STATISTICAL PROPERTIES OF NONSTATIONARITY IN FREQUENCY CONTENTS OF GROUND MOTIONS BASED ON ZERO-CROSSING RATE

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ABSTRACT :

Considerable attention has been paid to the nonstationarity in frequency contents of ground motions and many methods have been proposed to study this property. But most of these research findings are not suitable for studying the statistical properties and attenuation laws of nonstationarity in frequency contents due to their nonparametric nature, which also blocks their further applications in selection and synthesis of proper ground motions for seismic design. In this paper, zero-crossing rate is used to describe the nonstationary property in frequency contents and an exponential decay model is proposed to represent the zero-crossing rate. Then, the downhill simplex method is employed to estimate the corresponding model (zero-crossing) parameters. By applying the proposed method to ground motions for various magnitudes, epicentral distances and sites, the statistical properties and attenuation laws of zero-crossing parameters are explored and the site-specific zero-crossing parameters are suggested. The work of this paper not only provides a simple and effective approach to study the nonstationarity in frequency contents and its statistical properties, but also gives a more reasonable method to select and synthesize ground motions for seismic design.

KEYWORDS: earthquake ground motion, nonstationarity, frequency content, statistical property, attenuation law, zero-crossing rate

1. INTRODUCTION

The nonstationary properties in amplitude and frequency contents have been regarded as important properties of earthquake ground motions besides the conventional properties in amplitude, frequency and duration. Since last two decades, considerable research has been reported on the nonstationary property in amplitude, but not much work has been done on the nonstationary properties in frequency contents. Recent research work has shown that the nonstationary property in frequency contents of earthquake ground motions have nonnegligible influence on elastic-plastic response of structures under certain conditions (Li 1999). Though several notions have been proposed to describe the nonstationarity in frequency contents, most of them can not be easily used for practical purpose due to their nonparametric nature, which has blocked their further applications in selection and synthesis of proper ground motions for seismic design. The modeling, statistical properties and attenuation laws of nonstationarity in frequency contents of earthquake ground motions have been the open issues in earthquake engineering.

As a simple but effective approach to solving the difficulties mentioned above, zero-crossing rate is used to describe the nonstationary property in frequency contents in this study. And an exponential decay model is proposed to represent the zero-crossing rate. Then, the downhill simplex method is employed to estimate the corresponding model (zero-crossing) parameters. Lastly, by applying the proposed method to ground motions with various magnitudes, epicentral distances and sites, the statistical properties and attenuation laws of zero-crossing parameters are studied, and the site-specific zero-crossing parameters are suggested. The work of this paper not only demonstrates the effectiveness of the proposed method in studying the nonstationarity in frequency contents and its statistical properties, but also gives a new criterion in selection and synthesis of ground motions for seismic design.

2. NONSTATIONARITY IN FREQUENCY CONTENTS

2.1. Modeling of Nonstationarity in Frequency Contents

As has been discussed above, the nonstationary properties in amplitude and frequency contents, which have nonnegligible influence on elastic-plastic response of structures under certain conditions, are important properties of earthquake ground motions besides the conventional properties in amplitude, frequency and duration. Since last two decades, considerable attention has been paid to the nonstationary property in amplitude, e.g. in Li (1999) the nonstationarity in amplitude is described by a three-segment piecewise envelop function and the site-specific envelop parameters are suggested. But not much significant progress has been achieved in the nonstationary properties in frequency contents. Though several approaches are available to analyze such nonstationary properties i.e. the evolutionary spectrum (Priestley 1965), physical spectrum (Mark 1970), time-frequency analysis based on Hilbert transform (Cohen 1989), the parametric method based on time varying autoregressive moving average model (Conte 1990 and Dong 2004) and the recently developed Hilbert-Huang transform (Huang 1998), their analysis results still need to be parameterized if they are used to study the statistical properties and attenuation laws of the nonstationarity in frequency contents, which is really a difficult task and has blocked their further applications for practical purpose.

To circumvent above difficulties in a simple but effective way, the zero-crossing rate is used here to describe the nonstationary property in frequency contents and the zero-crossing rate is represented in an exponential decay form. For a given earthquake ground motion $x(t)$, the cumulate zero-crossing number $\mu_0(t)$ can be designated as

$$m_0(t) = \frac{h_0}{g_0}\left(1 - e^{-g_0 t}\right) \tag{2.1}$$

where h_0 is initial zero-crossing rate and g_0 is the decay coefficient. Accordingly, the derivative form of $\mu_0(t)$, i.e. zero-crossing rate $v_0(t)$, is given by

$$n_0(t) = h_0 e^{-g_0 t} \tag{2.2}$$

A detailed physical interpretation of the zero-crossing rate $v_0(t)$ has been made in Li (1999) which stated that the zero-crossing rate $v_0(t)$ was related to first- and second-order spectral moments of the earthquake ground motion $x(t)$.

