DISSIPATIVE BRACES BASED ON HIGH DAMPING RUBBER: ANALYSIS AND DESIGN

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ABSTRACT :

Viscoelastic devices have proven to be very efficient devices for new buildings and seismic retrofitting of existing structures. They are usually obtained by using copolymers or glassy substances, though recently High Damping Rubber (HDR), already used in vibration or seismic isolators, has also been employed. The behaviour of HDR is quite complex and both stiffness and damping devices significantly depend on the strain amplitude and strain rate. Equivalent linear models may however be used to simulate their behaviour at a fixed displacement amplitude and frequency, with an acceptable approximation level. This paper proposes a design method for dissipating braces inserted in reinforced concrete (r.c.) frames with limited ductility. The dissipative behaviour of both the two components (r.c. frame and HDR dampers) are taken into account, by considering an equivalent linear model for HDR-based devices and an equivalent viscous damping coefficient for the r.c. frame to allow for its hysteretic behaviour.

KEYWORDS: Passive control, HDR dampers, seismic upgrading.

1. INTRODUCTION

Passive control systems have proven to be very efficient devices for new buildings and seismic retrofitting of existing structures. The usual classification of these devices is by elastic-plastic dampers, viscous fluid dampers**,** friction dampers and viscoelastic dampers (Soong and Dargush 1997). This paper focuses on the latter device. These devices generally provide lower energy dissipation with respect to elastic-plastic dampers but may be preferable because they withstand a large number of cycles and no permanent deformation remains after a seismic event. Moreover, compared to viscous devices, they provide stiffness as well as damping to the structure in which they are used. Viscoelastic devices are usually obtained by using copolymers or glassy substances, though recently High Damping Rubber (HDR), already used in vibration or seismic isolators, has also been employed (Fuller et al. 2000, Giacchetti and Bartera 2004, Lee et al. 2005).The behaviour of HDR is quite complex and both stiffness and damping significantly depend on the strain amplitude and strain rate to which they are subjected (Dall'Asta and Ragni 2006). It has however been demonstrated in the case of simple dynamical systems that linear viscoelastic models may be used in order to simulate their behaviour at a fixed displacement amplitude and frequency, with an acceptable approximation level (Dall'Asta and Ragni 2008a,b). Several methods are proposed in technical literature (Soong and Dargush 1997, Kasai et al. 1998) for the analysis or design of structures equipped with linear viscoelatic dampers, which usually are based on a linear behaviour of the main frame and static or dynamic linear analyses of the coupled system (main frame in parallel with viscoelastic damper system). These methods may also be applied to HDR-based devices and may be extended to take into account the non linear behaviour of the frame in which they are used.
This paper proposes a design method for dissipating braces inserted in reinforced concrete frames with limited ductility , taking into account the nonlinear behaviour of the r.c. frame. In particular, the method is based on the assumption of a deformation shape of the coupled system that coincides with the first vibration mode of the original frame, in the case of regular post-elastic frame behavior, whereas in the case of irregular post-elastic frame behavior, the deformation shape may be arbitrary chosen in order to regularize the frame behaviour. The case of regular frames is considered in this paper and the design procedure for the r.c. frame is based on the classical non linear static analysis (push-over analysis) and the concept of equivalent viscous damping coefficient (Priestley et al. 2007). The solution for the equivalent linear Single Degree of Freedom (SDOF) system is achieved by using the capacity spectrum method (Freeman 1998) in order to satisfy a given limit state

for a given level of seismic excitation. In this paper Collapse Limit State is taken into account for existing structures, as suggested by several technical codes (EN1998-3 2005, OPCM3431 2005, DM 2008). Finally a distribution criterion of the HDR dampers along the frame height is furnished and nonlinear time history analyses are performed in order to validate the design procedure.

