Numerical Simulation of Biot-wave equation in porous medium

Xinming Zhang[1] Chaoying Zhou[2] Jiaqi Liu[3] Ke'an Liu[4]

[1] *Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen 518055, China*

[2] *Professor, Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen 518055, China*

[3, 4] *Professor, Department of Mathematics, Harbin Institute of Technology, Harbin, 150001, China*

Email: xinmingxueshu@hitsz.edu.cn，xinmingused@hit.edu.cn

ABSTRACT:

In this paper, a wavelet Galerkin finite element method is proposed by combing the wavelet analysis with traditional finite element method to analyze wave propagation phenomena in fluid-saturated porous medium. The scaling functions of Daubechies wavelets are considered as the interpolation basis functions to replace the polynomial functions, and then the wavelet element is constructed. In order to overcome the integral difficulty for lacking of the explicit expression for the Daubechies wavelets, a kind of characteristic function are introduced. The recursive expression of calculating the function value of Daubechies wavelets on the fraction nodes is deduced, and the rapid wavelet transform between the wavelet coefficient space and the wave field displacement space is constructed. The results of numerical simulation demonstrate the method is effective.

KEYWORDS: Porous Medium; Wavelet Galerkin Finite Element Method; Daubechies Wavelet; Scaling Function; Rapid Wavelet Transform

1. INTRODUCTION

The fluid-saturated porous medium is modeled as a two-phase system consisting of a solid and a fluid phase. Compared with the single-phase medium theory, fluid-saturated porous medium theory can describe the formation underground more precisely and the fluid-saturated porous medium elastic wave equation can bring more lithology information than ever. For these reasons, fluid-saturated porous medium theory can be used widely in geophysics exploration and engineering surveying.

In 1956, a theory was developed for the propagation of stress waves in a porous elastic solid containing compressible viscous fluid by Biot [1, 2]. After that, many researchers paid their attentions to the propagation characters of elastic wave in saturated porous medium and obtained many achievements [3, 4]. Most dynamic problems in fluid-saturated porous medium are solved using numerical methods, especially using finite element method. Yadkin [5] combined the finite element method with the boundary element method, constructed the finite-boundary element method to deal with the two-phase model in lateral extensive field and obtained better result. Shao xiumin [6] discussed the wave propagation in the saturated porous medium and developed a new kind of non-reflecting boundary conditions on the artificial boundaries. Zhao [7, 8] proposed an explicit finite element method for Biot dynamic formulation in fluid-saturated porous medium. For the problem of local high gradient, finite element method improves the calculation precision by employing the higher order polynomial or the denser mesh. However, the increment of polynomial order and mesh knots inevitably need more computational work. Meanwhile, the condition of numerical dissipation will limit the frequency range that can be obtained. To overcome the disadvantages, wavelet analysis is introduced to the finite element method in this paper. Its desirable advantages are the multi-resolution analysis property and various basis functions for structure analysis. According to different requirement, the corresponding scaling functions and wavelet functions can be adopted to improve the numerical calculation precision. Especially, those wavelets with compactly supported property and orthogonality, such as Daubechies wavelets, can play an important role in many problems [9]. Because of the compactly supported property, if the Daubechies wavelets are considered as the interpolation functions of the finite element method, the coefficient matrices obtained are sparse matrices and their condition number can be proved independent to the dimension [10]. Moreover, a new method could be provided because of the existence of various basis functions, which can increase the resolution without changing mesh.

In this paper, the wavelet Galerkin finite element method is applied to the direct simulation of the wave equation in the fluid-saturated porous medium. The scaling functions of Daubechies wavelets are considered as the interpolation basis functions instead of the polynomial functions and the wavelet element is constructed. Because a kind of characteristic function is introduced, the integral difficulty for lacking of the explicit expression for the Daubechies wavelets is solved. Based on the recursive expression of calculating the function value of Daubechies wavelets on the fraction nodes, the rapid wavelet transform between the wavelet coefficient space and the wave field displacement space is constructed and reduces the computational cost. The results of numerical simulation demonstrate the method is effective.

