PRICING CAT BONDS FOR SOUTHEASTERN CITIES

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ABSTRACT :

A pricing model for CAT bonds, based on engineering seismic risk assessment, is introduced. The occurring probability of a defined earthquake catastrophe, estimated by seismic risk assessment method, is an input of the pricing model. Yields and proportion of reinvestment, principal protected ratio, issuance fee, circulation, maturity period, claim payments of insurers and reinsurers, are designed as factors. The annual coupon rates of the CAT bonds are studied under the equilibrium between the incomes of investors and issuers, in which the earning from earthquake insurance premium and the payout for reinsurance premium in complete and incomplete markets are described by Geometric Brownian Motion and Jump-Diffusion processes respectively. Then, four southeastern cities in Fujian province are adopted as examples to illustrate the feasibility of the model.

KEYWORDS: CAT bonds, engineering seismic risk assessment, pricing model

1. INTRODUCTION

In areas with an increased probability of catastrophe activity, the insurance industry will have to adjust its risk assessments appropriately and pass on the increased risk in the form of higher premiums. At the beginning of 1990s, several catastrophes occurred, catastrophic losses rocketed. Insurance fund, accumulated by the industry in many years, might be nothing, and some insurers might go to bankrupt (David, 1999; Howard, 2000). As supplements, the capital market can provide (re)insurers far more financing capacity than that has been available previously and instruments for dispersing risk widely, like catastrophe derivatives.

CAT bond is one of the most active instruments in the securitization of catastrophic risk. One of the key points to issue CAT bonds successfully is to price them reasonably. For earthquake disaster, losses are mainly from damage of engineering structures, the amount of losses cannot represent the component of losses. The pricing model for the CAT bonds in this paper is based on engineering seismic risk assessment method. The occurring probability of a defined catastrophe is an input, and ratio and yields of reinvestment, principal protected ratio, issuance fee, circulation and maturity are designed as factors. A partial equilibrium circumstance among the cash flows on earthquake risk of the issuer is built to price the CAT bonds in complete and incomplete markets (Tao and Tao, 2007). For illustration, CAT bonds are priced for four southeastern cities, Nanan, Quanzhou, Zhangzhou and Xiamen, in Fujian Province.

2. PRICING MODEL

Catastrophe insurance and its derivatives widen the mitigation channels against natural disasters. There might be a complementary relation between the parameters of catastrophe insurance and those of CAT bonds to balance the issuer's seismic risk. To bridge them, our setup in this paper makes an insurer the issuer of CAT bonds, who sale earthquake insurance policy also. If the insured amount is high, the circulation of the CAT bonds M needs to be increased to handle the high claim payments; otherwise, the circulation to be low to reduce the interest payment.

Let the period of validity of the CAT bonds be T, the occurring probability of the defined catastrophe p, estimated by engineering seismic risk assessment methods, the catastrophe occurs at t, then the expected return of the issuer at maturity is

$$E_I = E^P\left[I(t)\cdot 1_{A\{0\le t\le T\}} + I(T)\cdot 1_{A\{t>T\}}\right] \tag{1}$$

where, P is the probability measure; 1_A is an indicator function; $I(\cdot)$ is the return of the issuer, and is presented as

$$\begin{cases} I(t) = P(t) + C_{Rt} + k\left[M + P(0)\right]\cdot e^{r_1 t} + (1-k)\left[M + P(0)\right]\cdot e^{r_0 t} - P_R(t), 0 \le \mathrm{t} < \mathrm{T} \\ I(T) = P(T) + C_{RT} + k\left[M + P(0)\right]\cdot e^{r_1 T} + (1-k)\left[M + P(0)\right]\cdot e^{r_0 T} - P_R(T), t \ge T \end{cases} \tag{2}$$

where, $P(t)$ and $P(T)$ are the issuer's earning from earthquake insurance premium at t and T respectively; C_{Rt} and C_{RT} are the claim payment from reinsurance at t and T respectively, which can be estimated by seismic risk assessment and the distribution of reinsurance policy-holders; k is the ratio of reinvestment to the circulation M and the earned premium at 0; r_1 is yields of reinvestment; r_0 is risk-free interest rate; $P_R(t)$ and $P_R(T)$ are the payout for reinsurance premium at t and T.

