

## TECHNOLOGY AND ASSESSMENT QUESTIONS ON REINFORCED CONCRETE BEAMS BUILT AT THE BEGINNING OF THE 20<sup>TH</sup> CENTURY

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### ABSTRACT:

In European cities there are many R.C. buildings and constructions belonging to the early 20<sup>th</sup> century and, in most cases, these pieces of work so rich in historical, architectural and cultural significance urgently require maintenance and rehabilitation. It appears particularly important to study in detail the historical examples, and to recognize those characters and technical details of the solutions in order to keep the trace of the history and preserve the memory of the architectural and structural 20<sup>th</sup> century buildings.

Reinforced concrete works, from the origins until the release of the first national technical standards, were built by applying patented systems (i.e. the Monnier's or the Hennebique's system) that were often the result of individual intuitions more than the product of coherent and established scientific and technical knowledge. This is one of the reasons why many of structures built in those years, and still surviving, could not be considered reliable with regard to the structural safety, as it is presently intended.

In this scenario, the evaluation of R.C. beam shear capacity becomes crucial. In this paper, starting from the tests carried out in Stuttgart in the early 20<sup>th</sup> century, an analysis of shear capacity evaluation is presented. The aim is to show that the relations given for 'new beams' in the present codes cannot be used, without any modification, for the verification of 'historical beams'.

### KEYWORDS:

Reinforced Concrete, Rehabilitation, Shear Capacity, Load Path Method

## 1. REHABILITATION OF REINFORCED CONCRETE ARCHITECTURE

On a scientific and technological standpoint, the knowledge of reinforced concrete as a material vulnerable to the deteriorating action of time is well established, and is widely proved by several R.C. buildings, whose poor performances are very well known by technicians. Only in the last few years a perception of the problems of this endangered heritage, despite its youth, is beginning to spread on a larger scale (unfortunately, because of several and repeated collapses). The extensive technical debate about structural safety of reinforced concrete buildings is in fact opposed to a 'popular' imagery where still the reinforced concrete is considered a forefront technology, an everlasting and indefinitely resistant material. In many cases, this became the symbol of the social and economic redemption: the main desire for many people was to abandon their old and crumbling masonry house for brand new, concrete buildings.

Unfortunately, the too steep climbing is the reason for the partial failure of R.C. technology: the speculative impulse, together with an excessive confidence in the standardization of the building process, actually induced to disregard the quality and accuracy of the work, severely undermining the performance and the durability of structures. Also from a merely technological standpoint, even the slogan coined by the forerunners, "*Reinforced concrete is for forever*", suddenly seems to be unsuited. In a few years, the maintenance and restoration of R.C. structures have grown into a fundamental question.

The poor durability of the current R.C. structures has brought this issue to the forefront, a new challenge for the contemporary design activity. At present, entire chapters of the most recent codes of practice are devoted to the definition of durability requirements, which have nowadays the same importance as the resistance requirements. The question of the ageing of concrete is then the real crucial point of a modern theory of Reinforced Concrete.

## 2. ASSESSMENT OF EARLY R.C. STRUCTURES: SHEAR CAPACITY

The main task to be faced in the restoration of the early R.C. constructions is the assessment of their actual structural capacity, in order to provide the proper guidelines for rehabilitation and conservation.

It is not so straightforward to apply to ancient concrete structures the same methods of calculation that are used for new designs, and this is particularly true with regard to shear behaviour due to the following reasons:

- the chemical characteristics of the steel reinforcement and, in particular, the low Carbon content that makes the collapse cracking pattern be typical of concrete structures with high ductility reinforcement;
- the technology and, in particular, the type of shear reinforcement that consisted of open “U” shaped plates which are not able to confine the inclined struts of the resisting internal truss.

In the evaluation of shear capacity, models and formulations which are used in the standard design practice are founded on the experimental observation of the behaviour of real scale structural elements. The constructive technique and the structural details concerning shear reinforcement have changed much in the last century and much more than the ones regarding longitudinal reinforcements.

Therefore, the approach to the question should necessary start from a critical review of the different formulations for the shear capacity, and from an analysis of the available experimental tests on ancient structures. In this sense, a great help is given by the work of Emil Mörsch and the experimental campaign that he performed in Stuttgart, which is widely described in his writings [Mörsch, 1923].

