BUILDING LONG PERIOD ATTENUATION RELATION WITH SUPPLEMENTARY DATA FROM SEISMIC SOURCE SPECTRUM THEORY

Shi Shuzhong[1] **Shen Jianwen**[2] **Liu Xiliang**[3]

[1] *Senior Engineer, Zhejiang Seismological Bureau，Hangzhou，China*
[2] *Professor, Shanghai Seismological Bureau，Shanghai，China*
[3] *Engineer, Zhejiang Seismological Bureau，Hangzhou，China*
Email: shiszt@126.com, jianwenshen@126.com

ABSTRACT:
A new statistical method is proposed to establish long period attenuation relation by using the supplementary data derived from seismic source spectrum theory. The results show that the attenuation relations of long period ground motion obtained by the method proposed in this article are relatively reasonable and agree well with the actual observation data, while that obtained directly from the strong motion records have poor reliability.

KEY WORDS: Seismic source spectrum, Long period, Acceleration spectra, Attenuation relations

1 INTRODUCTION

Seismic safety evaluation, as a foundation work of ground motion parameter zonation and seismic fortification for major engineering projects, has already attracted great attention of China government. Seismic hazard analysis is an important part of seismic safety evaluation and may be directly influenced by attenuation law of ground motion, so research work of attenuation laws of ground motion is of practical significance.

As the basic means estimating ground motion, the experience statistical method is applied extensively in the engineering seismic field. It’s based on the seismology observation data, so its results are clear and reliable to some extent. But the attenuation laws of ground motion still have uncertainty because the observation data are very limited. For the regions lack of strong motion records, the use of the method is restricted. Even for regions where the strong motion records are relatively abundant, data are still not enough, especially the long period part.

In present strong ground motion records, the quality of long period part of early analogue records is poor; the maximal reliable period bound is only 2-4 seconds [1, 2]. Digital strong earthquake records are better than analogue records, with maximal reliable period bound 7-10 seconds [3-5]. The long period part is relatively weak in the study of attenuation law of ground motion. On the other hand, with the appearing of large quantity of long period structures (long span bridges etc.) in economy construction, it’s urgent to obtain long period design ground motion parameters. So the research of long period attenuation catches more and more attention in engineering seismic field.

To obtain reliable data of long period ground motion, some experts suggest making use of broadband digital earthquake records. Wang Suyun[6], for example, established long period attenuation law with the records of China Digital Seismograph Network. Unfortunately most of these records are that of small or far earthquakes, records of large earthquake and near field needed by engineering seismology are very lack, because the broadband digital earthquake observation is a new technology developed a few years ago, there are few seismic stations with short time of observation. In order to meet the needs of statistics, Yu Yanxiang[7] has applied the method which utilizes magnitude definition to complement records of long period response spectra. This method attracts attention of engineering earthquake field, and the attenuation law of long period ground motion obtained by this method has been used in seismic safety evaluation in China.

Based on the discussion to the method of above magnitude definition, this paper suggests complementing records of long period response spectra by using seismic source spectrum theory, and establishing attenuation relations of long period ground motion by using experience statistical method.

2 THE APPROACH ESTIMATING LONG PERIOD RESPONSE SPECTRA BY SURFACE MAGNITUDE DEFINITION

Foundation item: key scientific research social development projects of Zhejiang Science and Technology Plan （2005C23075）

From the view of research purpose, method and the object, seismology emphasizes long period ground motion in far fields to study seismic source and characteristics of earth medium; while the engineering seismology emphasizes high frequency ground motion in near fields to provide evidence for structure design of earthquake resistance. Each of them has its advantages, so their combination is a development trend in the ground motion research.

Yu Yanxiang[7] analyzed the reliability of long period motion observation records systematically. Collecting strong motion and broadband digital records in the region of Western U.S., and estimating response spectra according to the definition of the magnitude, he established attenuation relations of long period acceleration response spectrum by using statistical regression method. This method opens up a new way for the research of long period ground motion.