2. HDR-BASED DISSIPATING BRACE MODELING

HDR-based dissipating braces generally consist of coupled metallic frames and HDR-based dampers. In dissipating braces, the HDR material is subjected to shear deformations and usually shows a nonlinear behavior. A linear description may be adopted by defining a shear modulus $G_0(\omega,\gamma)$ and an equivalent damping factor $\xi_0(\omega,\gamma)$, which nonlinearly depend on the deformation amplitude γ and on the frequency input ω. In particular, HDR is characterized by a value of $G_0(\omega,\gamma)$ which is very sensitive to the strain amplitude and quite sensitive to the strain rate, while the variation of the damping factor $\xi_0(\omega,\gamma)$ is less significant (Dall'Asta and Ragni 2008a). Once the strain amplitude $\overline{\gamma}$ and frequency $\overline{\omega}$ are fixed, a Kelvin-type model may be used for describing the HDR-based damper behavior. The spring stiffness K_0 and the dashpot coefficient c_0 of the Kelvin-type model are given by the following expressions:

$$K_0 = \frac{G_0(\overline{\omega},\overline{\gamma})A_0}{h_0} \qquad c_0 = \frac{2\xi_o(\overline{\omega},\overline{\gamma})G_0(\overline{\omega},\overline{\gamma})A_0}{\overline{\omega}h_0} \tag{2.1, 2.2}$$

where A_0 and h_0 are the rubber area and the rubber thickness of the device. The length of the HDR device is generally small and the dissipation brace may be considered as a system of two elements coupled in series: the dissipation device and the metallic brace link. The global behavior of the diagonal brace still represents a viscoelastic element which is described by means of the stiffness K_c and the damping factor ξ_c, for assigned frequency and strain amplitude. These two parameters may be expressed as a function of K_0, ξ_0 and the brace stiffness K_b, according to the following functions:

$$K_c = \frac{K_b K_o\left(\left(1+4\xi_0^2\right)K_o + K_b\right)}{\left(K_b + K_o\right)^2 + \left(2\xi_0 K_o\right)^2} \qquad \xi_c = \frac{\xi_0 K_b}{\left(1+4\xi_0^2\right)K_o + K_b} \tag{2.3, 2.4}$$

The brace stiffness is proportional to its transverse section area A_b, according to $K_b = EA_b/L_b$, where E is the steel elastic modulus and L_b is the length of the metallic brace, given by the difference between the total diagonal length (L_c) and the device length (L_0). The design procedure of dissipating braces generally gives an assigned value of K_c and ξ_c, for each brace. Once the equivalent damping coefficient of the rubber ξ_o is known, these values may be obtained by adopting the following values of device and link brace stiffness (K_0 and K_b):

$$K_0 = \beta(\xi_0, \xi_c)K_c \tag{2.5}$$

$$K_b = \alpha(\xi_0, \xi_c)K_0 = \alpha(\xi_0, \xi_c)\beta(\xi_0, \xi_c)K_c \tag{2.6}$$

where $\alpha(\xi_0, \xi_c)$ and $\beta(\xi_0, \xi_c)$ may be obtained by inverting Eqns. 2.3 and 2.4 and are:

$$\alpha(\xi_0, \xi_c) = \frac{(1+4\xi_0^2)\xi_c}{(\xi_0 - \xi_c)} \tag{2.7}$$

$$\beta(\xi_0, \xi_c) = \frac{(1+\alpha)^2 + 4\xi_0^2}{\alpha^2 + \alpha(1+4\xi_0^2)} \tag{2.8}$$

Figure 1 shows how the ratios K_0/K_c and K_b/K_c vary when the ratio ξ_c/ξ_0 spans between 0 and 1. A value of the damping ratio ξ_c/ξ_0 close to 1 results in values of $K_0/K_c \to 1$ and $K_b/K_c \to \infty$ which means that maximum damping may be obtained for very high values of the metallic brace stiffness. It should be observed that in many practical situations minimum dimensions of the metallic brace depend on the buckling check, therefore values of the dissipating bracing system damping ξ_c very close to the HDR damping ξ_0 are usually assumed in the design procedure.