In complete market, $P(t)$ and $P_R(t)$ are described by Geometric Brownian Motion with two different sets of parameters respectively. By Ito formula, these can be represented as

$$P(t) \approx P(0)e^{\left(\mu_P - \frac{\sigma_P^2}{2}\right)t + \sigma_P W_P(t)} \tag{3}$$

$$P_R(t) \approx P_R(0)e^{\left(\mu_R - \frac{\sigma_R^2}{2}\right)t + \sigma_R W_R(t)} \tag{4}$$

where, μ_P is the mean value of earthquake insurance premium; σ_P is the standard deviation; $W_P(t)$ is a Weiner process, in which the mean value is 0 and the deviation is 1; μ_R is the mean value of reinsurance premium; σ_R is the standard deviation; $W_R(t)$ is a Weiner process, in which the mean value is 0 and the deviation is 1.

A market is said to be complete if every pattern of cash flows can be replicated by some portfolio of securities that are traded in the market (Cox, 2000).

Catastrophe insurance premium will increase after a catastrophe because of the increased purchase quantity and the risk-averse attitude of insurers. The jump will be clear as the economic development. It is supposed that the market is arbitrage-free, and Jump-Diffusion process with two different sets of parameters is adopted to describe the earning from insurance premium and the payout for reinsurance premium in incomplete market. By Ito formula, $P(t)$ and $P_R(t)$ can be represented as

$$P(t) \approx P(0)e^{\left(\mu_P - \lambda_P k_P - \frac{\sigma_P^2}{2}\right)t + \sigma_P W_P(t)} e^{J_P q_P(t)} \tag{5}$$

$$P_R(t) \approx P_R(0)e^{\left(\mu_R - \lambda_R k_R - \frac{\sigma_R^2}{2}\right)t + \sigma_R W_R(t)} e^{J_R q_R(t)} \tag{6}$$

where, J_P and J_R are jump sizes, which follow standard log normal distribution; $q_P(t)$ and $q_R(t)$ are jump times in $(0,t)$, which follow an independent Poisson process with intensity λ_P and λ_R.

The expected payout of the insurer is

$$E_O = E^P\left[O_t \cdot 1_{A\{0\le t\le T\}} + O_T \cdot 1_{A\{t>T\}}\right] \tag{7}$$

where, $O(\cdot)$ is the payout of the insurer, and can be represented as

$$\begin{cases} O_t = C_t + BM + nM, & 0 \le t < T \\ O_T = C_T + BM + Me^{rT}, & t \ge T \end{cases} \tag{8}$$

where, C_t and C_T are the claim payment from earthquake insurance at t and T respectively, which can be estimated by seismic risk assessment and the distribution of insurance policy-holders; B is issuance fee, which is in a proportion of the circulation of the bond; n is the proportion of protected principal; r is the annual coupon rate of the bond.

The CAT bonds is a supplement of earthquake insurance to keep the issuer safe against earthquake risk, so the expected return should be more than the expected payout, which is also the expected return of investors. According to the equilibrium theory, the market is clear when the expected utility of all investors is maximized. In our model, the expected payout of the issuer is maximized when the market is in equilibrium, that is, the expected payout of the issuer equals to the expected return, E_I=E_O. Then, the annual coupon rates of the bond can be calculated by

$$r = \frac{1}{T}\ln\left[\frac{E_I - E_{0_t}}{M(1-p)} - \frac{C_T}{M} - B\right] \tag{9}$$

where, E_{Ot} is the expected payout when the catastrophe occurs.

3. PRODUCTIONS

In this model, the probability of catastrophic earthquake occurrence is an input. Investors will loss interest, part even whole principle if a catastrophic earthquake occurs; otherwise, investors will be repaid whole principle and higher interest than that of risk-free bonds. So it is important to define the "earthquake catastrophe", there are many different standard, such as ISO's and PCS's etc. The definition, adopted in our model, is related with the reflected region, the ratio of losses to IVA (Industry Value Added) in last year and its population, and an event will be a catastrophe if its loss ratio measures up with each of the indices listed in Table 1 (Tang, 1999).