This method is put forward firstly by Academician Hu Yuxian [8], as follows is its keystone:

(1) The surface wave magnitude is defined by the maximal displacement of ground motion and corresponding period, as follows is its definition:

$$M = \lg(\frac{A}{T})_{\max} + 1.66\lg\Delta + 3.3 \qquad (2.1)$$

where, A is the maximal displacement, T is corresponding period, M is surface wave magnitude, Δ is epicenter distance. The period is originally regulated as $T = 20s \pm 3s$. For practical use, the range of epicenter distances and periods have been extended to $\Delta \geqslant 1^{o}$ and $T \geqslant 3s$.

Base on the definition of magnitude, if magnitude and distance are given, the ground motion displacement of corresponding period can be evaluated.

$$\lg(\frac{A}{T})_{\max} = M - 166\lg\Delta - 3.3 \qquad (2.2)$$

If $(A/T)_{max}$ is approximately equal in periods T=3～20s, the surface wave displacement can be estimated according to different periods. So the maximal surface wave displacement of given period T can be denoted:

$$A_{max}(T, M, \Delta) = T \cdot 10^{M-1.66\lg\Delta-3.3} \qquad (2.3)$$

the maximal displacement of ground motion at the given period T ($T \geqslant 3s$), magnitude M and distance($\Delta \geqslant 1°$) can be estimated approximately by formulary (2.3).

(2) To get response spectra from the maximal displacements of corresponding periods, it is a key problem to determine the amplification rates between the maximal displacements of different periods and the response spectra. Because the relations between the maximal displacements and response spectra of different periods are very complex, the amplification rates is artificially taken 2.5 for period 2s and less than 2s, fixed as 1 for period 20s, and adjusted for the periods 2s to 20s by linear interpolation according to logarithm period. So amplification rate AMP can be calculated with the following formula:

$$AMP = 1 + 1.5 * \frac{\lg 20 - \lg T}{\lg 20 - \lg 2} \qquad (2.4)$$

(3) The maximal displacement A_{max} multiplies amplification rate of corresponding period to get the displacement response spectrum Sd（T）of period T.

(4) According the relation between the displacement response spectrum and absolute acceleration response spectrum, acceleration response spectrum Sa (T) can be calculated by formula (2.5).

$$Sa(T) = \left(\frac{2\pi}{T}\right)^2 Sd(T) \qquad (2.5)$$

In fact, the definition of surface wave magnitude reflects attenuation law of peak displacement around period of 20s, and has been applied for the past decades. So the relation between ground motion and magnitude-distance represented by surface magnitude has sound foundation.

However this method includes a lot of simplification and supposition contains many subjective factors. Such as formula (2.4) regulates the displacement amplification rates of different periods, actually regulates the shape of response spectrum. Because AMP in formula (2.4) is independent of magnitude and distance, the shape of response spectra is also independent of magnitude and distance. Meanwhile, the supposition that the amplification rates may use linear interpolation between 2s and 20s according to logarithm periods lacks reasonable demonstration. On the other hand, because the surface wave magnitude is measured by teleseismic surface wave, above formula can only be applied for evaluation of maximal displacement of distance larger than 100 km and periods longer than 3s, the evaluation of displacement at near field concerned by engineering is obviously insufficiency.

3 COMPLEMENTING GROUND MOTION DATA WITH SEISMIC SOURCE SPECTRUM THEORY

Seismology of strong motion studies ground motion properties by fracture mechanics and wave theory. This method can overcome, in some degree, the limit of lack of strong motion data, and is relatively mature for the research of long period motion [9-11]. So it becomes an active research field at present.

Since Brune[12] put forward the disk rupture model based on dislocation theory in 1970, the researches of seismic wave spectrum have greatly developed. Among which the application of ω^2 seismic source spectrum model is the most extensive. Boore[13] brought forward the stochastic motion model to forecast ground motion parameters in 1983. The research result coincides well with the actual observation data. The method has been applied successfully in the research of strong motion in other regions, especially in the Eastern U.S. [14, 15] where strong motion data is lack. Although it has been one of the preferable ways to establish attenuation law of ground motion, it’s a pity if abundant observation records are abnegated. Therefore it is a natural thinking to establish attenuation laws of ground motion using statistical method, by generating data with stochastic seismic source spectrum model to complement ground motion records of long periods.

