WAVELET-BASED ESTIMATION OF SITE RESPONSE

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ABSTRACT :

Seismic damage is highly dependent on the rate of energy input from the earthquake waves. Ground motions with the same amplitude spectrum can cause substantially different levels of seismic damage on the same structure, depending on the time-frequency localization of the energy imparted to the structure. In this study, a wavelet-based approach to detect nonlinearities and characterize site response is presented. To account for the nonstationary behavior of the ground motion processes, the traditional soil amplification ratio is extended to include the time parameter. The wavelet-based definition of soil amplification factor can easily be compared to its Fourier-based counterpart, by simply taking the root mean square of the time dependent amplification factors, for each frequency. Ground motion records from the main shock and aftershock of the 2001 Nisqually earthquake measured at three sites have been analyzed using Morlet wavelets. The comparison of energy localization patterns observed in the wavelet spectra from different sites is used to detect nonlinear soil behavior and characterize soil damping. Analysis results show the advantage of a wavelet-based approach over its Fourier-based counterpart, where time domain information is lost. This study concludes that the energy localization patterns observed in the wavelet spectra from different sites and events can be used to detect nonlinear effects and characterize soil damping.

KEYWORDS: Wavelet transform, nonlinearity, site amplification, soil damping

1. INTRODUCTION

Seismic damage is highly dependent on the rate of energy input from the earthquake waves. The rate of energy input at a given site is traditionally expressed in terms of the Fourier amplitude spectrum of the ground motion, with the inherent assumption that the ground motion is stationary. This assumption allows engineers and researchers perform seismic analysis in the frequency domain only, replacing time domain convolution by multiplication in the frequency domain.

Fourier transform, although a very powerful tool for analyzing Linear Time Invariant (LTI) systems, fails to accurately describe nonlinear or nonstationary behavior. When a nonstationary signal is decomposed into a series of trigonometric basis functions with infinite support, time varying features cannot be captured. Unfortunately, site response during an earthquake is both nonlinear and nonstationary.

In general, soil nonlinearity is associated with strength reduction and ground motion amplitude increase at soft soils (Hardin and Drnevich 1972). Site amplification, in addition to its direct effect on structures, increases liquefaction susceptibility of saturated soft soils, especially in long-duration earthquakes (Seed and Idriss 1982; Raoof, Herrmann et al. 1999; Osinov and Gudehus 2003). Soil nonlinearity, in addition to shifting resonant frequencies, causes a frequency-dependent increase in damping (Field, Johnson et al. 1997; Beresnev, Field et al. 1998), further complicating site response analysis. Considering the nonstationary and nonlinear nature of ground motion processes, the need for a joint time- frequency analysis becomes apparent.

Although the limitations of Fourier analysis have been long recognized, the ground motion models developed so far, as well as the current structural design codes, still heavily rely on Fourier-based tools. The ground motion spectrum at a given site is conveniently expressed as a product of source, path and local site effects (Boore 2003). The local site effects can then be expressed in terms of the spectral ratio between the signal recorded at the sedimentary site and a reference one, preferably recorded at a nearby bedrock site.

This study uses the wavelet transform to project the recorded accelerograms into the joint time-frequency domain. To account for the nonstationarity of the ground motion processes, the traditional soil amplification ratio is redefined to include the time parameter. The wavelet-based definition of soil amplification factor can easily be compared to its Fourier-based counterpart, by simply taking the root mean square of the time dependent amplification factors, for each frequency. Analysis results show the advantage of a wavelet based approach over the traditional Fourier transform, where time domain information is lost. The comparison of energy localization patterns observed in the wavelet spectra from different sites and can be used to detect nonlinear effects and characterize soil damping.

2. SITE RESPONSE ESTIMATION USING WAVELETS

This section starts with a brief introduction to the time - frequency analysis and the wavelet transform. Next, in Section 2.2, we discuss how the wavelet amplitude spectra can be used to estimate site response. Finally, in Section 2.3, we present an application using the data from the 2001 Nisqually main shock (M=6.8) and aftershock (ML=3.4) recorded at three stations.

