ESTIMATION OF THE LOW-FREQUENCY *Q*-VALUE (BELOW 1HZ)

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ABSTRACT :

Studies that estimate low-frequency *Q*-value are very scarce. In most of them estimations are made by eye or by trial and error, without any systematic inversion approach. In this study, we propose nonlinear waveform inversion methodology and estimate low-frequency *Q*-value from the observation data. Results will help us to improve accuracy of the numerical simulations of the long-period waves. The inversion methodology used in this study consists of three steps. Step 1: Compile 1-D velocity models for every observational site in the target area, source region of the 2000 West Tottori earthquake. Step 2: In order to test velocity models, we compared synthetic and observed waveforms. Step 3: Use grid search method to estimate *Q*-value and linear inversion to estimate corrections to the source and site effects. Results of inversion show that estimated value in low-frequency range, Q = 80-180, agrees well with the results of spectral inversion in high frequency range, in case of a realistic ray-theory geometrical spreading is considered.

KEYWORDS: Long-period ground motions, *Q*-value, waveform inversion, receiver function

1. INTRODUCTION

Studies that estimate low-frequency *Q*-value are very scarce. In most of them estimations are made by eye or by trial and error, without any systematic inversion approach. In this study, we propose to use nonlinear waveform inversion methodology and estimate low-frequency *Q*-value from the observation data. Results will help us to improve accuracy of the numerical simulations of the long-period waves that become important in recent years.

In order to estimate parameter of the path attenuation, *Q*-value, first, it is necessary to calculate and remove effect of the geometrical spreading. In our previous studies (Petukhin et al., 2004), for the high-frequency range, f > 1Hz, geometrical spreading was calculated for realistic 3-D velocity model using ray theory. Advantage of the employing ray theory is that it allows simple linear inversion to estimate *Q*-value from the observation data. However, this approach is not valid for low-frequency range, f <1Hz, where the wave effects become essential and the ray approach doesn't work.

For the estimation of *Q*-value we will use self-consistency principle. According to this principle, to reduce additional errors related to a slight difference of simulation models, for the estimation (inversion) of some parameter, it is necessary to use the same model that will be used later for strong ground motion simulation. In the hybrid Green function method, the discrete wave-number method (Bouchon, 1981) is frequently used for the low-frequency waveform simulation. Moreover, this method is fast and for this reason is better for non-linear inversion problem, which require many direct simulations. For these reasons we prefer fast and accurate discrete wave-number method for the estimation of the low-frequency *Q*-value. It is easy demonstrate that for 1-D velocity model, the discrete wave-number method simulates the same waveform as a more accurate finite-difference method. The only problem is to select data for which the 1-D velocity model is valid.

2. METHOD

In order to estimate low-frequency *Q*-value from the observational data, at first we form dataset of records of earthquakes in the target area, source area of the 2000 West Tottori earthquake, Japan, for which the F-net (http://www.hinet.bosai.go.jp/fnet) estimations of the source mechanism are available. Then we selected records of Kik-net (http://www.kik.bosai.go.jp) at the rock sites only, for which basin effects can be neglected and the 1-D approximations of the shallow velocity structure are reliable. Finally we applied low-frequency filter, 0.5-1Hz, and selected records having small low-frequency noise.

Q-value estimation procedure consists of 3 steps:
Step 1. Compile 1-D velocity models for every observational site in the target area. For this, in depth range more than 3km we use velocity model estimated by the regional seismic exploration studies. From another side, for shallow depth, less than 3km, we will use results of the receiver function inversion (Miyakoshi et al., 2003) and borehole logging data.
Step 2. To test results of the velocity modeling, it is necessary to compare synthetic waveforms with the observed waveforms. On this step, we apply 0.5 – 1Hz filter both to the observed and simulated record, compare them, and finally selected only records that have similar waveforms.
Step 3. By comparing observed and simulated waveforms, make nonlinear inversion for the *Q*-value and for the amplitude correction coefficients of source and site effects (*Q*-value - by grid search, correction coefficients – by linear inversion).