2. Wavelet Galerkin Finite Element Solution of 1-D elastic wave equation in fluid-saturated porous medium

From the Biot theory, the 1-D differential equation governing wave propagation in the fluid-saturated porous medium, without fluid viscosity, can be expressed as:

$$\frac{\partial}{\partial x}\left((\lambda+2\mu+\alpha M)\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial x}\left(\alpha M\frac{\partial \omega}{\partial x}\right)=\rho\ddot{u}+\rho_f\ddot{\omega}-f_1 \tag{2.1}$$

$$\frac{\partial}{\partial x}\left(\alpha M\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial x}\left(M\frac{\partial \omega}{\partial x}\right)=\rho_f\ddot{u}+m\ddot{\omega}-f_2 \tag{2.2}$$

β is the porosity, $\rho=(1-\beta)\rho_s+\beta\rho_f$ is the bulk density of solid-fluid mixture, and ρ_s and ρ_f are the densities of solid and fluid, respectively. t is time and λ_b,μ are the Lame coefficients, $\lambda=\lambda_b+\alpha^2 M$ where α is the effective stress parameter and M is the compressibility of pore fluid. $\alpha=1-K_b/K_s$, $M=K_s/[\alpha+\beta(K_s/K_f-1)]$ where K_s,K_f,K_b are the bulk change modulus of the solid, fluid and skeleton, respectively. $K_b=\lambda_b+2\mu/3$

Multiplying both sides of the fluid-saturated porous medium wave equation by the Daubechies wavelets basis function $\phi_{jk}(x)=2^{\frac{j}{2}}\phi(2^j x-k)$, and integrating them at $[0,L]$, we can get

$$\int_0^L\left(\frac{\partial}{\partial x}\left((\lambda+2\mu+\alpha M)\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial x}\left(\alpha M\frac{\partial \omega}{\partial x}\right)\right)\phi_{jk}(x)\mathrm{d}x=\int_0^L\left(\rho\ddot{u}+\rho_f\ddot{\omega}-f_1\right)\phi_{jk}(x)\mathrm{d}x \tag{2.3}$$

$$\int_0^L\left(\frac{\partial}{\partial x}\left(\alpha M\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial x}\left(M\frac{\partial \omega}{\partial x}\right)\right)\phi_{jk}(x)\mathrm{d}x=\int_0^L\left(\rho_f\ddot{u}+m\ddot{\omega}-f_2\right)\phi_{jk}(x)\mathrm{d}x \tag{2.4}$$

By using integration by part

$$(\lambda+2\mu+\alpha M)\frac{\partial u}{\partial x}\phi_{jk}(x)\Big|_0^L-\int_0^L(\lambda+2\mu+\alpha M)\frac{\partial u}{\partial x}\frac{\partial\phi_{jk}(x)}{\partial x}\mathrm{d}x+\alpha M\frac{\partial \omega}{\partial x}\phi_{jk}(x)\Big|_0^L-\int_0^L\alpha M\frac{\partial \omega}{\partial x}\frac{\partial\phi_{jk}(x)}{\partial x}\mathrm{d}x$$
$$=\int_0^L\left(\rho\ddot{u}+\rho_f\ddot{\omega}-f_1\right)\phi_{jk}(x)\mathrm{d}x \tag{2.5}$$

$$\alpha M\ \frac{\partial u}{\partial x}\phi_{jk}(x)\Big|_0^L - \int_0^L \alpha M\ \frac{\partial u}{\partial x}\frac{\partial \phi_{jk}(x)}{\partial x}\mathrm{d}x + M\ \frac{\partial \omega}{\partial x}\phi_{jk}(x)\Big|_0^L - \int_0^L M\ \frac{\partial \omega}{\partial x}\frac{\partial \phi_{jk}(x)}{\partial x}\mathrm{d}x$$
$$= \int_0^L \left(\rho_f \ddot{u} + m\ddot{\omega} - f_2\right)\phi_{jk}(x)\mathrm{d}x \tag{2.6}$$

Set $$u(x) = \sum_{l=2-2N-2^j L}^{0} a_l(t)\phi_{jl}(x) \qquad \omega(x) = \sum_{l=2-2N-2^j L}^{0} b_l(t)\phi_{jl}(x) \tag{2.7}$$