In this paper, starting from the tests performed in Stuttgart in the early 20<sup>th</sup> century [Mörsch, 1923], a preliminary analysis of shear capacity evaluation of old R.C. beams is showed.

### 2.1 Some comments on the tests performed by Mörsch in Stuttgart

In table 2.1 data and results of some shear tests performed by Mörsch in Stuttgart from 1906 to 1921 are summarised.

In table 2.1:

- $R_c$  is the cubic concrete compressive strength;  $f_t$  is the reinforcement tensile strength;
- $b_w$  is the web width;  $h$  is the overall depth of the cross section;
- $\phi_l$  is the longitudinal reinforcement diameter;  $n_l$  is the number of longitudinal bars;
- $\phi_w$  is the shear reinforcement diameter;  $n_w$  is the number of links of shear reinforcement;
- $s$  is the longitudinal spacing of shear reinforcement;
- $V_{u,test}$  is the ultimate shear strength of the tests.

The test results in table 2.1 have been analysed using the models of present codes. In particular, the last release of both Eurocode 2 [CEN, 2004] and Italian technical standards [Ministero delle Infrastrutture, 2008] adopt the ‘truss model’ with variable strut angle  $\theta$ .  $V_{rd,max}$  is the design value of the maximum shear force which can be sustained by the member, limited by crushing of the compression struts and  $V_{rd,s}$  is the design value of the shear force which can be sustained by the yielding shear reinforcement:

$$V_{rd,max} = v f_{cd} b_w z \frac{ctg \theta + ctg \alpha}{1 + (ctg \theta)^2} \quad (2.1)$$

$$V_{rd,s} = f_{ywd} \frac{A_{sw}}{s} z \sin \alpha (ctg \theta + ctg \alpha) \quad (2.2)$$

where:

- $\alpha$  is the angle between shear reinforcement and the beam longitudinal axis perpendicular to the shear force;
- $\theta$  is the angle between the concrete compression strut and the beam axis perpendicular to the shear force;
- $v f_{cd}$  is the compressive strength of the concrete inclined struts;
- $z$  is the internal lever arm;

- $f_{ywd}$  is the design yield of shear reinforcement;
- $A_{sw}$  is the cross sectional area of the shear reinforcement.

$\theta$  should be chosen between the following recommended limits:  $1 \leq \text{ctg}\theta \leq 2.5$ .

The value  $\theta_d$  of  $\theta$  that makes the stirrups yield and, at the same time, the web concrete reach its limiting compression can be obtained setting the relations (2.1) and (2.2) equal and, in the case of  $\alpha=90^\circ$ , it results:

$$\text{ctg}\theta_d = \sqrt{\frac{\nu f_{cd} b_w}{f_{ywd} \frac{A_{sw}}{s}} - 1} \quad (2.3)$$

Coefficient  $\nu$  is an efficiency factor which take account of the actual distribution of the stress within the inclined struts and the effect of cracking. This factor is defined by technical standards and it is calibrated on the reinforcement detailing prescribed in the codes.

Table 2.1 Some shear tests performed in Stuttgart [Mörsch, 1923]