Table 1 compiles in a brief form all settings necessary for simulations, i.e. velocity model, source mechanism, source location, simulation frequency range, rise time, and simulation methodology. Velocity modeling was already described above. Source mechanism and seismic moment were fixed according to the F-net estimations. It is necessary to notice that F-net has estimated source mechanism parameters using surface waves in the frequency range 0.02 – 0.07Hz, but in this study we will use a higher frequency range, 0.5 – 1Hz. Due to this, a slight errors of source parameters, especially of the effective seismic moment, are possible. Coordinates of epicenters, reported by the JMA and F-net are practically the same, however the hypocenter depth, estimated by JMA, has better accuracy. For this reason we use coordinates estimated by F-net and hypocenter depth estimated by JMA. According to our previous experience (Petukhin and Miyakoshi, 2006), such combination gives better waveform fit. Rise time was calculated according to the formula of Somerville et al. (1999). Simulation method is fast and accurate discrete wave-number method of Bouchon (1981).

Table 1 Parameters for the waveform simulation.

PARAMETER	SETTINGS
Shallow velocity model	Receiver function inversion (Miyakoshi et al., 2003)
M_0, strike, dip, rake	F-net (0.02 - 0.07Hz)
Depth of the source	JMA
Frequency range	0.5 - 1Hz
Trise, sec	$2.03e\text{-}9*(1e7M_0)^{1/3}$ (Somerville et al., 1999)
Waveform simulation method	Discrete wave-number method (Bouchon, 1981)

3. ONE-DIMENSIONAL VELOCITY MODELS FOR TARGET SITES

In order to calculate synthetic waveforms we compiled 1-D velocity models for each target site in the target area. Velocity models were compiled from three segments. (1) Deep segment - the same regional crustal model (H > 3km, upper crust and lower crust, Ito et al., 1995) was assumed for all target sites. (2) Shallowest segment - for all Kik-net sites well-logging velocity models are available up to 100 - 200m depth; we used these data without changes. (3) Intermediate shallow segment - for the depth range in between two segments above, i.e. 0.1 – 3km depth, we used results of the high-frequency receiver function inversion of Miyakoshi et al. (2003).

Shallow velocity models for all sites used in this study are shown in Figure 1.

Table 2 shows example of the total 1-D velocity model for site TTRH02, used to calculate synthetic waveforms. *Q*-values shown by the italic numbers are adjusted during the nonlinear inversion in the following way. We assumed Q_S value for the upper crust (underlined in Table 2) as a reference value. During the inversion, we changed this value in the range from 10^1 to 10^4. Other *Q*-values were changed proportionally to the reference *Q*-value with the coefficient of proportionality *k*.

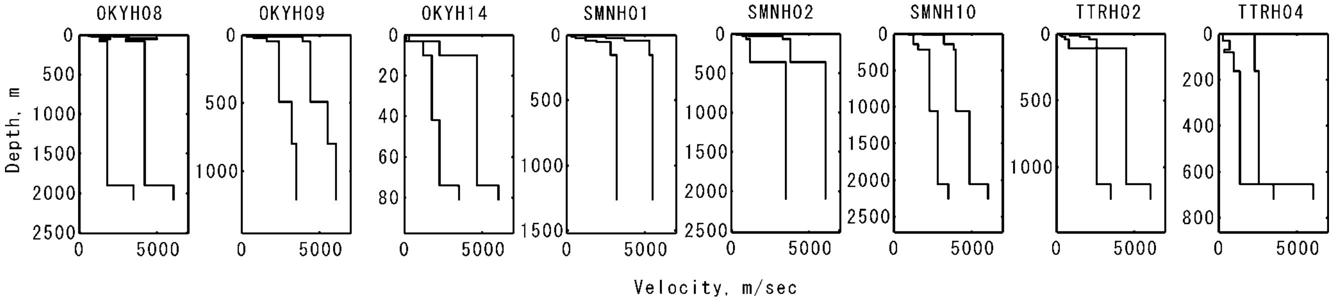

Figure 1 Shallow velocity structures for all target sites in the Tottori region.