2.1. Time-frequency Analysis and Wavelets

The Wavelet Transform (WT) is a linear transform, which decomposes a signal x(t) into basis functions that are dilations and translations of a selected mother wavelet $\psi(t)$through the convolution (Daubhechies 1990):

$$W_{\psi}x(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}x(t)\psi^{*}(\frac{t-b}{a})dt \tag{2.1}$$

where ψ^* is the complex conjugate of the wavelet, a and b are the scale and the time parameters of the wavelet, respectively. The coefficients $W_\psi x(a,b)$ measure the similarity between the signal and the scaled and translated wavelet. The square of the wavelet coefficient $\left|W_\psi x(a,b)\right|^2$ is proportional to energy of $x(t)$ contained in the time-frequency grid represented by the scaled and translated wavelet (Spanos, Tezcan et al. 2005).

The WT analyzes different frequencies with different resolutions. The time-scale decomposition offered by wavelet transform becomes very useful as in most practical applications, high frequencies (low scales) do not last for a long duration, but instead, appear as short bursts, while low frequencies (high scales) usually last for entire duration of the signal (Mallat 1989; Daubhechies 1990). In this study, we use the Morlet wavelets defined as (Goupillaud, Grossmann et al. 1984):

$$\psi(t) = \cos(5t)e^{-t^2/2} \tag{2.2}$$

Figure 1 shows the Morlet wavelet function:

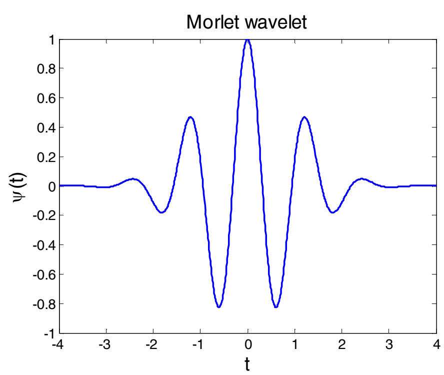

Figure 1 Morlet wavelet basis function

The Wavelet Transform of seismic signals produces two dimensional coefficients, time and scale. Depending on the selected wavelet basis function, the coefficients can be real or complex. The squared absolute values of the wavelet coefficients represent the local energy, or equivalently the amplitude spectra. If a complex wavelet is used in the analysis, amplitude and phase spectra are generated. The scale represents the frequency, although the relationship between the two depends on the wavelet basis used in the analysis.

Figure 2 shows an accelerogram (top), its power spectrum obtained from its Fourier coefficients (bottom left) and its wavelet amplitude spectrum (bottom right). The wavelet spectrum shows how the total energy is distributed both in time and frequency domains.