According to the stochastic motion model of seismic source spectrum brought forward by Boore[13], this paper uses the present model parameters in California, estimates parameters of long period motion in California to complement ground motion records for the magnitude-distance combinations where the records are absent, then establishes long period attenuation law for the Western U.S..

3.1 Basic ideas and methods

The basic ideas using stochastic motion model based on seismic source spectrum to estimate ground motion parameters is: firstly, the seismic source spectrum of acceleration is calculated by seismic source spectrum theory; secondly, the stochastic vibration theory is combined to synthesize time histories; finally, ground motion parameters that structure seismic analyzing needs are determined. The concrete process is as follows:

(1) Suppose strong motion is a narrow-band Gaussian process, use trigonometric series method to build a smooth Gaussian process with mean value zero.

$$X(t) = \sum_{k=0}^{n-1} F(\omega_k) e^{i\omega_k t} \tag{3.1}$$

where $F(\omega_k) = A(\omega_k) e^{i\phi_k}$, $A(\omega_k)$ is Fourier amplitude spectrum, ϕ_k is stochastic Fourier phase spectrum。

(2) Multiply the envelope function of time history W (t) to acquire non-stationary stochastic motion S(t):

$$S(t) = X(t) \cdot W(t) \tag{3.2}$$

(3) Use Fourier transform to calculate amplitude and phase spectra of non-stationary stochastic process, adjust its Fourier spectrum until the mean value of amplitude spectrum equals to 1.

(4) Determine the Fourier spectrum of ground motion by using Brune seismic source spectrum model, as spectra of targets

(5) Multiply spectrum of targets by Fourier amplitude spectrum of stochastic time history, and use the inverse Fourier transform to build time history.

(6) Using time history to calculate motion parameters that structure seismic analyzing needs, such as peak acceleration and response spectra.

3.2 Determine parameters of seismic source spectrum

The key of the stochastic motion model based on seismic source spectrum is to use seismic source spectrum to determine Fourier amplitude spectrum of ground motion. This paper applies the present research results to determine Fourier amplitude spectrum of ground motion by Brune seismic source spectrum model which is used

extensively in the research of seismology.
Acceleration seismic source spectrum based on Brune point source model can be denoted:

$$A(f) = C * S(f) * D(f) * R(f) \tag{3.3}$$

where C is seismic source effect coefficient,

$$C = R_{\theta\phi} \cdot F \cdot H / 4\pi\rho\beta^2 \tag{3.4}$$

where $R_{\theta\varphi}$ is radiation coefficient relate to earthquake source mechanism (take 0.63 for shear wave), F is enlarge coefficient of freedom surface (2.0) , H is amplitude coefficient of two horizontal components (the geometry mean value equals to 0.71), ρ is medium density and β is velocity of shear wave in seismic source zone, take 2.7g/cm^3 and 3.2km/s[16] for California.

$S(f)$ is Brune seismic source function,

$$S(f) = (2\pi f)^2 M_0 / \left[1 + (f / f_c)^2\right] \tag{3.5}$$

where f_c is corner frequency. In Brune model, for $f < f_c$, the spectra attenuate at the rate of f^2 along with the increase of the periods. The corner frequency can be calculated by stress drop σ and seismic moment M_0, the statistical experience formula is:

$$f_c = 4.9 * 10^6 \beta(\sigma M_0)^{1/3} \tag{3.6}$$

where M_0 is seismic moment (dyne*cm), σ is stress drop (bar), β is shear wave velocity(km/s).
Seismic moment reflects the total moment caused by the dislocation of the seismic fault. Because the concept of seismic moment was put late, it was rarely given in earlier seismic catalogues. Generally, it can be calculated by the experience statistical relation from other magnitudes. In California, the seismic moment can be calculated with the formula $\lg M_s = 1.5M_0 + 16.05$ [17].
Stress drop σ is an important parameter of seismic source spectrum model. In Brune model, stress drop controls high frequency components. Based on the analysis of a mass of strong earthquake records in California, seismic source spectra calculated theoretically agree with the actual seismic records, when the stress drop equals to 100 bars. So this paper’s stress drop takes 100 bars.
$D(f)$ is high frequency cut-off function, it’s related commonly to the characteristics of strong-motion seismograph, focal process and propagation media. In California,