Table 2 Example of the 1-D velocity model for TTRH02 site.

	H, m	V_P, m/s	V_S, m/s	ρ, g/cm^3	Q_P	Q_S
Kik-net borehole logging data	0	860	210	1680	26	13
	11	1500	340	1930	44	22
	20	2100	560	2100	74	37
	42	2600	790	2210	104	52
Receiver function inversion	107	4460	2580	2530	344	172
	1128	6050	3500	2730	*k550*	*<u>k270</u>*
Regional crustal model	16000	6600	3820	2800	*k800*	*k400*
	38000	8000	4620	3100	*k1000*	*k500*

4. GRID ANALYSIS OF *Q*-VALUE AND FINAL DATA SELECTION

Q-value is the only nonlinear parameter used in the inversion. For this reason it is convenient to use grid search method to invert *Q*-value. In this step, we simulated waveforms for grid of 20ty *Q*-values in the logarithmic scale, in the range of values 10^1 – 10^4. Than, all waveforms were plotted to the same plot and compared with the observed waveforms. For each waveform (for each grid *Q*-value), residual value was calculated according to Eqn. 5.2 below and printed near the waveform. Figure 2 shows an example of the grid analysis.

As expected, with increasing of *Q*-value, amplitude of simulated waveforms increases from a value much smaller than the amplitude of observed waveform, and than exceeds it. This is also reflected by the residual values. Plot and *Q*-value corresponding to the minimum residual, are the best fit.

In most cases, variation of the *Q*-value in the reasonable range of values, 10^1 – 10^4, was enough to get acceptable agreement between amplitudes of synthetic and observed waveforms. In some cases, it was impossible. We rejected such records and selected only records that have both similar waveforms and similar amplitudes of waves on two horizontal components (see example in Figure 2). Total number of the finally selected records is 102; see Figure 3.

