A NEW MAGNITUDE ESTIMATION METHOD BASED ON PREDOMINANT PERIOD AND PEAK AMPLITUDE

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ABSTRACT:

Earthquake Early Warning (EEW) is a useful tool for practical real-time seismic hazard mitigation at present. The critical technology of EEW is determining the size of an earthquake and the predicted ground motion at given site, from the first few seconds of the P waves. Currently, there are two different approaches to the EEW magnitude estimation, predominant period method and amplitude method. However, both methods have some disadvantages, such as significant uncertainty and saturation at great magnitude. To improve the estimation of magnitude, a new united predominant period τ_c and amplitude P_{max} method is developed and the formula is

$$M = a*\log P_{max} + b*\log\tau_c + c*\log\Delta + d$$

Where a, b, c, and d are constants, Δ is epicentral distance. The magnitude estimation results of the first three second P wave of NSMP strong motion data indicate that, the estimation precision of new method is higher than those of the two methods mentioned above, and the saturation at great magnitude is improved. Meanwhile, for short hypocentral distances, a simplified united predominant period τ_c and amplitude P_{max} method is presented, the formula is

$$M = a*\log P_{max} + b*\log\tau_c + d$$

KEYWORDS:
Earthquake Early Warning (EEW), Magnitude, Predominant Period, Amplitude

1. INTRODUCTION

In the past several decades, research on earthquake early warning (EEW) has undergone a rapid development. At present EEW is becoming a useful tool for practical real-time seismic hazard mitigation, with its applications in Japan (Nakamura,1988; Horiuchi et al.,2005; Kamigaichi,2004,2008), Taiwan (Wu and Teng,2002; Wu and Kanamori,2005a), Mexico (Espinosa-Aranda et al.,1995,2003), United States (Bakun,1994; Allen and Kanamori, 2003; Allen,2007), Romania(Wenzel et al.,2005), and Turkey(Erdik et al.,2003). The characterization of an earthquake for early warning includes most importantly estimates of its size and location (Allen and Kanamori, 2003), in this two research areas a lot of research results have been made in recent years (Nakamura,1988; Allen and Kanamori,2003; Odaka et al.,2003; Rydelek and Pujol,2004; Kanamori, 2005; Wu and Kanamori, 2005a,2005b; Olson and Allen,2005; Horiuchi et al., 2005; Wu et al.,2006; Zollo et al.,2006; Satriano et al, 2008).

The deterministic relationships between the earthquake magnitudes and some waveform properties, such as the predominant period and peak amplitude, of the first few seconds of P waves, can be used to estimate magnitude. Olson and Allen (2005) suggested that the final magnitude of an earthquake is determined by the first few seconds of the rupture process and the state of stress in the region surrounding the fault plane. This model provides a physical basis for the deterministic nature of earthquake magnitudes and for EEW applications. Currently, there are two different approaches to the EEW magnitude estimation. One is the predominant period method (Nakamura, 1988; Allen and Kanamori, 2003; Kanamori, 2005; Wu and Kanamori, 2005a; Olson and Allen, 2005), another is the amplitude method (Odaka et al., 2003; Wu and Kanamori, 2005b; Wu et al., 2006; Zollo et al., 2006).However, both methods have significant uncertainty and the saturation at great magnitude. So, here we are concerned with obtaining the high-point of magnitude estimation and simultaneously the less uncertainty as time passes after the first triggered arrival from the event, improving the underestimate at great magnitude. To achieve the objective, a new single station earthquake magnitude estimation method united amplitude and predominant period is developed.

2. METHOD

In the single station approach, the relationships between the magnitudes of earthquakes and some observational properties of the first few seconds of the *P* waves, including the predominant period τ_c (Nakamura, 1988; Allen and Kanamori, 2003; Kanamori, 2005; Wu and Kanamori, 2005a; Olson and Allen, 2005) and the peak amplitudes *Pd* (Odaka et al., 2003; Wu and Kanamori, 2005b; Wu et al., 2006; Zollo et al., 2006) were find out.