$$D(f) = e^{-\pi f k} \tag{3.7}$$

where k, in general, is between 0.35 and 0.55. According to the research result of Silva etc., when k takes 0.44, high frequency characteristics of the motion on bedrock sites will be simulated preferably in California.
$R(f)$ is the distance attenuation function,

$$R(f) = \frac{1}{R} \cdot \exp(-\pi f R / Q(f)) \tag{3.8}$$

The first item 1/R denotes geometry attenuation, where R is focal distance. The second item $\exp(-\pi f R / Q(f)$ denotes inelastic attenuation of the crust media, where Q is quality factor of the propagation media and related to frequency.
According to the seismic wave propagation theory, besides direct wave, the seismic waves include the reflected wave and surface wave with the increasing of the distance, so the attenuation process of seismic wave will be slow. Two or three segments attenuation models are used to denote the attenuation of ground motion along with distance. In fact, the determination of geometry attenuation is related to the value of inelastic attenuation coefficient. Different inelastic attenuation coefficients are taken for different geometry attenuation models; the results are similar for attenuation curves of motion along with distance[17]. In this paper, in the light of the results of ground motion attenuation in California given by Chin etc., geometry attenuation takes two segments attenuation model. The geometry attenuation item takes 1/R for R<40km, and $1/\sqrt{R}$ for R⩾40km, the quality factor of corresponding inelastic attenuation coefficient takes $Q(f) = 360 f^{0.31}$.

3.2 Determination of intensity envelope function

When using stochastic motion model to evaluate design parameters of ground motion, the motion strength envelope function is an important factor. In this paper, the motion strength envelope model adopts the model proposed by Boore (1983):

$$w(t) = at^b e^{-ct} H(t) \tag{3.9}$$

where a, b and c are the parameters of the model, H (t) is unit step function,

$$a = (e / \varepsilon Tw)^b \tag{3.10}$$

$$b = -\varepsilon \ln \eta / (1 + \varepsilon (\ln \varepsilon - 1)) \tag{3.11}$$

$$c = b / \varepsilon Tw \tag{3.12}$$

where, ε is the control coefficient of strong stationary segment of time history, η is the strength attenuation coefficient, T_w is the total time of ground motion. $T_w = 1/2f_c$, f_c is the corner frequency of seismic source spectrum. According to the research results, when $\eta = 0.05$ and $\varepsilon = 0.02$, the intensity envelope function agrees with the actual seismic record in California.

4 BUILDING ATTENUATION RELATIONS OF LONG PERIOD GROUND MOTION

This article collected strong motion acceleration records of free bedrock sites in the Western U.S. and broadband digital seismic records in California. Limited by space, the detailed introduction is omitted.

In order to avoid acceleration response spectra evaluated by seismology method accounts for too large percentage, response spectra will only be estimated for the sections lacking of actual earthquake records, namely, the data is complemented only for the magnitude – distance combination points where actual earthquake record is zero. To consider the stochastic characteristic of the ground motion, two ground motion response spectra will be provided at each empty point.

Because the sources of seismic data, instrument performance and output mode of records are different, the records have different reliable period range for long period motion. This article adopts subsection statistical method for different frequency domains to build the attenuation relation in the Western U.S.. Because the sampling rate of the broadband digital records is low, acceleration response spectra are unreliable for high frequencies. Therefore, the strong motion records are selected only when attenuation relation of high frequency response spectrum (0.04-1.0s) are built. Both the strong motion acceleration records and the wideband digital records are used for 1.0-2.0 seconds. The wideband digital seismic recording and the acceleration response spectra estimated by seismology method are adopted for the low frequency band (2-15s).