Upon substituting Eqn.(2.7) into Eqns.(2.5) and (2.6), one gets

$$(\lambda+2\mu+\alpha M)\frac{\partial u}{\partial x}\phi_{jk}(x)\Big|_0^L + \alpha M\frac{\partial \omega}{\partial x}\phi_{jk}(x)\Big|_0^L - \int_0^L\left((\lambda+2\mu+\alpha M)\sum_{l=2-2N-2^j L}^{0} a_l(t)\frac{\partial\phi_{jl}(x)}{\partial x}\frac{\partial\phi_{jk}(x)}{\partial x} + \alpha M\sum_{l=2-2N-2^j L}^{0} b_l(t)\frac{\partial\phi_{jl}(x)}{\partial x}\frac{\partial\phi_{jk}(x)}{\partial x}\right)\mathrm{d}x$$
$$= \int_0^L\left(\rho\sum_{l=2-2N-2^j L}^{0} a_l''(t)\phi_{jl}(x) + \rho_f\sum_{l=2-2N-2^j L}^{0} b_l''(t)\phi_{jl}(x) - f_1\right)\phi_{jk}(x)\mathrm{d}x \tag{2.8}$$
$$\alpha M\ \frac{\partial u}{\partial x}\phi_{jk}(x)\Big|_0^L + M\ \frac{\partial \omega}{\partial x}\phi_{jk}(x)\Big|_0^L - \int_0^L\left(\alpha M\sum_{l=2-2N-2^j L}^{0} a_l(t)\frac{\partial\phi_{jl}(x)}{\partial x}\frac{\partial\phi_{jk}(x)}{\partial x} + M\sum_{l=2-2N-2^j L}^{0} b_l(t)\frac{\partial\phi_{jl}(x)}{\partial x}\frac{\partial\phi_{jk}(x)}{\partial x}\right)\mathrm{d}x$$
$$= \int_0^L\left(\rho_f\sum_{l=2-2N-2^j L}^{0} a_l''(t)\phi_{jl}(x) + m\sum_{l=2-2N-2^j L}^{0} b_l''(t)\phi_{jl}(x) - f_2\right)\phi_{jk}(x)\mathrm{d}x \tag{2.9}$$

By rearranging, Eqn.(2.8) and (2.9) become

$$(\lambda+2\mu+\alpha M)\frac{\partial u}{\partial x}\phi_{jk}(x)\Big|_0^L + \alpha M\frac{\partial \omega}{\partial x}\phi_{jk}(x)\Big|_0^L - \left((\lambda+2\mu+\alpha M)\sum_{l=2-2N-2^j L}^{0} a_l(t) + \alpha M\sum_{l=2-2N-2^j L}^{0} b_l(t)\right)\int_0^L\frac{\partial\phi_{jl}(x)}{\partial x}\frac{\partial\phi_{jk}(x)}{\partial x}\mathrm{d}x$$
$$= \left(\rho\sum_{l=2-2N-2^j L}^{0} a_l''(t) + \rho_f\sum_{l=2-2N-2^j L}^{0} b_l''(t)\right)\int_0^L\phi_{jl}(x)\phi_{jk}(x)\mathrm{d}x - f_1\int_0^L\phi_{jk}(x)\mathrm{d}x \tag{2.10}$$
$$\alpha M\ \frac{\partial u}{\partial x}\phi_{jk}(x)\Big|_0^L + M\ \frac{\partial \omega}{\partial x}\phi_{jk}(x)\Big|_0^L - \left(\alpha M\sum_{l=2-2N-2^j L}^{0} a_l(t) + M\sum_{l=2-2N-2^j L}^{0} b_l(t)\right)\int_0^L\frac{\partial\phi_{jl}(x)}{\partial x}\frac{\partial\phi_{jk}(x)}{\partial x}\mathrm{d}x$$
$$= \left(\rho_f\sum_{l=2-2N-2^j L}^{0} a_l''(t) + m\sum_{l=2-2N-2^j L}^{0} b_l''(t)\right)\int_0^L\phi_{jl}(x)\phi_{jk}(x)\mathrm{d}x - f_2\int_0^L\phi_{jk}(x)\mathrm{d}x \tag{2.11}$$

If select $L=1$， $j=0$, Eqn. (2.7) become

$$u(x) = \sum_{l=1-2N}^{0} a_l(t)\phi(x-l) \quad \omega(x) = \sum_{l=1-2N}^{0} b_l(t)\phi(x-l) \tag{2.12}$$