2.1 Predominant Period Method

The predominant period τ_c, that is similar to the one used by Nakamura (1988), is defined in terms of the waveforms of the first few seconds of the *P* waves as follows.
First we compute *r* by

$$r = \frac{\int_0^{\tau_0} \dot{u}^2(t)\,dt}{\int_0^{\tau_0} u^2(t)\,dt} \tag{2.1}$$

Where *u(t)* is the ground-motion displacement and the integration is taken over the time interval (0, τ_0) after the onset of the P wave. Usually, τ_0 is set at 3 s. Using Parseval's theorem,

$$r = \frac{4\pi^2 \int_0^{\infty} f^2 \left|\hat{u}(f)\right|^2 df}{\int_0^{\infty} \left|\hat{u}(f)\right|^2 df} = 4\pi^2 \left\langle f^2 \right\rangle \tag{2.2}$$

Where f is the frequency, $\hat{u}(f)$ is the frequency spectrum of *u(t)* and $\left\langle f^2 \right\rangle$ is the average of f^2 weighted by $\left|\hat{u}(f)\right|^2$. Then,

$$\tau_c = \frac{2\pi}{\sqrt{r}} = \frac{1}{\sqrt{\left\langle f^2 \right\rangle}} \tag{2.3}$$

Previous studies have shown that the predominant period reflect the sizes of earthquakes (Kanamori, 2005; Wu and Kanamori, 2005a) as follows.

$$M_{est}(\tau_c) = A * \log \tau_c + B \tag{2.4}$$

Where *A* and *B* are constants, $M_{est}(\tau_c)$ is the estimation magnitude determined by predominant periods τ_c.

2.2 Amplitude Method

The quantity P_{max} is the peak amplitude of waveform within the first few seconds (again usually 3 sec) after the arrival of the *P* wave. P_{max} is an amplitude parameter and reflects the attenuation relationship of the ground motion with distance. Therefore, if we can determine the attenuation relationship of P_{max}, then we can use P_{max} to estimate the magnitude when the hypocentral (or epicentral) distance is available.

Wu and kanamori (2005b), Wu et al. (2006) found a good linear relationship among the peak displacement amplitude *Pd*, the magnitude *M*, and the hypocentral distance *R* can be represented by

$$M_{Pd} = A^{'} * \log Pd + B^{'} * \log R + C^{'} \qquad (2.5)$$

Zollo et al. (2006) also obtained the similar results. Odaka et al. (2003) choose a novel constant '*B*' instead of hypocentral distance. They find that log*B* is linearly proportional to -logΔ, and the linear relationship between '*B*' and magnitude.

In this study, using formula

$$M_{est}(P_{\max}, \Delta) = A^{'} * \log P_{\max} + B^{'} * \log \Delta + C^{'} \qquad (2.6)$$

instead of equation (2.5).Where $A^{'}$, $B^{'}$ and $C^{'}$ are constants, P_{max} is the peak amplitude of acceleration waveform within the first three seconds after the arrival of the *P* wave, Δ is epicentral distance that can be determined using real-time location procedures as, for instance, the method proposed by Odaka et al. (2003) or Horiuchi et al.(2005) , $M_{est}(P_{max}, \Delta)$ is the estimation magnitude determined by P_{max} and Δ.

2.3 United Amplitude and Predominant Period Method

The relationships between magnitude and the two methods above-mentioned are deterministic. However, both methods have some shortcomings, such as significant uncertainty (Grecksch and Kumpel, 1997; Allen and Kanamori, 2003; Olson and Allen, 2005) and saturation at great magnitude (Wu et al., 2006). So, a new approach that can improve these disadvantages is necessary.

