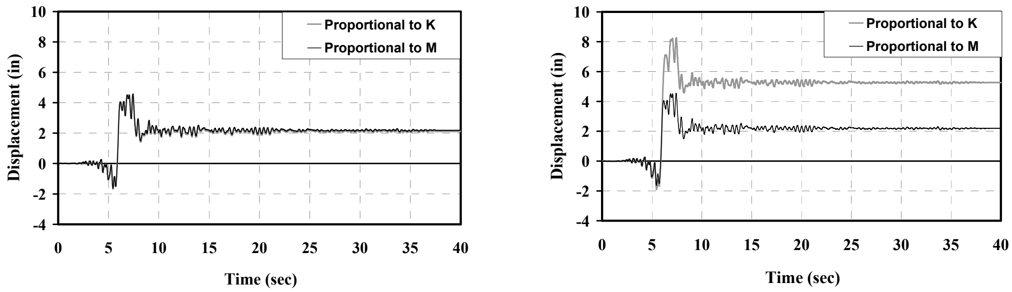

Figure 3: (a) Roof displacements of Model A, (b) Roof displacements of Model B

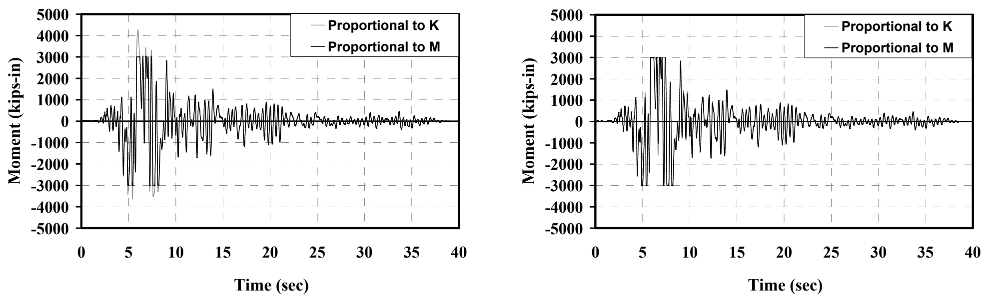

Figure 4: (a) Column end moments of Model A, (b) column end moments of Model B

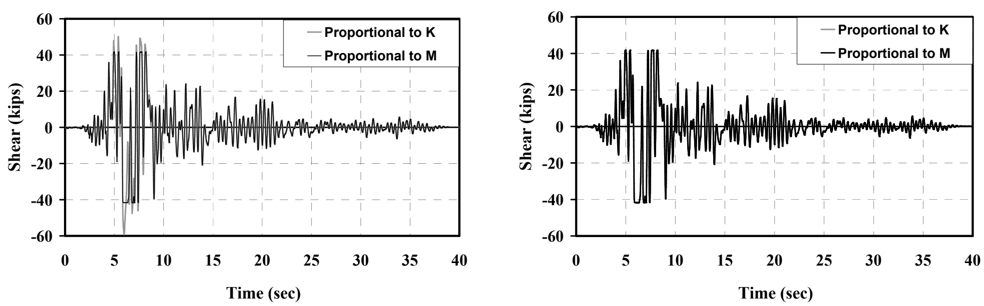

Figure 5: (a) Base shear of Model A, (b) Base shear of Model B

It is depicted in Figure 4 that the column moments in model A are exceeding the maximum amount of M_y=3000 kips-in whereas these moments are limited in Model B. The same trend is recognizable in base shear plots (Figure 5).A study has also been provided on the effect of *n* value on results of the analysis, as confirmed by Zareian (2006) the larger the *n* the harder the convergence of the analysis. To overcome the convergence problems with larger *n* we had to provide smaller time intervals. The reason for this inconvenience with increasing the value of *n* is that in this case the difference between the initial stiffness of the beam and the spring increases and results in stiffness instabilities.

In these models it is assumed that elements do not possess strain hardening properties ($\alpha_{s,member}$= 0). In case the original element has strain hardening ,which is the slope of the back bone curve shown in Figure 6 in post-yielding range, this property of the plastic hinge which is all concentrated in the spring must be adjusted to obtain the strain hardening coefficient for the moment rotation of the member, as suggested by Ibarra and Krawinkler (2005).

Since the beam and spring element are connected to each other in series as mentioned before, the changes in rotation of the total element in post yielding is the sum of the rotation changes in spring and beam, and the strain hardening coefficient of the spring will be:

$$\alpha_{s,s} = \frac{\alpha_{s,member}}{n+1-n\alpha_{s,member}} \tag{4.4}$$

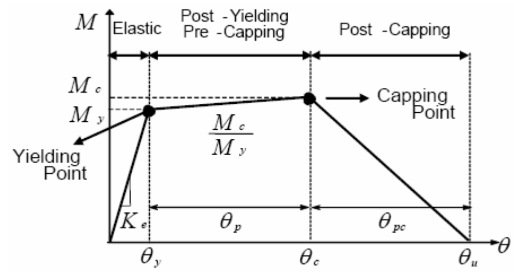

Figure 6: Structural component back bone curve

4.2. Two-DOF Systems

The comparison of the 2-DOF systems or generally speaking MDOF systems with this procedure is almost impossible. In MDOF cases both mass proportional and stiffness proportional damping are acting simultaneously and study/ comparison of these cases with this procedure is impossible.

5. CONCLUSION

A comprehensive study on available literature on drawbacks of using Rayleigh damping is conducted in this paper. It has also been shown that usage of Rayleigh damping in analysis of structures which undergo inelastic behavior may lead to ambiguous moments in members and unexpected behavior especially under seismic forces. The proposed method by Medina, Krawinkler and Zareian has been examined as an alternative to overcome the lack of accuracy resulted from Rayleigh damping. It can also be concluded that problems associated with application of this kind of damping is related to the usage of the linear viscous model in equation of motions which is used in nonlinear analysis of the structures. An easier remedy to this problem would be constructing equation of motion with a new damping matrix which also considers the real nature of damping in structures that is friction and Hysteresis model.

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