Set $\mathbf{A} = (a_{1-2N}, a_{2-2N}\cdots a_0)$ $\mathbf{B} = (b_{1-2N}, b_{2-2N}\cdots b_0)$ $\mathbf{R} = (a_{1-2N}, a_{2-2N}\cdots a_0, b_{1-2N}, b_{2-2N}\cdots b_0)^T$
Then, Eqns.(2.10) and (2.11) can be changed into an equation system of coefficient $\mathbf{R}$

$$\bar{\mathbf{M}}\ddot{\mathbf{R}} + \mathbf{P}\mathbf{R} = \mathbf{F} + \mathbf{Q} \tag{2.13}$$

Where

$$\bar{\mathbf{M}} = \begin{pmatrix} \rho E & \rho_f E \\ \rho_f E & mE \end{pmatrix} \qquad \mathbf{P} = \begin{pmatrix} -(\lambda+2\mu+\alpha M)G & -\alpha MG \\ -\alpha MG & -MG \end{pmatrix} \qquad \mathbf{F} = \begin{pmatrix} \mathbf{F_1} \\ \mathbf{F_2} \end{pmatrix} \qquad \mathbf{Q} = (\mathbf{Q_1}, \mathbf{Q_2})^{\mathrm{T}}$$

$$\mathbf{E} = \begin{pmatrix} \int_0^1\phi(x-1+2N)\phi(x-1+2N)\mathrm{d}x & \int_0^1\phi(x-2+2N)\phi(x-1+2N)\mathrm{d}x & \cdots & \int_0^1\phi(x)\phi(x-1+2N)\mathrm{d}x \\ \int_0^1\phi(x-1+2N)\phi(x-2+2N)\mathrm{d}x & \int_0^1\phi(x-2+2N)\phi(x-2+2N)\mathrm{d}x & \cdots & \int_0^1\phi(x)\phi(x-2+2N)\mathrm{d}x \\ \vdots & \vdots & \vdots & \vdots \\ \int_0^1\phi(x-1+2N)\phi(x)\mathrm{d}x & \int_0^1\phi(x-2+2N)\phi(x)\mathrm{d}x & \cdots & \int_0^1\phi(x)\phi(x)\mathrm{d}x \end{pmatrix}$$

$$\mathbf{G}=\begin{pmatrix} \int_0^1\phi'(x-1+2\mathrm{N})\phi'(x-1+2\mathrm{N})\mathrm{d}x & \int_0^1\phi'(x-2+2\mathrm{N})\phi'(x-1+2\mathrm{N})\mathrm{d}x & \cdots & \int_0^1\phi'(x)\phi'(x-1+2\mathrm{N})\mathrm{d}x \\ \int_0^1\phi'(x-1+2\mathrm{N})\phi'(x-2+2\mathrm{N})\mathrm{d}x & \int_0^1\phi'(x-2+2\mathrm{N})\phi'(x-2+2\mathrm{N})\mathrm{d}x & \cdots & \int_0^1\phi'(x)\phi'(x-2+2\mathrm{N})\mathrm{d}x \\ \vdots & \vdots & \vdots & \vdots \\ \int_0^1\phi'(x-1+2\mathrm{N})\phi'(x)\mathrm{d}x & \int_0^1\phi'(x-2+2\mathrm{N})\phi'(x)\mathrm{d}x & \cdots & \int_0^1\phi'(x)\phi'(x)\mathrm{d}x \end{pmatrix} \qquad \mathbf{F}_1=f_1\left(\int_0^1\phi(x-1+2N)\mathrm{d}x,\ \int_0^1\phi(x-2+2N)\mathrm{d}x,\ \cdots,\ \int_0^1\phi(x)\mathrm{d}x\right)^{\mathrm{T}}$$

$$\mathbf{Q}_1=\left((\lambda+2\mu+\alpha\mathrm{M})\frac{\partial \mathrm{u}}{\partial \mathrm{x}}+\alpha\mathrm{M}\frac{\partial\omega}{\partial \mathrm{x}}\right)\left(\phi(\mathrm{x}\text{-}1\text{+}2\mathrm{N}),\phi(\mathrm{x}\text{-}2\text{+}2\mathrm{N})\cdots\phi(\mathrm{x})\right)^{\mathrm{T}}\Big|_0^1 \qquad \mathbf{F}_2=f_2\left(\int_0^1\phi(x-1+2N)\mathrm{dx},\ \int_0^1\phi(x-2+2N)\mathrm{dx},\ \cdots,\ \int_0^1\phi(x)\mathrm{dx}\right)^{\mathrm{T}}$$