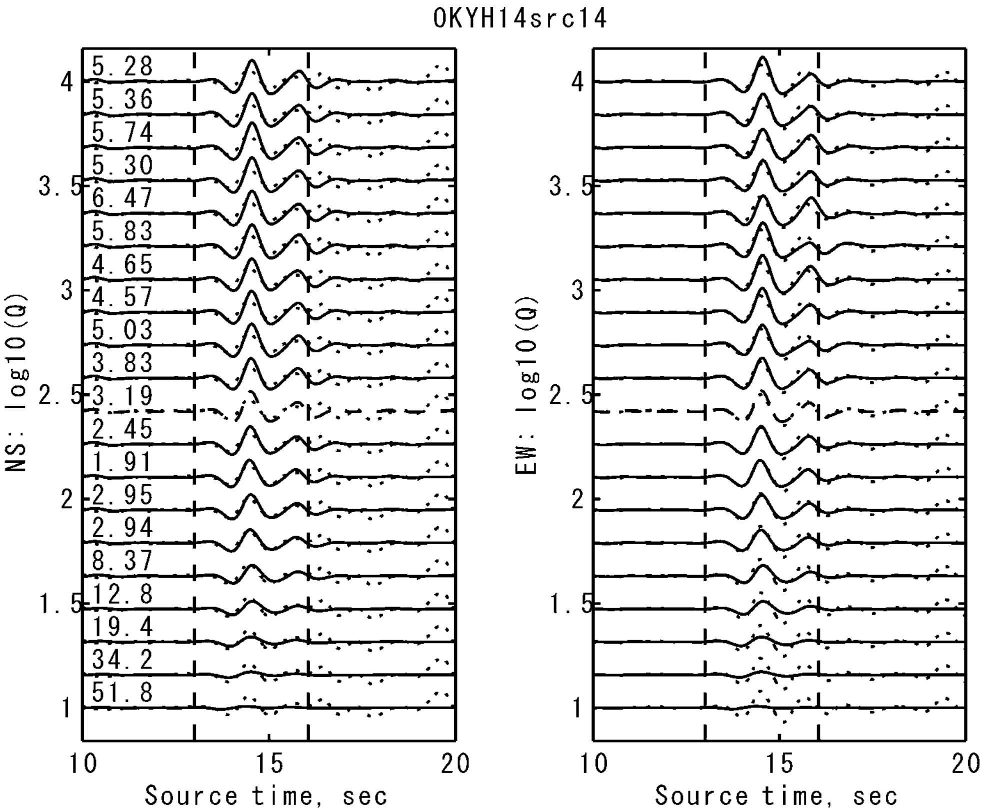

Figure 2 Example of the grid analysis for site OKYH14. Solid line – simulated waveform, dotted line – observed waveform, dashed line – waveform that corresponds to the reference *Q*-value in Table 2. Vertical dashed lines indicate segment of the “direct wave” having similar simulated and observed waveforms.

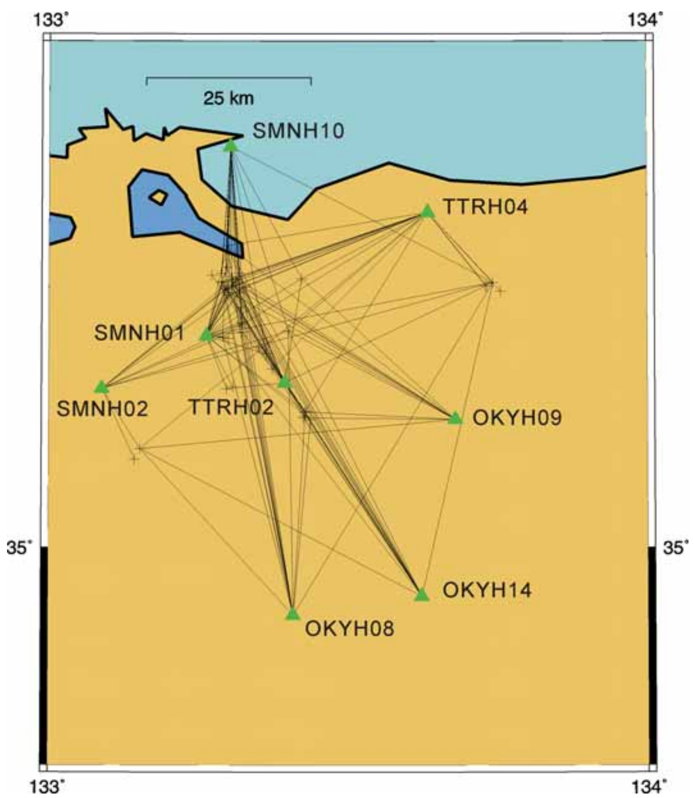

Figure 3 Target area of the 2000 West Tottori earthquake and selected source-site paths (lines).

5. INVERSION PROCEDURE

Flow-chart in Figure 4 illustrates inversion procedure, used in this study. Synthetic records are simulated by the discrete wave-number method (Bouchon, 1981), using site-specific 1-D velocity models above, and using source mechanism and seismic moment parameters estimated by the F-net. Records are synthesized for a set of grid *Q*-values in 10^1 – 10^4 range. Than, both synthetic and observed records are filtered by 0.5 – 1Hz band-pass filter, source and site amplitude correction coefficients are applied, and residual value is calculated according to Eqn. 5.2 below. To estimate correction coefficients, additional inversion is applied for each grid *Q*-value. Linear inversion method is used in this case. Grid *Q*-value that corresponds to the minimal residual value is assumed as the result of inversion.