Table.1 The definition of earthquake catastrophe

	The loss ratio of aggregate losses to IVA	The ratio of casualties to gross population（%）
Nation	>2×10^{-3}	>8×10^{-4}
Province / Larger city	>1×10^{-2}	>5×10^{-3}
City / County	>0.2	>3×10^{-2} (The population on million level) >1.0 (The population on 100 thousand level)

Losses of four southeastern cities, Nanan, Quanzhou, Zhangzhou and Xiamen, in Fujian Province are from vulnerability evaluation and seismic hazard analysis (Institute of Engineering Mechanics, 1999-2001). The direct economic losses, corresponding to intensity Ⅵ-Ⅹ, are listed in Table 2.

Table.2 Losses of four cities from vulnerability evaluation (RMB million)

	Ⅵ	Ⅶ	Ⅷ	Ⅸ	Ⅹ
Nanan	4.313	46.882	167.508	544.610	1184.225
Quanzhou	18.69	550	1571	3560	7928
Zhangzhou	46.968	171.072	571.314	1534.058	3281.369
Xiamen	464.03	1583.17	6976.10	24184.45	56849.91

Since the maturity period of a bond is 3 years, in general, the exceeding probabilities in 3 years are obtained from seismic hazard analysis. The losses in Table 1 are discounted to the values in 2006 by risk-free rate, and the effect of inflation is considered by the approach of Swiss Re. Then, an exceeding probability curve of loss rate, the rate of earthquake losses to IVA in last year, is constructed and shown in fig. 1.

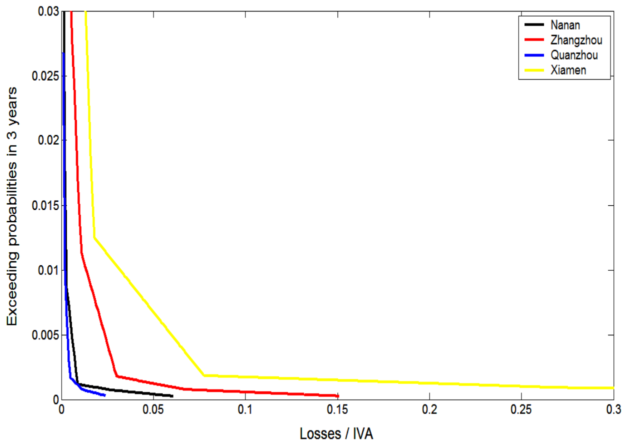

Figure 1 The exceeding probability curve of loss rate

Since earthquake insurance has not been launched in large-scale in China now, related data and information are not enough. We have to suggest that the insurance rate in this field is 5% and uniform distributed in space. Then, the probable maximum losses of buildings from the historical maximum earthquake in Fujian province (M=8, 1604) are RMB 59.38 million in Nanan, 704.36 million in Quanzhou, 468.15 million in Zhangzhou and 1813.07 million in Xiamen (2006 value), which is discounted from the values in 1999 and 2001, considering the influence of interest rate and inflation. So the probable maximum claim payments are RMB 2.9690, 23.4075, 35.2180 and 90.6535 million respectively.

In some years of 1990s, several insurers covered earthquake risk as extraneous risk of enterprise property insurance, and the premium rate is 10% that of main risk, which is limited strictly. It is supposed that 50% policy-holders of enterprise property insurance purchase the additional earthquake insurance, then the data is converted to those in four cities by the ratio of annual national IVA to the cities' IVA. Then, the earthquake premiums of the four cities are listed in Table 3.