$$\mathbf{Q}_2=\left(\alpha\mathrm{M}\frac{\partial \mathrm{u}}{\partial \mathrm{x}}+\mathrm{M}\frac{\partial\omega}{\partial \mathrm{x}}\right)\left(\phi(\mathrm{x}\text{-}1\text{+}2\mathrm{N}),\phi(\mathrm{x}\text{-}2\text{+}2\mathrm{N})\cdots\phi(\mathrm{x})\right)^{\mathrm{T}}\Big|_0^1$$

Using the 2-order center difference to approximate the two derivatives in the Eqn. (2.13), we can obtain

$$\bar{\mathbf{M}}\frac{\mathbf{R}^{n+1}-2\mathbf{R}^{n}+\mathbf{R}^{n-1}}{(\Delta t)^2}+\mathbf{P}\mathbf{R}^{n}=\mathbf{F}+\mathbf{Q} \tag{2.14}$$

Arranging above equation, we have

$$\bar{\mathbf{M}}\mathbf{R}^{n+1}=(2\bar{\mathbf{M}}-(\Delta t)^2\mathbf{P})\mathbf{R}^{n}-\bar{\mathbf{M}}\mathbf{R}^{n-1}+(\Delta t)^2\mathbf{F}+(\Delta t)^2\mathbf{Q} \tag{2.15}$$

Initial Conditions: $a_k(0)=b_k(0)=0 \qquad a_k(1)=b_k(1)=0$ (2.16)

So, we can obtain the wavelet coefficients at each time level by solving above Eqns. (2.15) and (2.16) with some boundary conditions, and then substitute the wavelet coefficients into the Eqn.(2.12), the wave field displacements can be obtained. The integral values of Daubechies wavelets in the coefficient matrix can be solved by introduce a kind of characteristic functions [11, 12].

3. Rapid Wavelet Transform

In order to obtain the wave field displacements conveniently and quickly, the fast wavelet transform between the wavelet space and the wave field displacements is constructed as follows:

$$U=\Phi P \tag{3.1}$$

U is the wave field displacement vector, P is the wavelet coefficient vector, Φ is the wavelet transform matrix.
For the sake of simplicity, take the DB2 wavelet as the example. There are 7 nodes in solution field.

$$\Phi=\begin{pmatrix} \phi(\frac{1}{8}+2) & \phi(\frac{1}{4}+2) & \phi(\frac{3}{8}+2) & \phi(\frac{1}{2}+2) & \phi(\frac{5}{8}+2) & \phi(\frac{3}{4}+2) & \phi(\frac{7}{8}+2) \\ \phi(\frac{1}{8}+1) & \phi(\frac{1}{4}+1) & \phi(\frac{3}{8}+1) & \phi(\frac{1}{2}+1) & \phi(\frac{5}{8}+1) & \phi(\frac{3}{4}+1) & \phi(\frac{7}{8}+1) \\ \phi(\frac{1}{8}) & \phi(\frac{1}{4}) & \phi(\frac{3}{8}) & \phi(\frac{1}{2}) & \phi(\frac{5}{8}) & \phi(\frac{3}{4}) & \phi(\frac{7}{8}) \end{pmatrix}^T \qquad \begin{gathered} U=(u(\tfrac{1}{8}),u(\tfrac{1}{4}),u(\tfrac{3}{8}),u(\tfrac{1}{2}),u(\tfrac{5}{8}),u(\tfrac{3}{4}),u(\tfrac{7}{8}))^T \\ P=(p_{-2},p_{-1},p_0)^T \end{gathered}$$

It is important for constructing the fast wavelet transform to solve the function value of the Daubechies wavelets on the fraction nodes. So, the recursive expression of calculating the function value of Daubechies wavelets on the fraction nodes is deduced to save the computational cost

$$\Phi(\frac{2^n i+p}{2^n})=\begin{cases} A\Phi(\frac{2^{n-1}i+q}{2^{n-1}}) & if\ 2\cdot\frac{p}{2^n}\le 1 \\ B\Phi(\frac{2^{n-1}i+q}{2^{n-1}}) & if\ 2\cdot\frac{p}{2^n}>1 \end{cases}$$

in which
$i=0,1,\cdots 2N-2 \quad p=1:2:2^n-1 \quad q=p \bmod 2^{n-1}$ n controls the mesh partition.