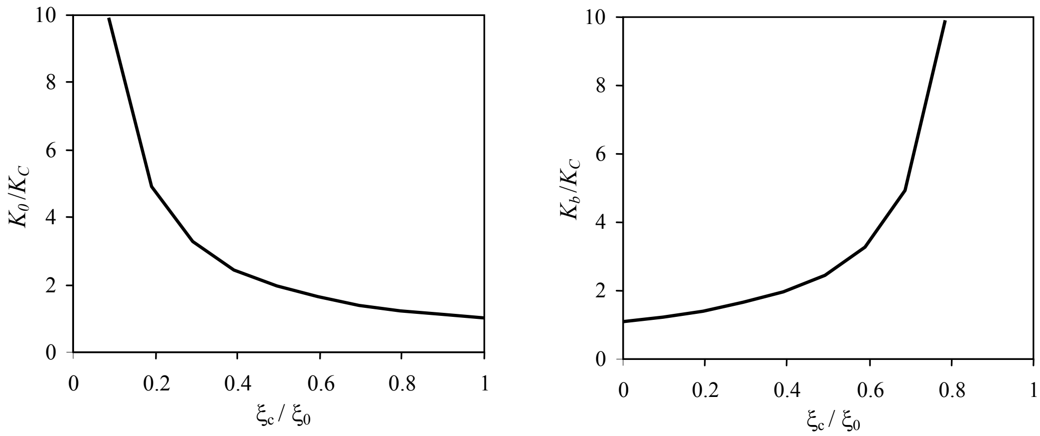


Figure 1 Values of K_0/K_c and K_b/K_c ratios versus ξ_c/ξ_0 ratio

3. DESIGN METHOD

In order to design the required dissipation system it is first necessary to reduce the behavior of an MDOF system, such as the r.c. frame or the coupled system, into an SDOF equivalent system. In this way the system capacity may be compared with the seismic demand in the ADRS plane and the required resistance and stiffness of the bracing system may be evaluated as a function of seismic intensity. For purposes of the displacement-based design approach this may be accomplished by assuming an objective displacement shape for the MDOF system, which may be considered as associated to the first vibration mode of the structure, in order to simplify the design procedure. If the original r.c. frame shows a regular behavior along its height (no localization of displacement demand is presumed to occur in the inelastic field), it is reasonable to assume the deformed shape of the first vibration mode of the original r.c. frame as the objective displacement shape of the coupled system. As a consequence, the two resisting systems acting in parallel (r.c. frame and dissipative bracing system) are forced to deform mainly according to the assumed vibration mode, at least until the frame remains within the elastic field. Different shapes could be chosen if the frame behaves irregularly along its height and the aim is to change the frame behavior.
This paper considers the case of regular frames assuming an objective displacement shape coinciding with the first vibration mode of the r.c. frame. The inter-storey displacements of the first vibration mode of the frame, addressed as Δ^i (i=1..n), are defined with exception of a constant, and displacements at the different heights may be expressed as:

$$U^1 = \Delta^1, \quad U^i = U^{i-1} + \Delta^i \qquad (i=2 \ldots n) \tag{3.1}$$

where n is the total floor number. It is convenient to normalize the distributions of the inter-storey and storey displacements with respect to the ultimate floor displacement, which usually coincides with the control node in a pushover analysis:

$$\delta^i = \Delta^i / U^n \tag{3.2}$$

$$u^i = U^i / U^n \tag{3.3}$$

The equivalent SDOF mass m^* and the modal participating factor Γ associated to the system deforming according to the assumed shape are:

$$m^* = \sum_i m^i u^i \tag{3.4}$$

$$\Gamma = m^* \Big/ \sum_i m^i \left(u^i\right)^2 \tag{3.5}$$

The distribution of shear forces at each floor may be deduced from equilibrium:

$$V^n = \omega^2 m^n u^n \quad , \qquad V^i = V^{i+1} + \omega^2 m^i u^i \qquad (i = 1 \ldots n-1) \tag{3.6}$$

where ω is the circular frequency corresponding to the vibration mode. The stiffness distribution may be expressed as a function of the shear and inter-storey displacement at each level according to $K^i = V^i / \Delta^i$. In order to get rid of the dependency on frequency, the shear forces and stiffness of each level may be normalized with respect to the base shear and stiffness:

$$v^i = V^i / V^1 \qquad k^i = K^i / K^1 \tag{3.7, 3.8}$$

Once the objective shape and the relevant quantities described above are fixed, the design procedure may be started and the following steps defined: evaluation of the bare frame capacity by means of the equivalent SDOF system (*step1*), definition of the coupled SDOF equivalent system according to the seismic demand (*step2*), distribution of the base shear and base stiffness obtained along the MDOF system floors (*step3*), in the plane distribution of the bracing systems and design of each dissipative brace component (*step4*). Each step is described in detail below.