	$R_c$ [MPa]	$f_t$ [MPa]	$b_w$ [m]	$h$ [m]	$\phi_l$ [mm]	$n_l$	$\phi_w$ [mm]	Shear reinf.	$n_w$	$s$ [m]	$V_{u, \text{test}}$ [daN]	Failure type
4	24.80	407.70	0.20	0.40	40.0	2.0	7.0	Stirrups	2	0.150	11000.00	Sliding of bottom longitudinal bars
5	24.80	407.70	0.15	0.40	40.0	2.0	7.0	Stirrups	2	0.150	8350.00	
6	24.80	407.70	0.30	0.40	40.0	2.0	7.0	Stirrups	2	0.150	13650.00	
8	24.80	407.70	0.20	0.40	40.0	2.0	10.0	Stirrups	2	0.200	18150.00	Stirrup failure
9	24.80	407.70	0.20	0.40	40.0	2.0	7.0	Stirrups	2	0.200	16350.00	
10	24.80	407.70	0.20	0.40	40.0	2.0	5.0	Stirrups	2	0.200	14900.00	Stirrup failure
11	24.80	407.70	0.20	0.40	40.0	2.0	10.0	Stirrups	2	0.150	18800.00	
12	24.80	407.70	0.20	0.40	40.0	2.0	7.0	Stirrups	2	0.150	18000.00	
13	24.80	407.70	0.20	0.40	40.0	2.0	5.0	Stirrups	2	0.150	16400.00	Stirrup failure
15	24.80	407.70	0.20	0.40	40.0	2.0	10.0	Stirrups	2	0.100	21350.00	
16	24.80	407.70	0.20	0.40	40.0	2.0	7.0	Stirrups	2	0.100	20000.00	
17	24.80	407.70	0.20	0.40	40.0	2.0	5.0	Stirrups	2	0.100	18150.00	Stirrup failure
18	24.80	407.70	0.20	0.40	40.0	2.0	5.0	Stirrups	2	0.050	20250.00	
19	24.80	407.70	0.20	0.40	40.0	2.0	7.0	Links	2	0.150	16300.00	Concrete failure caused by top longitudinal bars
20	24.80	407.70	0.20	0.40	40.0	2.0	7.0	Stirrups	2	0.150	16850.00	
21	24.80	407.70	0.20	0.40	40.0	2.0	7.1	"U" shaped links	4	0.150	19850.00	Stirrup failure
22	24.80	407.70	0.20	0.40	40.0	2.0	5.0	"U" shaped links	4	0.150	17500.00	
23	24.80	407.70	0.20	0.40	40.0	2.0	7.0	Stirrups	2	0.150	15350.00	straightening of the hooks
75	24.80	407.70	0.20	0.50	25.0	2.0	7.0	Stirrups	2	0.090	15250.00	Sliding of bottom longitudinal bars
77	24.80	407.70	0.20	0.50	25.0	2.0	10.1	"U" shaped links	6	0.140	18850.00	

## 2.2 Test ultimate shear strength vs. code ultimate shear strength

In order to verify the reliability of the relations recommended in the present codes with respect to failure loads measured in tests, numerical analyses have been performed using the ultimate resistance values of the materials. The following data have been adopted:

- ultimate values of the material strength;
- $R_c = 24.80 \text{ MPa}$ ;
- $f_c = 0.83 \cdot R_c = 20.58 \text{ MPa}$  (compressive cylinder strength of concrete);
- $f_t = 407.70 \text{ MPa}$ ;
- $c_{inf} = 2.0 \text{ cm}$  (bottom cover on longitudinal reinforcement).
- $z = 0.90 \cdot d$  (where  $z$  is the internal lever arm and  $d$  is the effective depth of the cross section).

From relations (2.1), (2.2), (2.3), assuming that theoretical shear strength is equal to test shear strength, the value  $\nu_{test}$  of  $\nu$  has been evaluated. Figure 1 shows that, almost in all the examined cases, the calculated theoretical value  $\nu_{theor.}$  of  $\nu$  is bigger than the one ( $\nu_{test}$ ) calculated from the test ultimate load.

In figure 2 the values of the test ultimate shear  $V_{u, test}$  and of the theoretical one  $V_{Rd, ctg\theta}$  (calculated according to

Eurocode 2 [CEN, 2004], ignoring the limits on  $ctg\theta$  are shown. The histogram in the figure highlights that the theoretical value overestimates, almost in all the cases, the test ultimate shear strength.