There are two reasons for which we need to apply correction coefficients to the source and site effects. First is an error of the seismic moment due to difference in frequency ranges, used for estimation of the seismic moment and for the waveform simulation (see above). The second is an error of the shallow velocity structure, due to the difference between the *P*-wave modeling, used for receiver function inversion, and the *S*-wave modeling, used for waveform simulation (slight model inconsistency, see above). Amplitude correction coefficients are applied according to the formula below.

$$Obs = S \cdot Syn(Q) \cdot G \,, \qquad (5.1)$$

where *Obs* - observed waveform, *Syn* - synthetic waveform, *S* - source correction coefficient, *G* - site correction coefficient. After taking logarithm, such simple formula allows linear inversion; we will take advantage of this. In order to calculate residual for record from source *i* at a site *k*, we used next equation:

$$Res_{ik} = \log\left(\int Obs^2_{ikNS}\,dt\right) - \log\left(\int Syn^2_{ikNS}\,dt\right) + \log\left(\int Obs^2_{ikEW}\,dt\right) - \log\left(\int Syn^2_{ikEW}\,dt\right) - 4\log S_i - 4\log G_k \,. \qquad (5.2)$$

This is simply the sum of differences between logarithm values of intensities of the observed and simulated waveforms, on the NS and EW components. Intensities are calculated within the “direct wave” phase, estimated like in the example in Figure 2. Finally, for each grid *Q*-value we calculated total RMS residual over all selected records, and then looked for its minimum, in order to select best *Q*-value

$$\sum Res^2_{ik} \to \min \qquad (5.3)$$

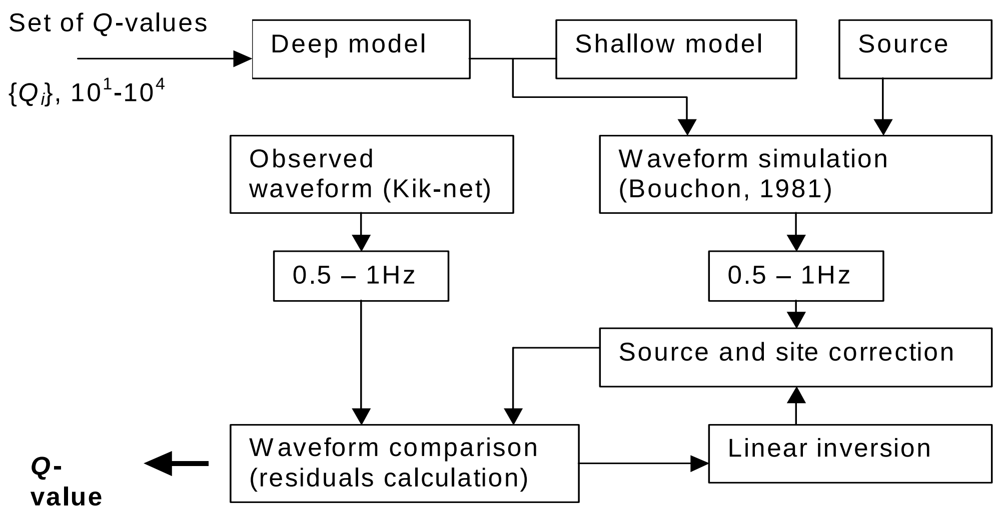

Figure 4 Flow-chart of the inversion procedure.

6. RESULTS OF INVERSION

In order to detect statistically if the shift of source or site effect exists or not, it is necessary to have at least 4-5 data records per each source or site. That is not the case for the sources; many sources in our data set have only 1-2 records. For this reason, we divided sources into groups, selecting sources having four and more data and grouping all other sources into one group “Others”. During the inversion, in order to remove trade-off between the source and the site effects, we assumed the “Others” group of sources as the reference group of data and fixed S = 1.0 for this group.

Figure 5 shows result of inversion – plot of the total RMS residual vs. grid Q-value. Clear minimum point at Q = 180 is observed in this case. However, RMS residual values are almost the same for values Q > 80. We took formal estimate Q = 180 as the inversion result and analyzed residuals of individual records.

Figure 6 shows average dependence of residuals vs. hypocenter distance. An inclined trend that was clearly observed in a preliminary inversion without source and site corrections disappears. Moreover, residuals separately for each source and site, see Tables 3 and 4, also do not show any source or site related anomalies. We conclude that the inversion produces results free from the trade-off between the Q-value and the source and/or site effects.