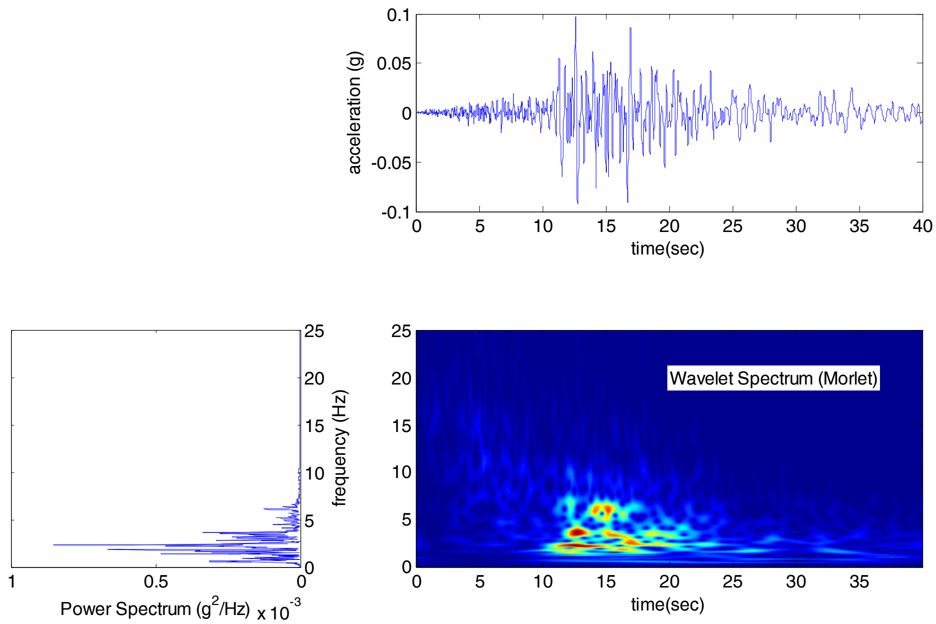

Figure 2 Time-frequency representation of a recorded accelerogram

2.2. Fourier vs. Wavelet-based Spectral Ratio

Most seismic codes and provisions use amplification factors to account for the soil amplification, including the effects of the resonance within the soil column, and the impedance difference between the bedrock and the overlying soils. Site amplification factor ($A(f)$) is defined as the ratio of the response spectrum of the soil, ($S_{a,site}(f)$), to that the underlying bedrock ($S_{a,rock}(f)$):

$$A(f) = \frac{S_{a,site}(f)}{S_{a,rock}(f)} \tag{2.3}$$

Alternatively, site amplification can be expressed in terms of the Fourier coefficients of the ground motion records. If the horizontal ground acceleration records for the site and the reference (rock) site are available, the amplification factor can be calculated from (Safak 1997):

$$A(f) = \frac{\left|F_{a,site}(f)\right|}{\left|F_{a,rock}(f)\right|} \tag{2.4}$$

In Eqn. 2.4, $\left|F_{a,site}(f)\right|$ and $\left|F_{a,rock}(f)\right|$ are the Fourier amplitudes of the ground acceleration at the soil site, and the rock site, respectively, calculated as the vector sum of the two horizontal components. Note that since $\left|F(f)\right|$ represents the root-mean-square energy contained in the frequency component f, Eqn. 2.4 can be thought to represent the ratio of the energy contained at frequency f. Adopting this energy ratio approach, a time dependent version of the soil amplification factor can be defined as a function of the wavelet coefficients of the acceleration records as follows:

$$A_{\psi}(f,t)=\frac{\left|W_{\psi}a_{site}(f,t)\right|}{\left|W_{\psi}a_{rock}(f,t)\right|}. \tag{2.5}$$

If data from the main shock and aftershock is available, comparison of the two amplification factors provide information on the soil nonlinearity only, since the source and site path effects will be the same for both locations (Zhang, Hartzell et al. 2005).

2.3. Application to records from the 2001 Nisqually Earthquake

In this section, we analyze the wavelet spectra from the 2001 Nisqually main shock (M=6.8) and aftershock (ML=3.4) recorded at three stations. Table 2.1 shows the average shear wave velocity in the top 30 m (Vs30) and the Peak Ground acceleration (PGA). In this study, the station SEW is chosen as the reference "rock" site, since its shear wave velocity is representative of typical rock sites in the western United States (Frankel, Carver et al. 2002).

Table 2.1 Station data

Station	Vs30 (m/sec)	PGA of main shock (cm/sec2)	
		EW	NS
SEW	433	106.9	153.7
SEU	367	95.2	96.1
SDS	148	202.8	278

Figure 3 and Figure 4 show the wavelet spectra of the horizontal ground acceleration form the main shock and the aftershock of the of the Nisqually earthquake, respectively, as calculated from the wavelet coefficients. Figure 3 shows that, for the soft soil site (third plot), most of the energy is concentrated in the low frequency components whereas the energy in the stiff soil (second plot) is distributed over a wide frequency range. This is an expected result, and can be easily obtained from a Fourier analysis. The information that Fourier transforms fails to provide is the time domain evolution of the energy, which is related to soil damping. Note that there is little energy contained in soft soil site motion after about 15 seconds, which indicates fast energy absorption and strong damping. On the other hand, the stiff soil still has a significant part of the total energy after 15 sec, and thus has less damping compared to the soft soil site. Comparison of Figure 3 and Figure 4 reveals that, in the soft soil, amplification at low frequency motion is very sensitive to the strength of the shaking, indicating nonlinear behavior. On the other hand, the frequency components carrying the most energy are somewhat similar to the rock, for both the main shock and the aftershock, suggesting mostly linear behavior.