Step1

The capacity of the bare structure may be evaluated by nonlinear static analysis under a set of forces equal to the inertia forces ($F^i = \omega^2 m^i u^i$) of the first vibration mode. The pushover curve may be obtained by plotting the frame base shear as a function of the displacement of the control node, which is usually the top floor displacement. The frame base shear V_f^1 and the ultimate displacement s_u, corresponding to the failure of the frame section which first reaches its ultimate rotation capacity (if other fragile mechanisms are not met) may be determined from this analysis. The yielding displacement s_y of an elastic-perfectly plastic system equivalent to the frame may be obtained according to the code provisions (EN1998-3 2005). The nonlinear system may finally be substituted by an equivalent linear system having the following properties:

$$K_f = V_f^1 / s_u \qquad \xi_f = \mu^{0.34} \cdot 0.05 + 0.215 \cdot \left(1 - \frac{1}{\mu^{0.642}}\right) \tag{3.9, 3.10}$$

where $\mu = s_u / s_y$ is the ductility of the elastic-perfectly plastic system. The value of the equivalent viscous damping ξ_f is derived using the formulas to support the direct displacement based design, by assuming a Takeda-Thin hysteretic rule for concrete frames (Priestley et al. 2007).

Step2
The dissipative bracing system may also be represented by an equivalent viscoelastic system with stiffness K_d and equivalent damping ξ_d. The coupled systems is still a viscoelastic system whose total stiffness K is equal to the sum of the single stiffness and whose damping factor ξ is deduced by applying energy equivalence criteria (Soong and Dargush 1997):

$$K = K_f + K_d \qquad \xi = \frac{\xi_f K_f + \xi_d K_d}{K_d + K_f} \tag{3.11, 3.12}$$

The maximum acceleration that the equivalent SDOF system is able to withstand is $a^* = K s_u / m^* \Gamma = (K / m^*) s_u^*$, whereas the maximum displacement is $s_u^* = s_u / \Gamma$, where m^* and Γ remain unchanged despite the introduction of the braces. The capacity of the system must be compared with the seismic demand. A useful representation may be obtained by plotting the capacity and demand curves in the ADRS plane. The value of the bracing system stiffness K_d may be varied until the capacity curve intersects the demand curve at the ultimate frame displacement. The base shear required by the dissipating bracing system is $V_d^1 = K_d s_u$, while its stiffness at the first floor is $K_d^1 = V_d^1 / s_u \delta^1$.

Step3
The shear and stiffness that must be provided by the dissipating bracing system at each level may be determined according to Eqns. 3.7 and 3.8, and are:

$$V_d^i = V_d^1 v^i \quad , \quad K_d^i = K_d^1 k^i \tag{3.13, 3.14}$$

Step4
Once the damper number at each floor is fixed, the design stiffness values of a single dissipation diagonal brace (K_c) may easily be determined according to geometrical considerations. Other geometric relationships may be introduced to calculate the ultimate displacements required for the diagonal brace s_{cu}, from the ultimate inter-storey displacements $\Delta_u^i = s_u \cdot \delta^i$.
Finally, the geometry of the HDR-devices may be defined. In particular, the thickness of the HDR-based devices may be directly obtained from the expression $h_0 = s_{cu} / \bar{\gamma}$, for the considered value of admissible maximum shear strain $\bar{\gamma}$, while the device area A_0 follows from Eqn. 2.1., since the fundamental vibration frequency of the upgraded structure ($\varpi = \sqrt{m^* / K}$) is known and the rubber shear modulus $G_0(\varpi, \bar{\gamma})$ is defined. By assuming that the damping value of a single diagonal brace ξ_c is equal to the total damping provided by the bracing system ξ_d, the stiffness of the elastic link brace (K_b) may be defined by applying Eqn. 2.6. The dashpot coefficients of the single device (c_c) may also be determined, according to Eqn. 2.2, and applied in structural models to perform time history analyses.