4. Numerical Simulation

To verify the correctness and accuracy of the wavelet Galerkin finite element method, an example is given to compare the results obtained by this method with an analytical solution. A one-dimensional column of length l as sketched in Fig.1 is considered. It is assumed that the side walls and the bottom are rigid, frictionless, and impermeable. At top, the stress σ_y and the pressure ρ are prescribed. The boundary conditions are

$$u|_{y=0} = \omega|_{y=0} = 0, \quad \sigma|_{y=l} = -P_0 f(t), \quad p|_{y=l} = 0$$

For this model, if the permeability tends to infinity i.e. $\kappa \to \infty$, the analytical solutions in time domain are [13]

$$\begin{aligned} u_y = &\frac{P_0}{E(d_1\lambda_2 - d_2\lambda_1)}\sum_{n=0}^{\infty}(-1)^{-n}\{d_2[(t-\lambda_1(l(2n+1)-y)H(t-\lambda_1(l(2n+1)-y)) \\ &-(t-\lambda_1(l(2n+1)+y))H(t-\lambda_1(l(2n+1)+y))]-d_1[(t-\lambda_2(l(2n+1)-y)) \\ &H(t-\lambda_2(l(2n+1)-y))-(t-\lambda_2(l(2n+1)+y))H(t-\lambda_2(l(2n+1)+y))]\} \end{aligned} \tag{4.1}$$

$$\begin{aligned} p= &\frac{P_0 d_1 d_2}{E(d_1\lambda_2 - d_2\lambda_1)}\sum_{n=0}^{\infty}(-1)^{-n}[H(t-\lambda_1(l(2n+1)-y)) + H(t-\lambda_1(l(2n+1)+y)) \\ &-(H(t-\lambda_2(l(2n+1)-y)) + H(t-\lambda_2(l(2n+1)+y))] \end{aligned} \tag{4.2}$$

Where E is Yang modulus, assuming a Heaviside step function as temporal behavior i.e. $f(t) = H(t)$, and together with vanishing initial conditions.

$$d_i = \frac{E\lambda_i^2 - (\rho - \rho_f)}{(\alpha - Q)\lambda_i} \qquad (i = 1, 2) \qquad Q = \frac{\beta^2 \rho_f}{\rho_\alpha + \beta\rho_f} \quad (\kappa \to \infty) \qquad \rho_\alpha = 0.66\beta\rho_f$$

λ_i are the characteristic roots of following characteristic equation

$$E\frac{Q}{\rho_f}\lambda^4 - (E - \frac{\beta^2}{M} + (\rho - Q\rho_f)\frac{Q}{\rho_f} + (\alpha - Q)^2)\lambda^2 + \frac{\beta^2(\rho - Q\rho_f)}{M} = 0$$

Supposing $A = E\frac{Q}{\rho_f} \quad B = E - \frac{\beta^2}{M} + (\rho - Q\rho_f)\frac{Q}{\rho_f} + (\alpha - Q)^2 \qquad C = \frac{\beta^2(\rho - Q\rho_f)}{M}$, one gets

$$\lambda_1 = -\lambda_3 = \sqrt{\frac{B + \sqrt{B^2 - 4AC}}{2A}} \qquad \lambda_2 = -\lambda_4 = \sqrt{\frac{B - \sqrt{B^2 - 4AC}}{2A}}$$

In the example, three very different materials, a rock (Berea sandstone), a soil (coarse sand), and a sediment (mud) are chosen. The material data are given in Table 1. The results, both the analytical method and the wavelet Galerkin finite element method developed in this paper, are shown in Fig.2, Fig.3, Fig.4 with results plotted in dash lines and dot. All the figures show that the numerical solutions are perfectly close to the analytical solutions, so the method developed in this paper has a very high degree of calculating accuracy.