Tables 3 and 4 show results of inversion for the S and G coefficients in case of Q = 180. A large value, log$S$ = -0.6, is observed for source number 16, which indicates possible shift of the estimation of the seismic moment for this source. However, total number of data recorded for this source is 4, which is small and cannot shift results of Q-value inversion significantly. Other values of S and G coefficients are small, 1.3 – 1.5 in average, that means that estimations of the seismic moment by F-net and the shallow velocity structures by the receiver function inversion are accurate enough (value 1.5 is a typical value of residuals for seismic amplitude data).

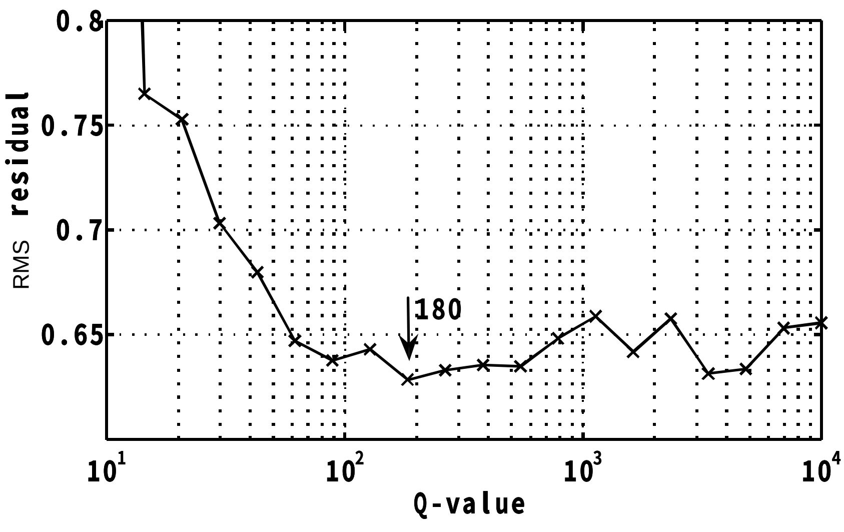

Figure 5 Results of second step of inversion: dependence of the RMS residuals vs. grid Q-values (crosses). Arrow shows formal inversion result, corresponding to minimum of the RMS residuals.

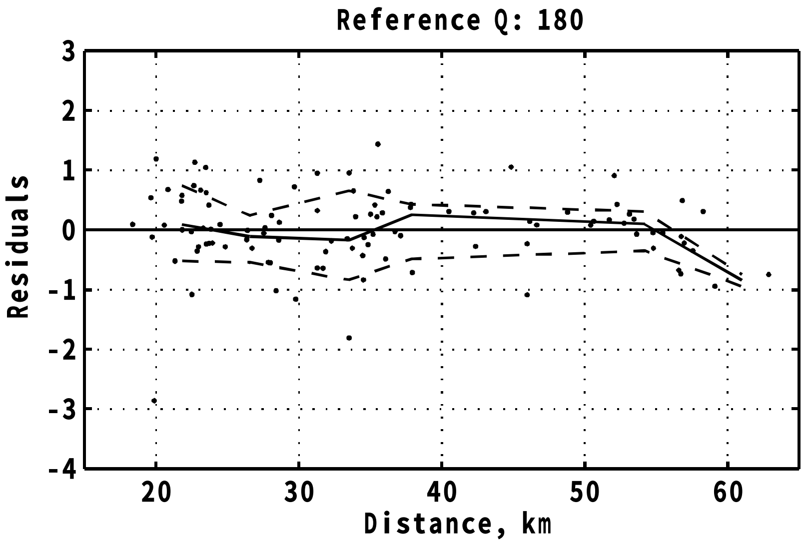

Figure 6 Second step of inversion: average dependence (solid line) of residuals (dots) vs. distance. Dashed lines show standard deviation of residuals. Notice zero average trend of residuals.

Table 3. Inversion results: source correction coefficients *S*.