"*The duration and the predominant period of the shaking is proportional to the earthquake's magnitude*" (Nakamura, 1988), which means that the larger the predominant period, the greater the magnitude. Also, under the same distances conditions, the previous studies (Wu and Kanamori, 2005a; Wu et al., 2006) indicate that the larger the amplitude, the greater the magnitude. Wu and kanamori (2005b) considered that $\tau_c * Pd$ is a good quantity of threshold EEW and Wu et al (2006) "*propose that a combination of Pd and* τ_c *analyses to be used for single station or onsite EEW operation*". Meanwhile, from the pioneer definition of the magnitude, it is a quantity determined by the amplitude and period (Ritcher, 1935). On the basis of the physical basis, a new magnitude estimation method united predominant period and amplitude is introduced as followed.

$$M_{est}(P_{\max}, \tau_c, \Delta) = a * \log P_{\max} + b * \log \tau_c + c * \log \Delta + d \qquad (2.7)$$

Where *a, b, c,* and *d* are constants, $M_{est}(P_{max}, \tau_c, \Delta)$ is the estimation magnitude determined by P_{max}, τ_c , and Δ.

3. DATA

The U.S. Geological Survey National Strong-Motion Project (NSMP) has the primary Federal responsibility for recording each damaging earthquake in the United States on the ground and in man-made structures in densely urbanized areas to improve public earthquake safety. The Project maintains a national cooperative instrumentation network, a national data center, and a supporting strong-motion data analyses and research center in support of this responsibility.

The waveforms used in this study are collected from 17 earthquakes occurring between 1999 and 2006 from NSMP. All of the events have magnitudes from 4.1 to 7.9 and focal depths of less than 60 km, and all of these epicentral distances are from 20 km to 100 km. Vertical component recordings were used in this study. Acceleration was integrated once to obtain velocity and twice to obtain displacement. A 0.075 Hz Butterworth high-pass filter was applied to remove the low-frequency drift after the integration. We used an automatic *P* wave picker similar to that described by Allen (1978) to detect the P wave arrival. To recognize the seismic arriving time automatically, on the basis of seismic phase recognition by using the method proposed by Allen (1978), a searching method in a window which is before arriving time at trigger threshold is developed. Then，we computed predominant periods τ_c from the first 3-second-lonsg filtered signals after the *P* wave arrival. The peak acceleration amplitudes P_{max} was also computed in the same time window.

Table 3.1 Events used in this study

Origin Time (UTC)		Lon. （W）	Lat. （N）	Depth （km）	M	N
2006/10/20	17:00:08.10	122.790	38.870	3.0	4.5	2
2006/05/12	10:37:29.31	122.820	38.820	2.0	4.7	2
2005/08/10	22:08:22.61	104.833	36.947	5.0	5.0	4
2005/07/26	04:08:37.16	112.615	45.365	12.9	5.6	2
2005/06/26	18:45:57.82	120.093	39.305	0.4	4.8	3
2005/04/16	19:18:13.00	119.178	35.027	10.8	4.6	3
2005/01/12	08:10:46.38	116.395	33.953	7.6	4.1	4
2004/11/29	01:54:14.51	120.492	35.945	10.2	4.2	2
2004/09/28	17:15:24.24	120.364	35.819	8.8	6.0	2
2002/11/03	22:12:41.00	147.444	63.517	4.9	7.9	2
2002/06/17	16:55:07.44	124.604	40.828	22.0	5.3	2
2002/02/22	19:32:41.75	115.322	32.319	7.0	5.7	9
2001/06/10	13:19:11.29	123.503	47.167	40.7	5.0	7
2001/02/28	18:54:32.83	122.727	47.149	51.9	6.8	3
2001/01/13	17:33:32.38	88.660	13.049	60.0	7.6	11
1999/08/18	01:06:18.95	122.686	37.907	7.0	4.5	2
1999/07/03	01:43:54.00	123.463	47.076	40.6	5.8	4

4. RESULTS

The results we obtained for the relationships among the peak amplitudes of acceleration within the first three seconds after the arrival of the *P* wave P_{max}, the predominant periods τ_c, epicentral distance Δ, and the magnitude *M* is shown in Figure 1a to Figure 1d.