Table 1 The parameters of Fluid Saturated Porous Medium

	K(Pa)	G(Pa)	ρ(kg/m^3)	β	K_s(Pa)	ρ_f (kg/m^3)	K_f (Pa)
rock	8.0×10^9	6.0×10^9	2548	0.19	3.6×10^{10}	1000	3.3×10^9
soil	2.1×10^8	9.8×10^7	1884	0.48	1.1×10^{10}	1000	3.3×10^9
sediment	3.7×10^7	2.2×10^7	1396	0.76	3.6×10^{10}	1000	2.3×10^9

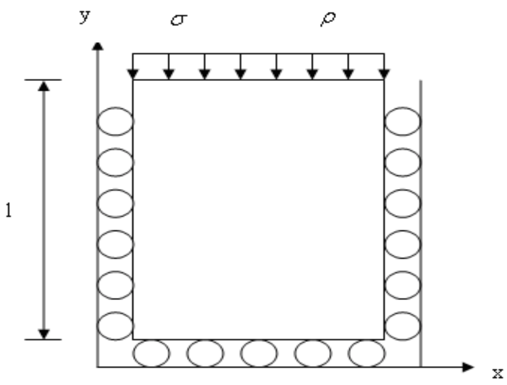

Fig.1 Model of Fluid Saturated Porous Medium

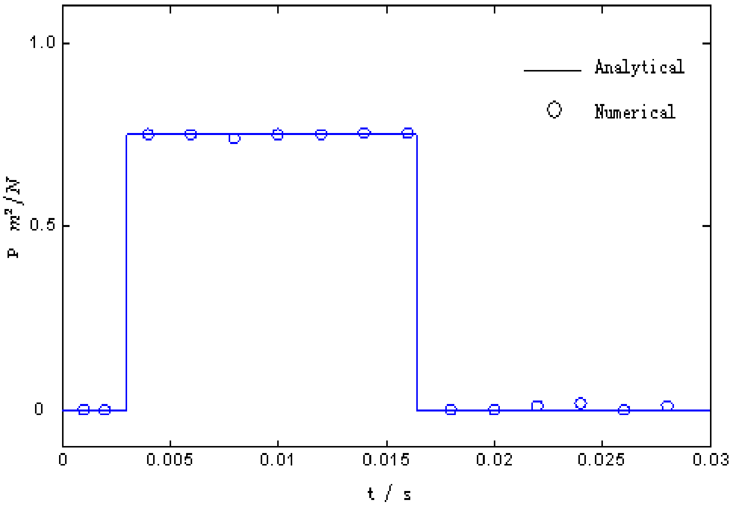

Fig.2 The Pressure of Rock ($l = 1000\text{m}$, $y = 995\text{m}$)

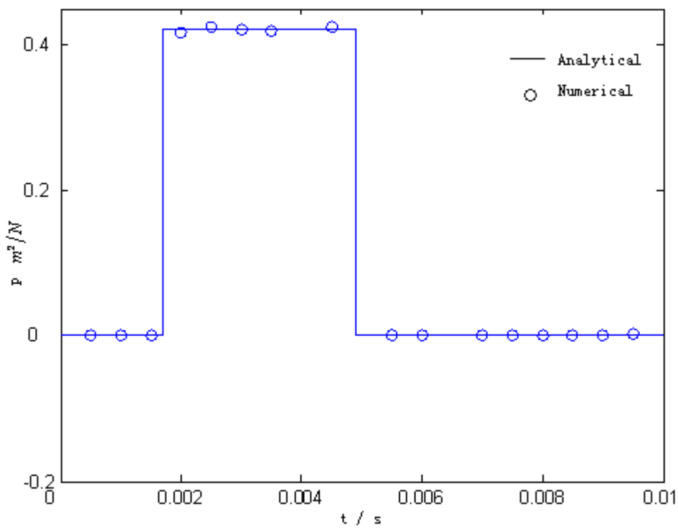

Fig.3 The Pressure of Soil ($l = 1000\text{m}$, $y = 995\text{m}$)