Figure 1 shows the attenuation curves of the acceleration response spectra in the Western U.S.. On the whole, the attenuation curves are smooth and continual and have no obvious sudden changes for different magnitudes and different distances. Such kind of continual curves prove, from one side, that the data is intrinsically reasonable.

Figure 2 shows that horizontal acceleration attenuation relations of periods T=10s contrast to the actual long period observation data. We can see the results of long period response spectrum attenuation relations obtained by this article are matched well with the actual observation records. This indicated that the long period response spectra complemented by the seismic sources spectrum theory are of certain rationality and reliability.

Figure 3 shows the attenuation relations of acceleration response spectra. Solid lines are the results given by the method introduced in this paper, and dashed lines are obtained only based on strong motion records. The figure shows that, for periods larger than 1s, the response spectra become smaller as periods grow larger. For dashed lines in near field (R=10km), the drop rates of large earthquakes are obviously larger than that of smaller earthquakes. And as the periods grow larger, the response spectra of different magnitudes have the trend of convergence. Such results are in contradiction with our general knowledge, namely, the long period ingredients should get richer along with the increase of magnitudes. The larger the magnitude, the richer the long period ingredients. For dashed lines at R=100km, the drop rates of the smaller earthquakes become slow along with the increasing of the periods, even appear platform and plump. It indicates that the signal-to-noise ratios of small earthquakes are very low in the long period part of strong motion records; the noises control the long period ingredients of the motion. Thus it can be seen, the long period motion attenuation relations only based on strong motion is unreliable; while long period acceleration response spectrum attenuation relations acquired by this article are relatively reasonable.

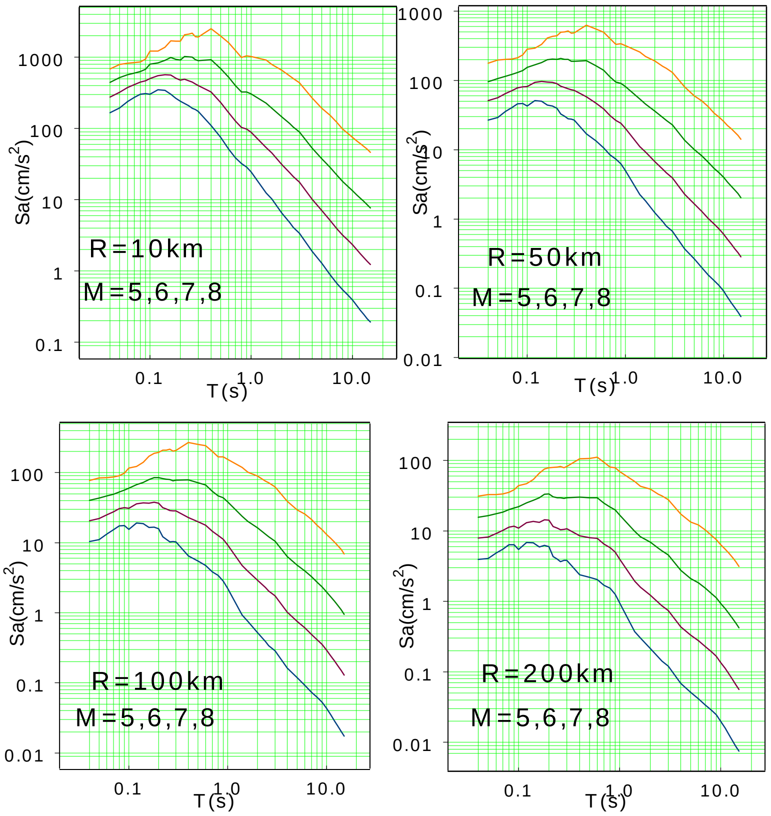

Fig. 1 Attenuation curves of absolute acceleration response spectra in the Western U.S.