Figure 1a shows the relationship between $M_{est}(\tau_c)$ determined from the predominant periods τ_c *versus* the final magnitude *M* of the earthquake. Estimation standard deviation level is 0.85 magnitude units, which is larger than the one obtained from the previous studies (Wu and Kanamori, 2005a; Wu et al, 2006). It is to be remarked, we choose single stations τ_c instead of multi stations average τ_c. The linear regression for the relationship between the $M_{est}(\tau_c)$ and τ_c leads to

$$M_{est}(\tau_c) = 2.60 * \log \tau_c + 5.72 \tag{4.1}$$

Figure 1b shows the relationship between $M_{est}(P_{max}, \Delta)$ determined from P_{max} obtained from the first three second *P* wave seismograms recorded by strong motion instruments and epicentral distance Δ *versus* the final magnitude *M* of the earthquake. Estimation standard deviation level is 0.56 magnitude units. According to the figure, the uncertainty of amplitude method is smaller than the one of predominant period method. The linear regression for the relationship among the $M_{est}(P_{max}, \Delta)$, P_{max} and Δ leads to

$$M_{est}(P_{\max}, \Delta) = 1.49 * \log P_{\max} + 3.10 * \log \Delta - 0.84 \tag{4.2}$$

Figure 1d shows the relationship between $M_{est}(P_{max}, \tau_c, \Delta)$ determined from united amplitude and predominant period method *versus* the final magnitude *M* of the earthquake. Estimation standard deviation level is 0.42 magnitude units. As is shown in the figure in evidence, the new method has less uncertainty and simultaneously