4. APPLICATION

In order to illustrate the design procedure a simple two dimensional r.c. frame is considered. The frame is typical of many structures designed and built during the 80s without any particular seismic detailing in Italy.
The frame consists of 6 spans and 4 stories. Columns have a 35x35 cm^2 square section at the base and a 30x30 cm^2 square section at the other levels. Beams are 30x50 cm^2 on the first and last floors and 80x20 cm^2 at the second and third floors. The frame geometry and some reinforcement detailing are shown by Figure 2.
The seismic combination of dead and live loads results in a uniformly distributed load equal to 35 *kN/m* at the

top floor and 26 *kN/m* at the other floors, while seismic masses are 95 kNs^2/m at the top floor and 72 kNs^2/m at the other floors. Structural analyses were carried out with the support of the finite element software SAP2000 (advanced version 10.1.1). Beams and columns were modelled as *beam elements* with reduced stiffness to take into account concrete cracking while the non linear behavior was lumped at the beam ends by means of two plastic hinges. The moment-rotation curves were calculated according to EN1998-3 (2005). Modal analysis of the bare frame showed a value of the first mode period equal to 1.57 *s* and a participating mass ratio above 77%. The results of modal analysis are summarized by Table 4.1.

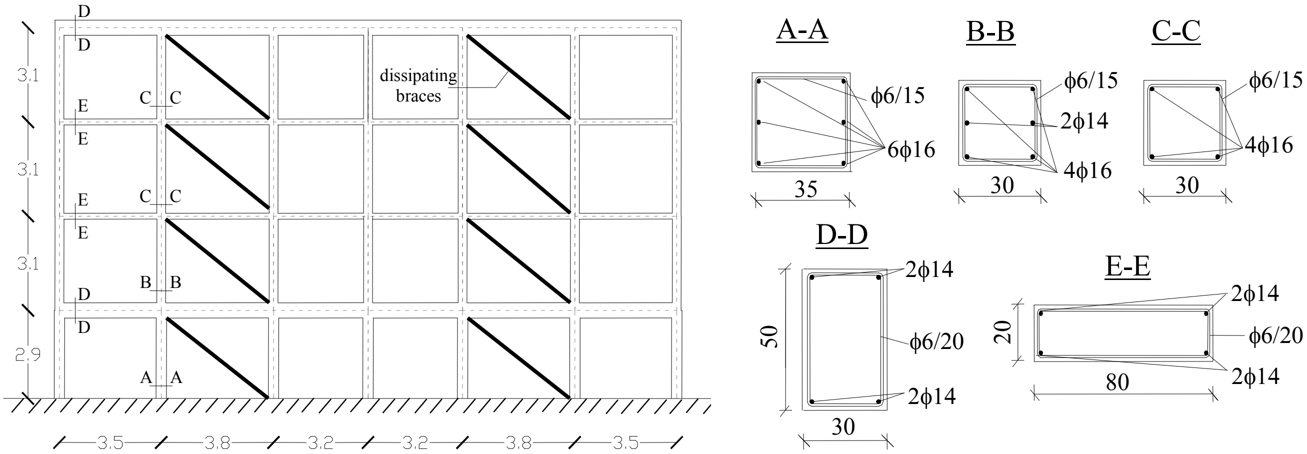


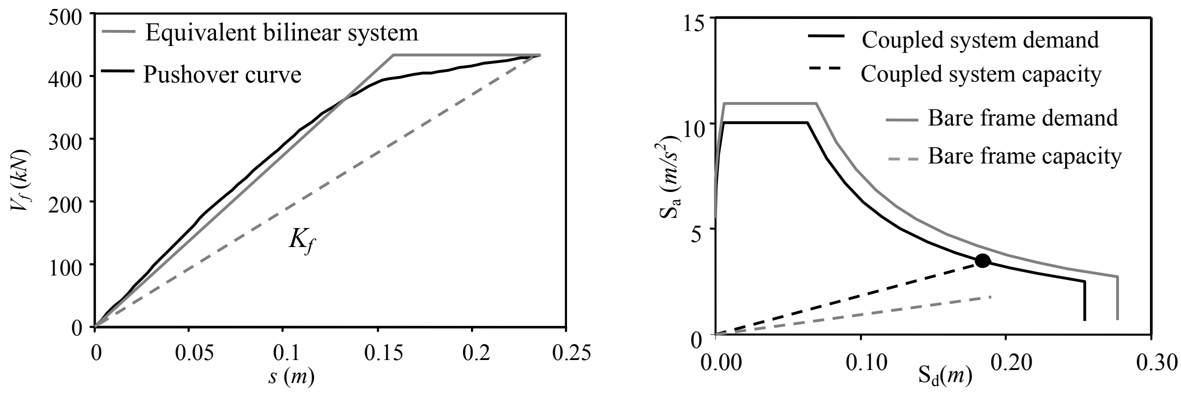
Figure 2 Frame and reinforcement details

Table 4.1 Frame modal analysis results

floor	*mass* (kNs^2/m)	U^i (*m*)	Δ^i (*m*)	u^i (*m*)	δ^i (*m*)	V^i (*kN*)	K^i (*kN/m*)
4	95.4	0.0800	0.017	1.000	0.208	95	459
3	72	0.0634	0.027	0.792	0.343	167	489
2	72	0.0359	0.025	0.449	0.316	239	758
1	72	0.0107	0.011	0.134	0.134	311	2330