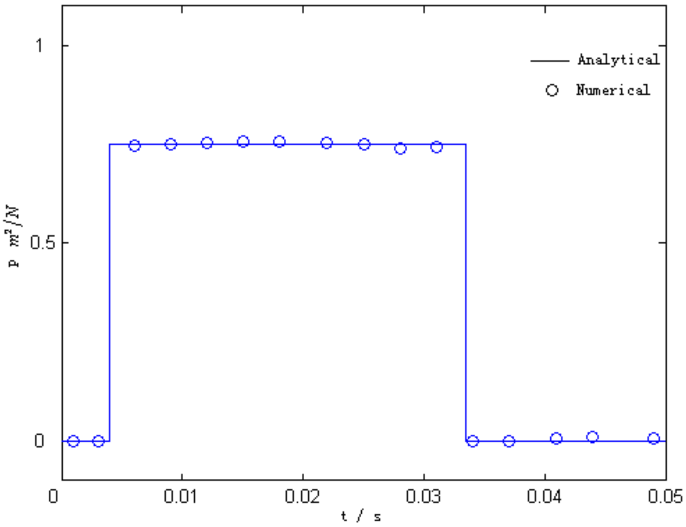

Fig.4 The Pressure of Sediment ($l = 1000\text{m}$, $y = 995\text{m}$)

5. Conclusion

In this article, the wavelet Galerkin finite element method is constructed by combining the finite element method with wavelet analysis, and is applied to the numerical simulation of the fluid-saturated porous medium elastic wave equation. For the beautiful and deep mathematic properties of Daubechies wavelets, such as the compactly supported property, vanishing moment property and so on, the wavelet Galerkin finite element method has the feature of quick iterative rate, high numerical precision and good stability. Moreover, contrasts to h- or p-based FEM, a new refine algorithm can be presented because of the multi-resolution property of the wavelet analysis. The algorithm can increase the numerical precision by adopting various wavelet basis functions or various wavelet spaces, without refining the mesh.

ACKNOWLEDGEMENT

The work was supported by the National Science Foundation of China, under Grant No. 40374046 and Project supported by the Natural Science Foundation of Guangdong Province, China, under Grant No. 07300059.

REFERENCE

1. Biot M A. (1956a). Theory of propagation of elastic waves in a fluid-saturated porous solid: Low-frequency range. J.Acoust. Soc. Amer. **28**:168-178.
2. Biot M A. (1956b). Theory of propagation of elastic waves in a fluid-saturated porous solid: Higher-frequency range. J. Acoust. Soc. Amer. **28**:179-191 .
3. Plona T J. (1980). Observation of a second Bulk compressional wave in porous media at ultrasonic frequences. Appl.Phys. **36,** 259-261
4. Kumar R, Hundal B S. (2005). Symmetric wave propagation in a fluid-saturated incompressible porous medium. Journal of Sound and Vibration. **288**,361-373
5 Yazdchi.M, Khalili.N, Vallippan.S. (1999). Non-linear seismic behavior of concrete gravity dams using coupled finite element-boundary element method. International Journal for Numerical Methods in Engineering. **44,**101-130
6. Shao X M, Lan Z L. (2000). Finite element method for the equation of waves in fluid-saturated porous media. Chinese J. Geophys. **43:2,**264-277.

7. Zhao C G, Li W H, Wang J T. (2005). An explicit finite element method for Biot dynamic formulation in fluid-saturated porous media and its application to a rigid foundation. Journal of Sound and Vibration. **282,** 1169-1181

8. Zhao C G, Li W H, Wang J T. (2005).An explicit finite element method for dynamic analysis in fluid saturated porous medium-elastic single-phase medium-ideal fluid medium coupled systems and its application. Journal of Sound and Vibration. **282,** 1155-1168

9. Daubechies I. (1988). Orthonormal basis of compactly supported wavelet, Commun. Pure Appl. Math. **41,** 909-996

10. Jaffard.S, Laurencop.P. (1992).Orthonormal wavelets, analysis of operators and applications to numerical analysis [A]. Chui.C. Wavelets: A Tutorial in Theory and Applications[C]. New York: Academic Press

11. Ko J, Kurdila A J, Pilant M S. (1995).A Class of Finite Element Methods Based on Orthonormal, Compactly Supported Wavelets. Computational Mechanics. **16,** 235-244

12. Latto A, Resnikoff H L, Tenenbaum E. (1991).The Evaluation of connection coefficients of compactly supported wavelets. Aware Inc, Technical Report AD910708.

13. Schanz M, Cheng A.H.-D. (2000).Transient wave propagation in a one dimensional poroelastic column. Acta Mechanica. **145,** 1-18