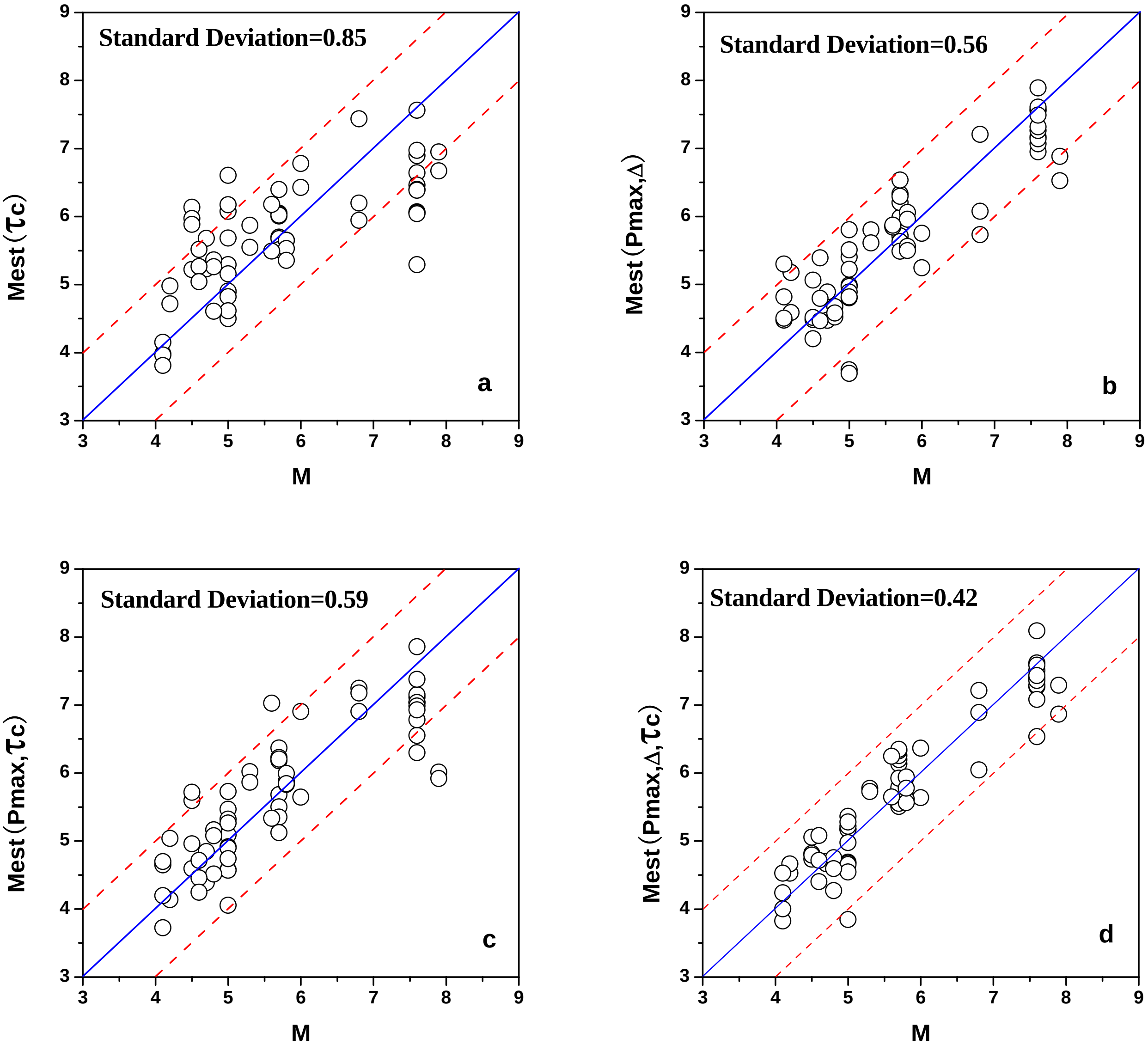

Figure 1 M_{est} (vertical axis) *versus* M (horizontal axis). Solid line shows the 1:1 linear relationship between M_{est} and M, two dashed lines show one magnitude units deviation.
(a) the relationship between $M_{est}(\tau_c)$ determined from the predominant periods τ_c *versus* the final magnitude M of the earthquake (b) the relationship between $M_{est}(P_{max}, \Delta)$ determined from P_{max} obtained from first three second P wave seismograms recorded by strong motion instruments and epicentral distance Δ *versus* the final magnitude M of the earthquake (c) the relationship between $M_{est}(P_{max}, \tau_c)$ determined only from peak amplitudes P_{max} and predominant periods τ_c *versus* the final magnitude M of the earthquake (d) the relationship between $M_{est}(P_{max}, \tau_c, \Delta)$ determined from united amplitude and predominant period method *versus* the final magnitude M of the earthquake

the saturation at great magnitude is improved. The linear regression for the relationship among the $M_{est}(P_{max}, \tau_c, \Delta)$, P_{max} , τ_c and Δ leads to

$$M_{est}(P_{\max}, \tau_c, \Delta) = 1.26 * \log P_{\max} + 2.16 * \log \tau_c + 1.34 * \log \Delta + 0.96 \qquad (4.3)$$

The characteristic of earthquake early warning is issue warning message only using first several triggered stations. The denser the seismological monitoring network in the source region, the less the epicentral distance of these stations. It will reduce the impact of amplitude attenuation caused by short hypocentral distances. For short hypocentral distances, equation (4.3) has a simplified version,

$$M_{est}(P_{\max},\tau_c) = 1.14 * \log P_{\max} + 1.97 * \log \tau_c + 4.74 \tag{4.4}$$

which is similar to Xu(2008) and by which we could estimate magnitude only using predominant period and amplitude, regardless of the impact of distances.

Figure 1c shows the relationship between $M_{est}(P_{max},\tau_c)$ determined only from peak amplitudes P_{max} and predominant periods τ_c *versus* the final magnitude M of the earthquake. Estimation standard deviation level is 0.59 magnitude units, which is similar to amplitude method, equation (4.2). Under the less epicentral distances, such as 50-km upper limit, the results of equation (4.4) must be continued to be considered.

5. DISCUSSION AND CONCLUSION

In this study, a new method united predominant period and amplitude is developed, which is based on the achievement of previous studies and definition of the magnitude. We determined the relationship among magnitude, predominant periods τ_c and peak acceleration amplitudes P_{max} observed from the first three seconds of P waves. Compared with predominant period method or amplitude method, the new method has less uncertainty and simultaneously the underestimate at great magnitude is improved, using the NSMP strong motion data.

The result of preliminary numerical validation of new method only using a small number of seismic records is presented in this study. It is unknown that the rationality of large amounts of data. Meanwhile, for short hypocentral distances stations data, a simplified united predominant period τ_c and amplitude P_{max} method is presented. The results of the rationality of large amounts of data and the simplified method must be continued to be considered.

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