Table.3 Earthquake premium (RMB million)

	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Nanan	0.0053	0.0111	0.0112	0.0116	0.0095	0.0094	0.0097	0.0101	0.0102	0.0102
Quanzhou	0.0168	0.0310	0.0325	0.0334	0.0313	0.0295	0.0309	0.0323	0.0262	0.0292
Zhangzhou	0.0287	0.0534	0.0577	0.0600	0.0621	0.0621	0.0625	0.0635	0.0672	0.0662
Xiamen	0.0169	0.0299	0.0306	0.0316	0.0331	0.0358	0.0388	0.0406	0.0431	0.0473

Earthquake reinsurance premium is adopted as 20% of the insurance premium. The mean values of the earning of insurance premium, in Nanan, Quanzhou, Zhangzhou and Xiamen, μ_P in Eqn. 3 and Eqn. 5 are 0.0098, 0.0583, 0.0293 and 0.0348 respectively, the standard deviations σ_P are 0.0018, 0.0111, 0.0048 and 0.0085, and the mean values of the payout of reinsurance premium μ_R in Eqn. 4 and Eqn. 6 are 0.0020, 0.0117, 0.0059 and 0.0070, the standard deviations σ_R are 0.0004, 0.0022, 0.0010 and 0.0017 respectively.

From Figure 1, one can see the loss from the maximum considered earthquake (M=8) is a catastrophe for Zhangzhou and Xiamen, but not for Nanan and Quanzhou. The occurring probabilities p of a defined earthquake catastrophe in Zhangzhou and Xiamen are 0.058% and 0.172%. By extrapolation, The occurring probabilities p in Nanan and Quanzhou are 0.002% and 0.0005%. If the defined catastrophe occurs, the losses, in Nanan, Quanzhou, Zhangzhou and Xiamen, should be RMB 2.52, 19.01, 7.17 and 11.62 billion at least, so the claim payments of earthquake insurance are RMB 1.26, 9.50, 3.59 and 5.81 billion, and those of reinsurance are RMB 0.25, 1.90, 0.72 and 1.16 million. If it does not occur, the claim payments of earthquake insurance are RMB 2.97, 96.84, 2.56 and 90.47 million, and those of reinsurance are RMB 0.59, 19.37, 0.51 and 18.09 million.

Let the proportion of reinvestment k 10%, the yield r_1 5%, the circulation of the CAT bonds M 100 million, the risk-free interest rate r_0 5.40%, the principal protected ratio n 50% and the issuance fee B 1% of the circulation. Then, the earning of insurance premium and the payout of reinsurance premium in complete and incomplete market can be obtained. The jumps of insurance and reinsurance premium from destructive earthquakes are considered in incomplete market. The annual coupon rates of the CAT bonds r, in Nanan, Quanzhou, Zhangzhou and Xiamen, are 6.31%, 9.13%, 7.51% and 8.13% in complete market, and 11.82%, 12.82%, 11.94% and 12.18% in incomplete market respectively from Eqn. 9. The principal and coupon will be refunded to investors, for a coupon bond at maturity, if the defined catastrophe does not occur, and the refundments per 100 yuan are listed in Table 4.

Table.4 Refundment of coupon bonds (yuan)

	Complete market	Incomplete market
Nanan	120. 8403	142.5610
Quanzhou	131.5083	146.9027
Zhangzhou	125.2698	143.0752
Xiamen	127.6217	144.1090

And the prices of zero coupon bonds, at t=0, are listed in Table 5. In other words, the investor purchases the CAT bonds at the prices and will get 100 yuan back at maturity.

Table.5 Price of zero coupon bonds (yuan)

	Complete market	Incomplete market
Nanan	82.7538	70.1454
Quanzhou	76.0408	68.0723
Zhangzhou	79.8277	69.8933
Xiamen	78.3566	69.3919

4. SENSITIVITY ANALYSIS

It is interesting to analysis the sensitivities of the annual coupon rate, proceeds at maturity of a coupon bond and the prices of a zero coupon bond, at t=0, by varying the different parameters. By assuming that (1) T=1, 2 or 3 years; (2) r_1=5%, 6% or 7%; (3) M=100, 200 or 300 million, the results for Xiamen are listed in Table 6.