The nonlinear static analysis of the bare frame under the inertia forces of the first vibration mode demonstrated regular and quite ductile behavior with plastic hinges reaching their ultimate capacity at the first storey column bases. The Pushover curve of the bare frame is characterized by a value of the maximum base shear $V_f = 432kN$ and an ultimate displacement $s_u = 0.236m$, as shown by Figure 3a. The ductility corresponding to the bilinear system is $\mu = 1.49$. The properties of the equivalent elastic SDOF are: $\Gamma = 1.2431$, $m^* = 194.4t$, $K_f = 1822.8kN/m$, $\xi_f = 0.109$, $s_u^* = 0.1906m$, $a_t^* = 0.1792$. By assuming a pseudo acceleration spectrum given by OPCM 3431 (2005) code with a soil class C (soil factor S=1.25) and a design acceleration equal to a_g= 0.30*g*, the frame is not able to withstand the seismic action at the collapse limit state, as shown by Figure 3b. A design rubber strain equal to $\bar{\gamma} = 1.5$ was chosen and a damping coefficient $\xi_d = 0.17$ was assumed for the dissipating braces, which is very close to the rubber equivalent damping coefficient for the design strain considered. The base shear the dissipative braces must provide was found to be V_d^1= 413 *kN* , which corresponds to a base stiffness K_d^1=1750 *kN/m*. Consequently the damping coefficient of the coupled system is ξ=0.139, the base shear is V^1=846 *kN* and the total stiffness is K^1=3579 *kN/m* (Figure 4.2b). Two braces are provided at each storey and placed in two symmetric spans (Figure 2). The application of the design method leads to values of stiffness of each diagonal brace at different stories (K_c^i) which are summarized by Table 4.3. The values of the ultimate displacement (s_{cu}^i), the dashpot coefficients (c_c^i) and the maximum axial force N_c^i are also reported. The modal analysis of the coupled system showed a value of the first mode period equal to

1.31 *s* and a first vibration mode coinciding with the fist mode of the bare r.c. frame, as expected according to the design procedure adopted.