2.2. Parameter Estimation of Zero-crossing Model

Based on above zero-crossing model in Eqns. 2.1 and 2.2, the optimization algorithm can be used to estimate the corresponding model parameters. In this study, the downhill simplex method (Nelder and Mead 1965) is used. In case that there are *n* parameters to be estimated, the downhill simplex method can be used to solve the minimization problem in *n* dimensions by maintaining at each iteration *n*+1 points that define a simplex. At each iteration, this simplex is updated by applying certain transformations to it so that it rolls downhill until it finds a minimum. Without any derivatives, this method only requires function evaluations. This makes it an elegant and robust method for function minimization.

In this paper, the weighted sum of square relative error between the actual and the estimated cumulate zero-crossing numbers is used as the objective function to be minimized. Then, the downhill simplex method is used to solve the problem

$$\begin{aligned} &\text{Minimize} && J(\hat{h}_0,\hat{g}_0)=\sum_{k=1}^{N} w(t_k)\left[1-\hat{m}_0(t_k)/m_0(t_k)\right]^2 \\ &\text{subjected to} && \hat{h}_0>0 \\ & && \hat{g}_0>0 \end{aligned} \tag{2.3}$$

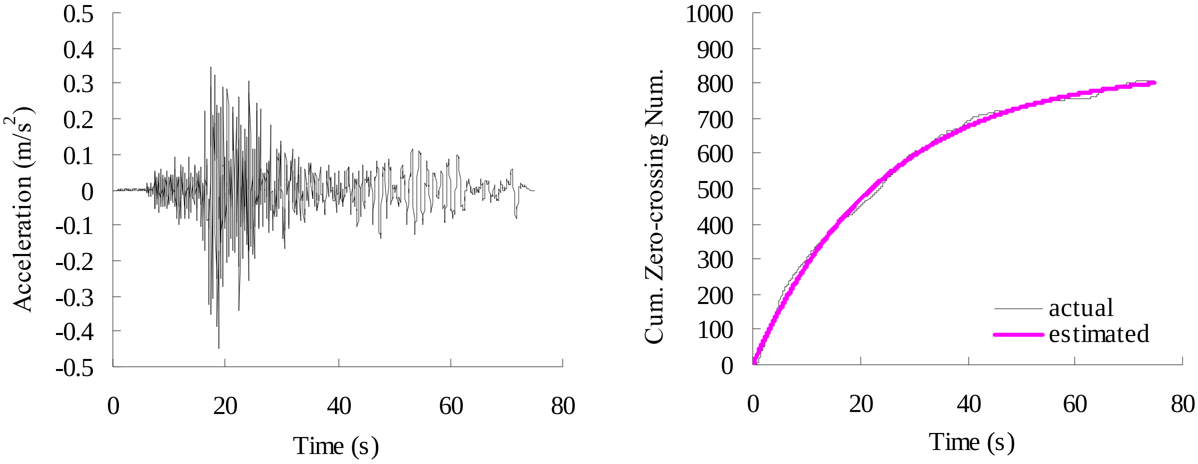
where $J(\hat{h}_0,\hat{g}_0)$ is the objective function corresponding to the estimated parameters $\hat{h}_0$ and $\hat{g}_0$, $w(t_k)$ is the weight function, t_k indicates the time $t=k\Delta t$ with Δt as the sampling time, N is the number of sampling points, $m_0(t_k)$ is the actual cumulate zero-crossing number, and its estimate $\hat{m}_0(t_k)$ is calculated according to Eqn. 2.1 with parameters as $\hat{h}_0$ and $\hat{g}_0$. Once the objective function is minimized, the optimum zero-crossing parameters are then obtained. For more details about the iteration process of downhill simplex method, the readers can refer to Nelder and Mead (1965).

3. RESULTS AND DISCUSSIONS

In this section, the proposed method is firstly applied to the horizontal component of ground acceleration recorded during the 1999 Chi-Chi earthquake at station TCU086 to demonstrate its effectiveness. Then, based on the records of the significant earthquake events since 1930 with various magnitudes, epicentral distances and sites, the attenuation laws and the correlation of zero-crossing parameters between various components are studied. Lastly, the site-specific zero-crossing parameters are suggested.

3.1. Estimation Result of Actual Acceleration Record

Figure 1 shows the accelerogram of the actual acceleration record and its estimation result of the cumulate zero-crossing number $\mu_0(t)$.