Figure 5 and Figure 6 compare the amplification factors calculated from the Fourier and wavelet approaches. To enable comparison between the two, wavelet-based amplification factors can be written as a function of the frequency only , using:

$$A_{\psi}(f)=\sqrt{\sum_{t}\left(A_{\psi}(f,t)\right)^{2}}. \tag{2.6}$$

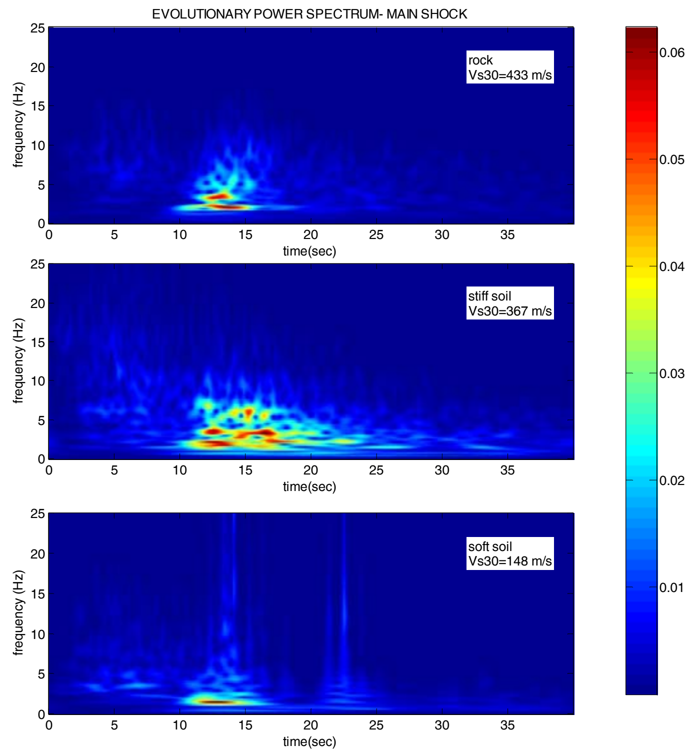

Figure 3 Wavelet Spectra of Nisqually main shock

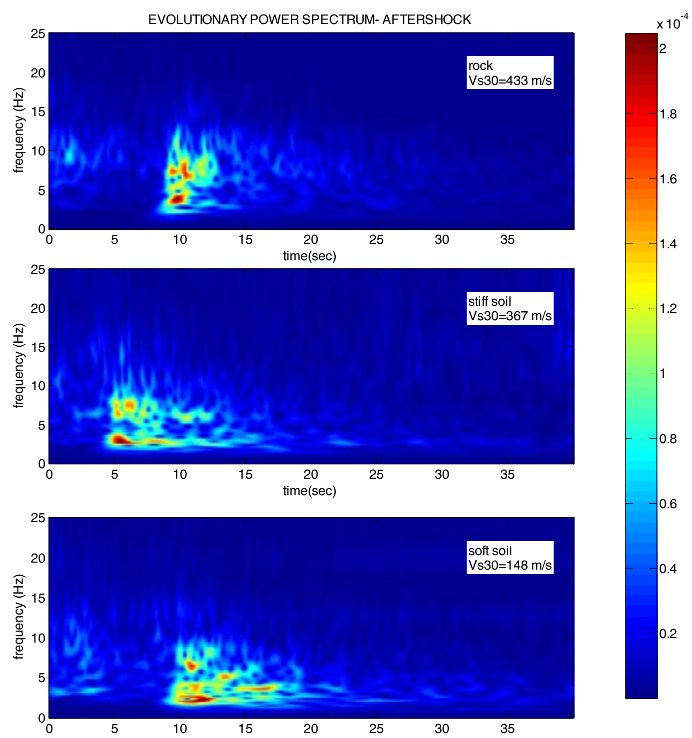

Figure 4 Wavelet Spectra of Nisqually aftershock

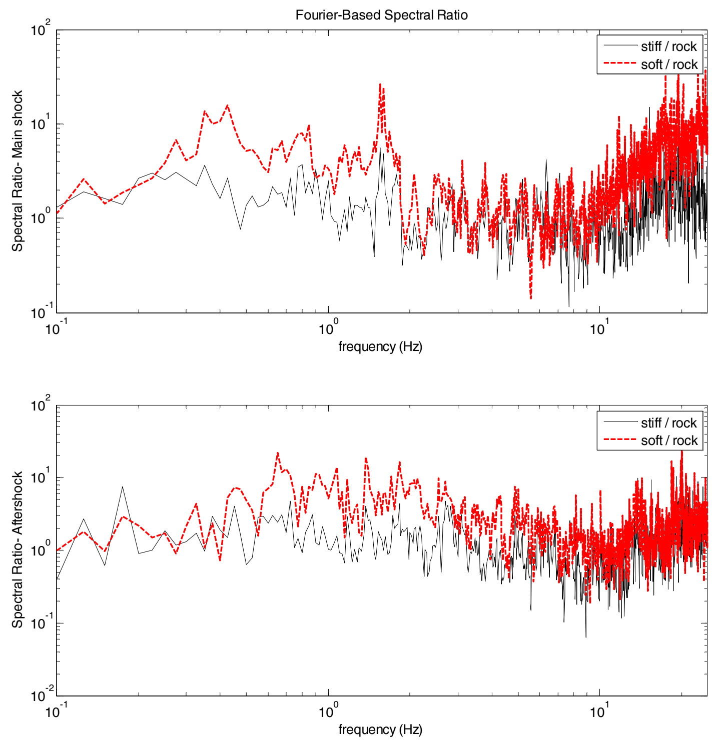

Figure 5 Soil amplification estimation from Fourier Spectral Ratios

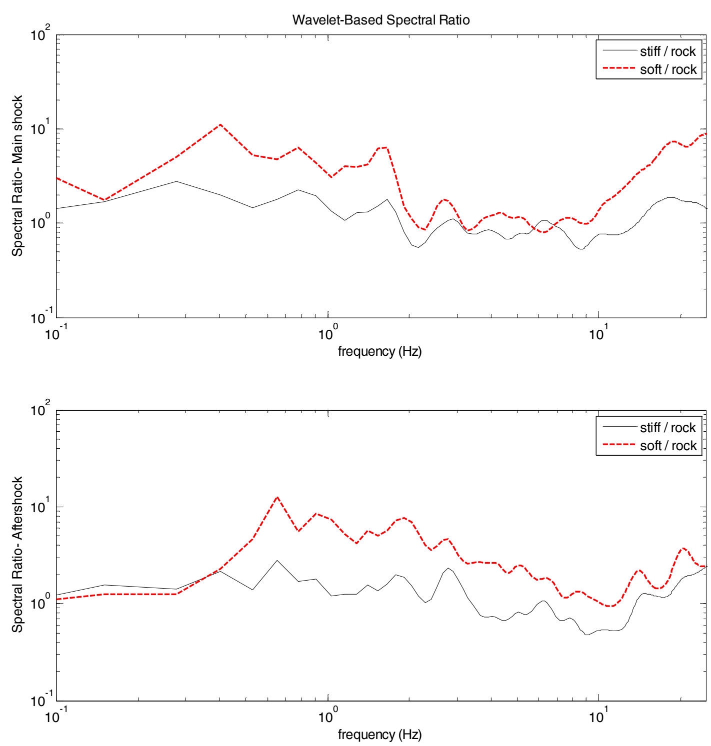

Figure 6 Soil amplification estimation from Wavelet Spectral Ratios

3. CONCLUSION

A wavelet-based approach to detect and characterize site response is presented. The traditional definition of soil amplification is modified to include the time domain, to account for the nonstationarity in the ground motion processes. Ground motion records from the main shock and aftershock of the Nisqually earthquake measured at three sites have been analyzed in the joint time-frequency domain using Morlet wavelets. The comparison of energy localization patterns observed in the wavelet spectra from different sites is used to detect nonlinear effects and characterize soil damping. Analysis results show the advantage of the wavelet-based approach over the traditional Fourier transform, where time domain information is lost.

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