Source number i	1	7	8	12	14	16	21	28	39	48	52	58
Correction $\log S_i$	0.16	-.19	0.06	-.05	0.03	-.60	-.18	-.21	-.10	0.03	0.04	-.11

Table 4. Inversion results: site correction coefficients *G*.

Site	OKYH08	OKYH09	OKYH14	SMNH01	SMNH02	SMNH10	TTRH02	TTRH04
Correction $\log G_k$	0.26	0.15	-0.12	0.04	0.07	0.28	0.27	0.22

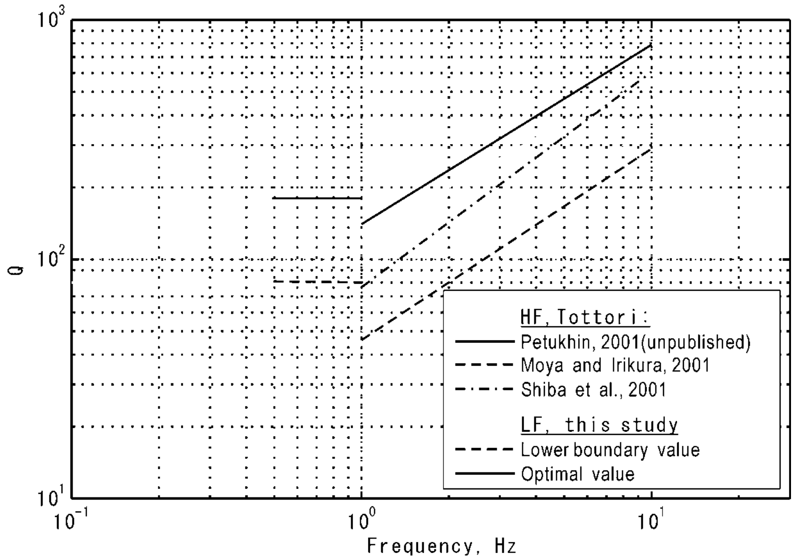

Figure 7 Comparison of the results of this study (low-frequency *Q*-value, waveform inversion) with other studies (high-frequency *Q*-value, spectral inversion). For details, see comments in the text.

7. COMPARISON OF THE RESULTS WITH OTHER STUDIES

Figure 7 shows comparison of the results of this study with other studies (high-frequency *Q*-value, spectral inversion). In this study, we estimated low-frequency *Q*-value employing the waveform inversion method.

This is the first study of this kind. For this reason, for comparison we chose results of the spectral inversion, which are valid in the high-frequency range. The question is if they are agree at a common frequency point f = 1Hz, or not.

For this study results, for low-frequency range, we plotted both formal estimate Q = 180 and minimal valid estimate Q = 80. For high-frequency range, we plotted several spectral inversion results for the same target region (source area of the 2000 West Tottori earthquake): considering realistic ray-theory geometrical spreading (calculated for the realistic 1-D velocity model, Petukhin, 2001, unpublished), and considering simplified spherical geometrical spreading (Moya and Irikura, 2001, Shiba et al., 2001) but corrected for the possible effect of 1-D geometrical spreading (Petukhin et al., 2008). In general, results considering realistic ray-theory geometrical spreading, agree well with results of this study.

8. CONCLUSIONS

- Method of the estimation of the low-frequency Q-value using the nonlinear waveform inversion is proposed.
- Using data from the source region of the 2000 West Tottori earthquake, low-frequency Q-value for the same region and the source and site correction coefficients are estimated.
- Estimated value Q = 80-180, agrees with the results of spectral inversion in case of a realistic ray-theory geometrical spreading is considered.
- Estimated source and site correction coefficients are small, 1.3 – 1.5 in average, which confirms correctness of the estimation of the seismic moment by F-net, and correctness of the shallow velocity structure estimated by the receiver function inversion.

Acknowledgements. We deeply appreciate the National Institute of Earth Science and Disaster Prevention for providing the Kik-net and F-net data.

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