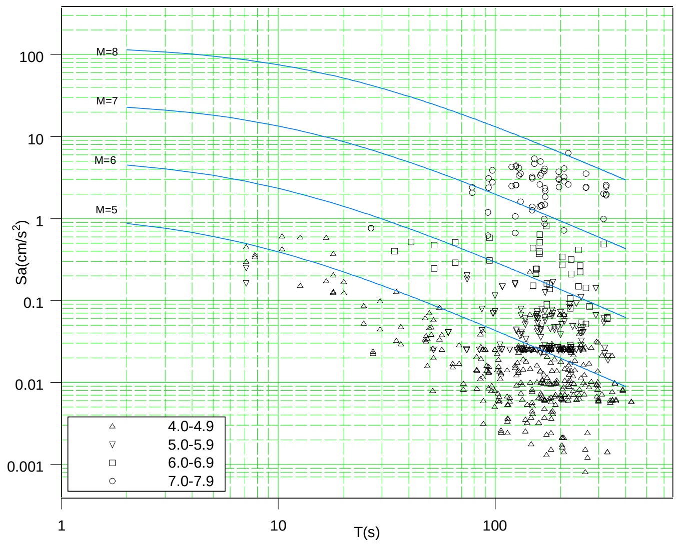

Fig. 2 Comparison of absolute acceleration response spectrum attenuation curves of period 10s with the observation data

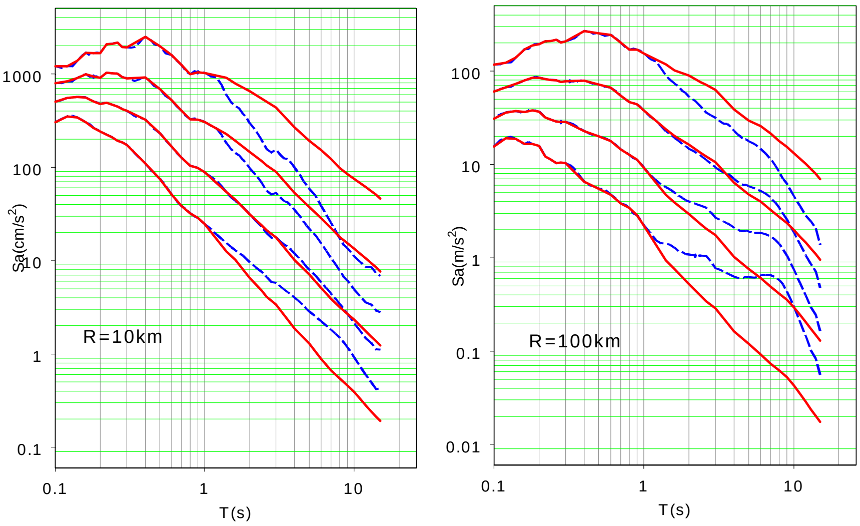

Fig.3 Comparison of acceleration response spectrum attenuation relations of this paper and that based only on strong motion and broadband digital seismic records

M＝5、6、7、8

5 CONCLUSION AND DISCUSSION

In this article, we analyze the insufficiency of the method using magnitude definition to supplement long period response spectra and establishing long period motion attenuation relations, propose a new way supplementing long period motion data based on seismic source spectrum theory, and establishing the attenuation laws of long period motion by empirical statistics method. From research and analysis of the long period motion attenuation relations in the Western U.S., we show that the method proposed in this article is relatively reasonable, concordant with the actual records; while the long period attenuation relations obtained directly by strong ground motion record is not reliable.

The lack of seismic data is one of the bottleneck factors to apply empirical statistics method for establishing ground motion attenuation relations, but this condition couldn't be improved at present or even for a long time. Therefore, it will be a good technical thinking to supplement the data with the seismology method. Along with the unceasing development of seismology research, the method to study long period ground motion attenuation relations with the seismology model will be improved further, more reasonable and reliable long period design ground motion parameters will be provided for the engineering seismic resistance design.