(a) (b)

Figure 3 Pushover curve of the bare frame (a) - Capacity vs Demand in ADRS plane (b)

Table 4.3 Distribution of dissipative braces properties along the height

floor	s_u^i (m)	Δ_u^i (m)	V_d^i (kN)	K_d^i (m)	K_c^i (kN)	s_{cu}^i (m)	c_c^i (kNs/m)	N_c^i (kN)
4	0.236	0.049	126.7	2576.6	2145.7	0.038	170.0	81.7
3	0.187	0.081	222.3	2746.1	2286.8	0.063	181.1	143.4
2	0.106	0.075	317.9	4261.2	3548.5	0.058	281.1	205.1
1	0.032	0.032	413.6	13095.5	10361.2	0.025	820.9	260.1

In order to validate the design procedure results, nonlinear time history analyses were performed under three artificially generated ground motions that match the seismic spectrum according to the criteria given by OPCM 3431 (2005). The results obtained were enveloped. The maximum roof displacement of the damped structure under design peak ground acceleration (at collapse limit state) is 0.183 *m* and the maximum base shear is 821 *kN*, which are very close to the predictions. Additionally, Figure 4 reports the maximum axial displacements experienced by the diagonal braces at each floor comparing these with the design displacements. Here again the values obtained are very close to the predictions. Finally, the introduction of the dissipating braces also resulted in a major diffusion of inelastic demand which also affects the higher storeys of the frame. Ultimate rotation capacity of the plastic hinges was not reached

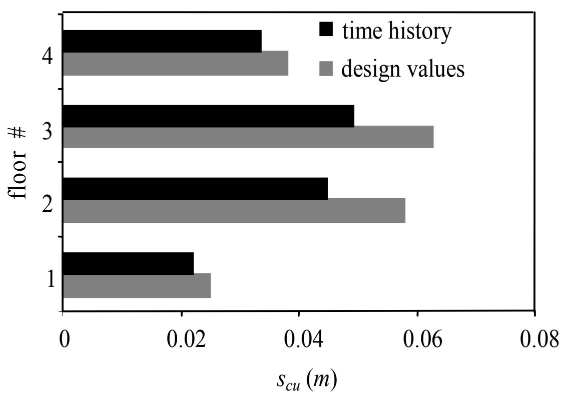


Figure 4 Diagonal brace displacements

Finally, the geometry of the elastic braces and HDR-based devices may be defined. The devices generally consist of two rubber layers having the same area and thickness separated by a steel plate. Since the fundamental period of vibration of the upgraded structure is T=1.31s and the maximum strain is $\bar{\gamma} = 1.5$, the

HDR damping factor is about $\xi_0(\overline{\omega},\overline{\gamma}) = 0.18$ while the elastic modulus is about $G_0(\overline{\omega},\overline{\gamma}) = 0.875MPa$ (Dall'Asta and Ragni 2008a). The required areas and thickness of the single rubber layers are shown by Table 4.4., while dimensions of metallic braces (diameter D_b and thickness t_b of the tubular profiles) are summarized by Table 4.5. Buckling checks were carried out with positive results.

Table 4.4 HDR devices properties

floor	ξ_0	G	$\overline{\gamma}$	α	β	K_0	h_0	A_0
	-	(N/mm^2)	-	-	-	(kN/m)	(m)	(m^2)
4	0.18	0.875	1.5	19.2	1.046	1121.9	0.025	0.033
3	0.18	0.875	1.5	19.2	1.046	1195.6	0.042	0.057
2	0.18	0.875	1.5	19.2	1.046	1855.3	0.039	0.082
1	0.18	0.875	1.5	19.2	1.046	5417.3	0.017	0.104

Table 4.5 Metallic braces properties

floor	L_0	L_b	K_b	A_b	D_b	t_b
	(m)	(m)	(kN/m)	(mm^2)	(mm)	(mm)
4	0.60	4.30	43088.5	883.1	101.6	3.60
3	0.60	4.30	45921.6	941.1	114.3	4.00
2	0.60	4.30	71258.2	1460.4	139.7	4.00
1	0.60	4.18	208062.6	4141.6	219.1	6.30

ACKNOWLEDGMENTS
The study presented in this paper was developed in the framework of the ReLUIS National Research Project. Support from the ReLUIS-DPC, Italian University Network of Seismic Engineering Laboratories and Italian Civil Protection Agency, is gratefully acknowledged.

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