(a) Accelerogram

(b) Estimation result of $\mu_0(t)$

Figure 1 Actual acceleration record and its estimation result of $\mu_0(t)$

It is clear that there exists notable nonstationarity in frequency contents of the actual acceleration record. In the initial 30s, the actual acceleration record consists of more high frequency contents. And in the latter 40s, it consists of more low frequency contents. There is a gradually decreasing tendency in the slopes of the cumulate zero-crossing curves. The zero-crossing model fits the actual result perfectly and the precision is satisfactory.

3.2. Attenuation Laws of Zero-crossing Parameters

To study the attenuation laws of zero-crossing parameters, 703 actual acceleration records, which are associated with the vast majority of significant earthquake events since 1930, are used in this paper. Each record consists of two horizontal components and one vertical component. The general form of attenuation model is designated as $\lg Y = c_1 + c_2 M + c_3 \lg(R+10) + c_4 T_g + e$, where Y indicates the zero-crossing parameters of various components, M is the magnitude, R is the epicentral distance, T_g is the characteristic period which is used to take into account the effects of M, R and sites and can be obtained using the method described in Li (1999), ε is the residual with covariance as s_e^2. By using regression analysis, the attenuation laws of zero-crossing parameters for various components as given in Table 3.1 are obtained.

Table 3.1 Attenuation laws of zero-crossing parameters

Component	Y	c_1	c_2	c_3	c_4	s_e
Horizontal	h_0	1.232	0.008	-0.035	-0.288	0.179
	g_0	-1.419	0.010	-0.363	0.486	1.129
Vertical	h_0	1.332	-0.006	-0.001	-0.187	0.157
	g_0	-0.974	-0.054	-0.239	0.244	1.079

It can be seen from Table 3.1 that the statistical property of h_0 is better than g_0 for its root mean square error s_e is smaller. M and R has more effect on g_0 than h_0, and T_g has an equal effect on h_0 and g_0.

3.3. Correlation of Zero-crossing Parameters

3.3.1 Two horizontal components

By using linear and log-linear regression analysis, the correction of zero-crossing parameters between two horizontal components are explored, and the corresponding results are given in Table 3.2. The superscript *hs* indicates the horizontal component with smaller peak ground acceleration (PGA), the superscript *hl* indicates the one with larger PGA, and the superscript *h* indicates the mean of the two horizontal components. It shows that the correlation for h_0 is better than g_0. And for h_0 the log-linear correlation is better than linear correlation.

Table 3.2 Correction of zero-crossing parameters between two horizontal components

Y	X	Linear regression $Y= c_1+c_2X$			Log-linear regression $\lg Y= c_1+c_2\lg X$		
		c_1	c_2	Rank	c_1	c_2	Rank
h_0^{hs}	h_0^{hl}	1.0527	0.9634	8	0.0172	1.0014	7
h_0^{hl}	h_0^{h}	-1.2353	1.0683	3	-0.0498	1.0336	2
h_0^{hs}	h_0^{h}	-0.0722	1.0246	4	-0.0320	1.0344	1
g_0^{hs}	g_0^{hl}	0.0020	1.0607	9			
g_0^{hl}	g_0^{h}	-0.0034	1.0549	6			
g_0^{hs}	g_0^{h}	-0.0014	1.1125	5			

3.3.2 Two horizontal components

In the same way, the correction laws of zero-crossing parameters between vertical and two horizontal components are studied, and the corresponding results are given in Table 3.3. The superscript v indicates the vertical component. It shows that for h_0 the correlation of zero-crossing parameters between vertical and two horizontal components is obviously better than g_0. And for h_0 the log-linear correlation between vertical and two horizontal components is better than the corresponding linear correlation.

Table 3.3 Correction of zero-crossing parameters between vertical and two horizontal components

Y	X	Linear regression $Y= c_1+c_2X$			Log-linear regression $\lg Y= c_1+c_2 \lg X$		
		c_1	c_2	Rank	c_1	c_2	Rank
h_0^v	h_0^{hl}	3.5117	1.0943	6	0.2451	0.9059	3
	h_0^{hs}	2.4387	1.1265	5	0.2383	0.8971	2
	h_0^h	2.3431	1.1554	4	0.2101	0.9273	1
g_0^v	g_0^{hl}	0.0010	1.0757	8			
	g_0^{hs}	-0.0011	1.0096	9			
	g_0^h	-0.0024	1.1208	7			