REFERENCES

[1] Trifunac, M.D. (1993). Long period Fourier amplitude spectra of strong motion acceleration. *Soil Dynamics and Earthquake Engineering* **12:4,** 363-382

[2] YU Yanxiang, HU Yuxian. (2000). Discussion on the long-period design spectrum in "Aseismic Design Code for Buildings" of Shanghai, *Earthquake Engineering and Engineering Vibration* **20:1,** 25-34

[3] Chiu, H. C. (1997).Stable baseline correction of digital strong motion data, *Bull. Seism. Soc. Am.* **87:4**, 932-944

[4] Zhou Yongnian, Zhang Wenbo, Yu Haiying. (1997). Analysis of long period error for the digital strong motion seismograph record (in Chinese). *Earthquake Engineering and Engineering Vibration* **17:2,** 1-9

[5] Wang Suyun, Yu Yanxiang, Lu Hongshan. (1998). Study of characteristics of long-period ground motion response spectra by using broad-band records of the Chinese Digital Seismograph Network (in Chinese). *Acta Seismologica Sinica* **11:5**, 557-564

[6] Wang Suyun, Yu Yanxiang, Gao Ajia, Yan Xiujie. (2000). Determination of ground motion attenuation relations in Chinese. *Earthquake Research in China* **16:2,** 99-106

[7] Yu Yanxiang., (2002). Study on attenuation relationships of long period ground motions (in Chinese). Institute of Geophysics, China Earthquake Administration.

[8] Hu Yuxian, Yu Yanxiang. (2000). A combine approach of evaluation of long period design spectrum, Proc.12th WCEE, Upper Hutt, New Zealand.
[9] Trifunac M.D. (1994). Fourier amplitude spectra of strong motion acceleration: extension to high and low frequencies.*Earthquake Engineering and Structural Dynamics* **23,** 389-411
[10] Du Xiuli Chen Houqun. (1995). Study of long period strong motion amplitude spectra in near field (in Chinese). *Journal of Natural Disasters* **4:2**, 23-30
[11] Lu Hongshan. (1998). Estimation of long period response spectrum by semi-experiment and semi-theory (in Chinese). *Fifth national earthquake academic conference paper* 43-46
[12] Brune, J.N. (1970). Tectonic stress and spectra of seismic shear waves from earthquakes. *Journal of Geophysical Research* **96:7,** 4997-5009
[13] Boore, D.M. (1983). Stochastic simulation of high frequency ground motions based on seismological models of the radiated spectra. *Bull. Seism. Soc. Am.* **73:6**, 1965-1894
[14] Beresnev A.Igor, G. M. Atkinson. (1997). Modeling finite fault radiation from the ω^2 spectrum. *Bull. Seism. Soc. Am.* **87:1**, 67-84
[15] Atkinson, G. M. (2001). An alternative to stochastic ground motion relations for us in seismic hazard analysis in eastern north America. *Seismological Research Letters* **72:2**, 299-305
[16] Atkinson, G.M., Walt Silva. (1997). An empirical study of earthquake source spectra for California earthquakes. *Bull. Seism. Soc. Am.* **87:1,** 97-112
[17] Hanks, T.C. The Lompoc. (1979). California, earthquake (November 4, 1927; M equal to 7.3) and its aftershocks. *Bull. Seism. Soc. Am.* **69:2**, 451-462
[18] Hanks, T.C., Robin K. McGuire. (1981). The character of high frequency strong ground motion, *Bull. Seism. Soc. Am.* **71:6,** 2071-2095
[19] Boore, D.M., G.M. Atkinson. (1987). Stochastic prediction of ground motion and spectral response parameters at hard rock sites in eastern North America. *Bull. Seism. Soc. Am.* **77:2,** 440-467
[20] Anderson, John G. (1997). Seismic energy and stress drop parameters for a composite source model, *Bull. Seism. Soc. Am.* **87:1**, 85-96
[21] Silva, W., R. Darragh. (1995). Engineering characterization of earthquake strong ground motion recorded at rock site, Report TR-102261, California: Electric Power Research Inst.
[22] Chin, B.H. and K. Aki. (1991). Simultaneous study of the source, path and site effects on strong ground motion during the 1989 Loma Prieta earthquake: preliminary result on Pervasi, *Bull. Seism. Soc. Am.* **81:5,**1859-1884
[23] Saragoni, G.R. and G.C. Hart. (1974). Simulation of artificial earthquakes. *Earthquake Engineering and Structural Dynamics* **2**,249-267