Table.6 The sensitivities of the CAT bond for Xiamen

T	1 year		2 years		3 years	
Market	Complete market	Incomplete market	Complete market	Incomplete market	Complete market	Incomplete market
r	0.2141	0.3063	0.1132	0.1766	0.0813	0.1218
proceeds (yuan)	23.8747	35.8390	25.4077	42.3616	27.6217	44.1090
price (yuan)	80.7268	73.6166	79.7399	70.2437	78.3566	69.3919
r_1	5%		6%		7%	
r	0.0813	0.1218	0.0819	0.1293	0.0824	0.1357
proceeds (yuan)	27.6217	44.1090	27.8516	47.3882	28.0435	50.2454
price (yuan)	78.3566	69.3919	78.2157	67.8480	78.0984	66.5578
M	100 million		200 million		300 million	
r	0.0813	0.1218	0.0709	0.1115	0.0662	0.1015
proceeds (yuan)	27.6217	44.1090	23.7013	39.7242	21.9694	35.5947
price (yuan)	78.3566	69.3919	80.8399	71.5696	81.9878	73.7492

It turns out, as expected, that the bond price declines as the period of validity and the yields of reinvestment are increased, and climbs as the circulation are increased. The analysis confirms also there are relationships among earthquake insurance premium $P(t)$, the yield of reinvestment r_1 and coupon rate r, the proceeds of coupon bonds or the price of zero coupon bonds. As an example, the relationship, as r_1 increases from 5% to 7%, is shown in fig. 2.

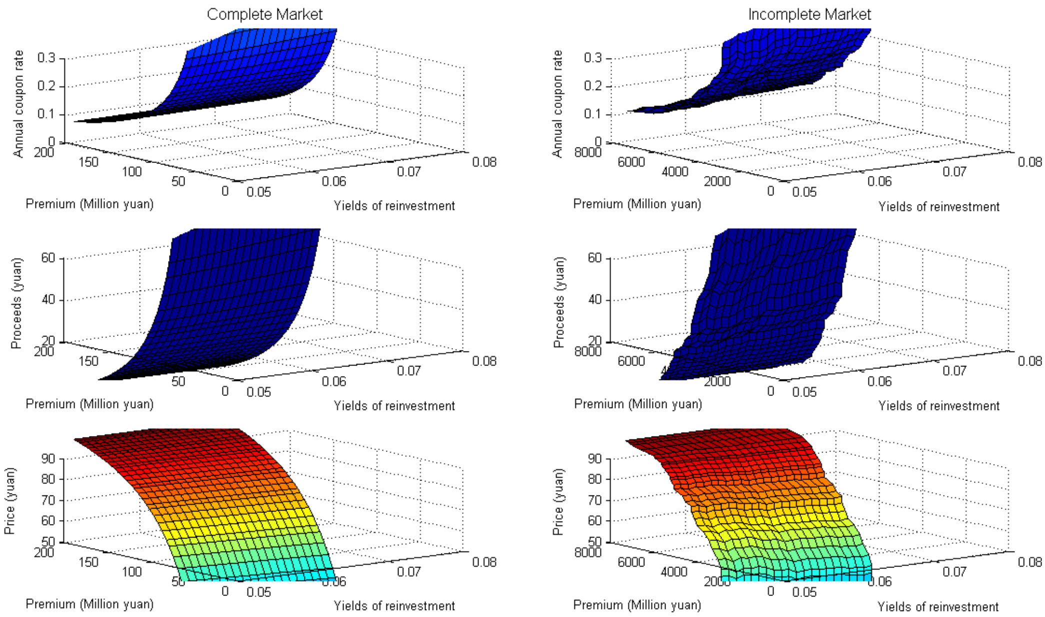

Figure 2 The relationships among premium, yields and rate, proceeds or price

5. CONCLUSIONS

A pricing model for CAT bonds, built on engineering seismic risk assessment, is introduced. As one of supplements for earthquake insurance, the complementarity of the cash flows from (re)insurance and the bonds is represented. The occurring probability of a defined earthquake catastrophe, estimated by seismic risk assessment, is an input, yields and proportion of reinvestment, principal protected ratio, issuance fee, circulation and maturity period are the factors of the model. Claim payments of (re)insurers can be also estimated by seismic risk assessment, and premiums in complete and incomplete markets are described by two kinds of stochastic processes respectively. As productions, the annual coupon rates and the prices of the CAT bonds are calculated for four southeastern cities.

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