3.4. Site-specific Zero-crossing Parameters

Based on the attenuation laws and the correlation of zero-crossing parameters demonstrated above, the site-specific zero-crossing parameters, adapting the Code for Seismic Design of Buildings in China, are suggested. The corresponding results for 1-D horizontal ground motion and 3-D ground motion (two horizontal components and one vertical component) are listed in Tables 3.4 and 3.5 respectively. In both of them, intensity and zone are used to take into account the effects of M and R, and site type is used to indicate the effect of site. From zone I to zone III epicentral distance R increases accordingly, and from site type I to site type IV characteristic period T_g increase from 0.25s to 0.9s. From Tables 3.4 and 3.5, it can be seen that h_0 increases as intensity is increasing and decreases as R and T_g are increasing. g_0 decreases as R is increasing, increases as T_g is increasing, and slightly increases as intensity is increasing.

Table 3.4 Site-specific zero-crossing parameters for 1-D horizontal ground motion

Intensity	Zone	Site type I		Site type II		Site type III		Site type IV	
		h_0	g_0	h_0	g_0	h_0	g_0	h_0	g_0
7	I	14.90	0.018	13.90	0.020	13.00	0.023	11.00	0.030
	II	14.60	0.016	13.40	0.019	12.35	0.022	10.30	0.029
	III	14.30	0.014	12.90	0.017	11.70	0.021	9.60	0.028
8	I	15.00	0.018	14.10	0.021	13.20	0.023	11.20	0.030
	II	14.75	0.017	13.60	0.020	12.50	0.023	10.45	0.030
	III	14.50	0.016	13.10	0.018	11.80	0.022	9.70	0.030
9	I	15.20	0.018	14.20	0.021	13.30	0.023	11.30	0.031

4. CONCLUSIONS

The nonstationarity in frequency contents is an important property of earthquake ground motions besides the conventional properties in amplitude, frequency and duration. In this study, zero-crossing rate is used as a simple but effective approach to describing the nonstationary property in frequency contents. An exponential decay model is proposed to represent the zero-crossing rate and the downhill simplex method is employed to estimate the model parameters. Then, based on the records of the significant earthquake events since 1930 with various magnitudes, epicentral distances and sites, the attenuation laws and the correlation of zero-crossing parameters are studied and discussed. Lastly, the site-specific zero-crossing parameters are suggested. The work of this paper not only demonstrates the effectiveness of the proposed method in studying the nonstationarity in frequency contents and its statistical properties, but also proposes a new criterion in selection and synthesis of ground motions for seismic design.

Table 3.5 Site-specific zero-crossing parameters for 3-D ground motion

Intensity	Zone	Component	Site type I		Site type II		Site type III		Site type IV	
			h_0	g_0	h_0	g_0	h_0	g_0	h_0	g_0
7	I	*hl*	14.70	0.018	13.60	0.018	12.70	0.018	10.50	0.028
		hs	15.20	0.021	14.20	0.021	13.20	0.021	11.20	0.032
		v	19.60	0.020	18.40	0.020	17.40	0.020	15.10	0.031
	II	*hl*	14.35	0.018	13.05	0.018	12.00	0.018	9.75	0.028
		hs	14.90	0.021	13.65	0.021	12.55	0.021	10.50	0.032
		v	19.25	0.020	17.80	0.020	16.65	0.020	14.25	0.031
	III	*hl*	14.00	0.018	12.50	0.018	11.30	0.018	9.00	0.028
		hs	14.60	0.021	13.10	0.021	11.90	0.021	9.80	0.032
		v	18.90	0.020	17.20	0.020	15.90	0.020	13.40	0.031
8	I	*hl*	14.80	0.018	13.80	0.018	12.90	0.018	10.70	0.028
		hs	15.30	0.021	14.40	0.021	13.50	0.021	11.40	0.032
		v	19.70	0.020	18.60	0.020	17.60	0.020	15.30	0.031
	II	*hl*	14.55	0.018	13.30	0.018	12.15	0.018	9.90	0.028
		hs	15.05	0.021	13.85	0.021	12.75	0.021	10.65	0.032
		v	19.40	0.020	18.05	0.020	16.80	0.020	14.45	0.031
	III	*hl*	14.30	0.018	12.80	0.018	11.40	0.018	9.10	0.028
		hs	14.80	0.021	13.30	0.021	12.00	0.021	9.90	0.032
		v	19.10	0.020	17.50	0.020	16.00	0.020	13.60	0.031
9	I	*hl*	15.00	0.018	13.90	0.018	13.00	0.018	10.80	0.028
		hs	15.50	0.021	14.50	0.021	13.60	0.021	11.50	0.032
		v	19.90	0.020	18.80	0.020	17.70	0.020	15.40	0.031

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