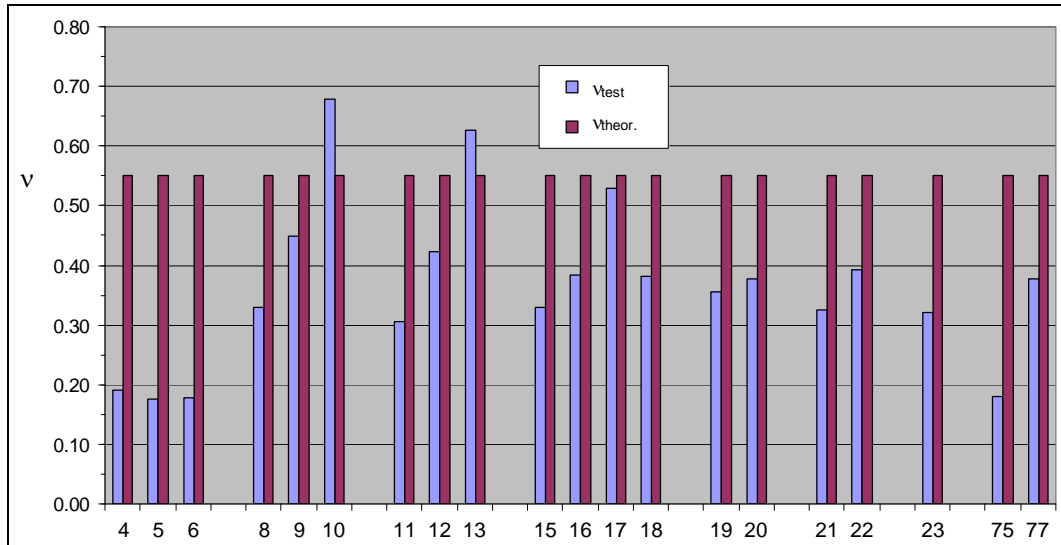


Figure 1  $v_{test}$  and  $v_{theor}$ .

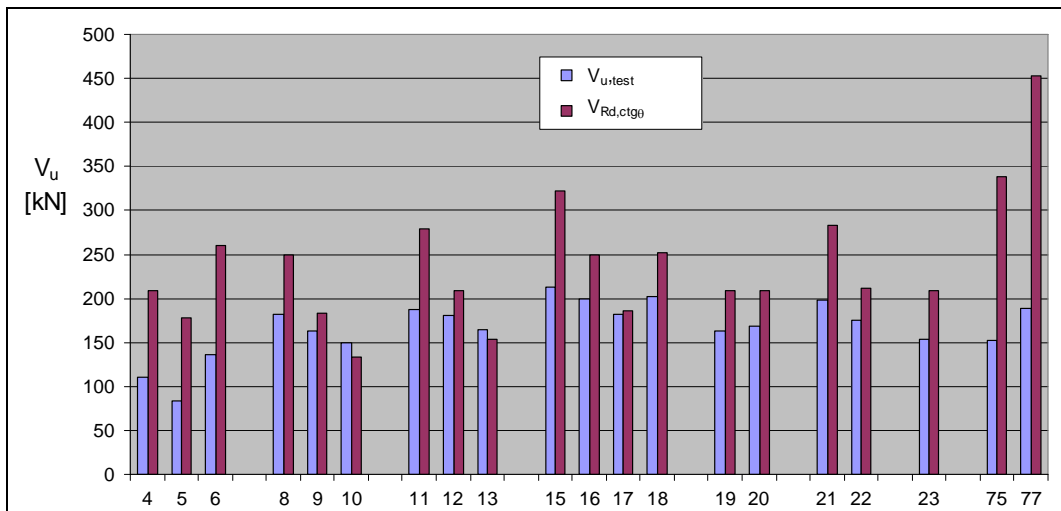


Figure 2  $V_{u,test}$  and  $V_{Rd,ctg\theta}$

Figure 3 shows that, almost in all the examined cases, theoretical value ( $ctg\theta_{theor}$ ) of  $ctg\theta$  is higher than the one ( $ctg\theta_{test}$ ) calculated from the test ultimate load. Thus, it follows that beams tested by Mörsch in the early 20<sup>th</sup> century did not have the capacity to reach the values of  $ctg\theta$  recommended by the present codes.

It is worth noting that the ‘truss model with variable strut angle’, that in the Eurocode [CEN, 2004] (but also in the ‘new’ Italian code [Ministero delle Infrastrutture, 2008]) has substituted the ‘modified hyperstatic truss model’, is principally based on the following assumptions:

- the ultimate resistance of the inclined struts should be reached when the shear reinforcement has yielded;
- shear reinforcement should have the capacity to limit the opening of the cracks in order to make them be crossed by struts having an inclination  $\theta$  lower than the one corresponding to first cracking.

The second assumption could not be satisfied by R.C. beams of the early 20<sup>th</sup> century. Steel reinforcement used in the past had ductility characteristics higher than the present reinforcement. This means that, because of the large deformations consequent to yielding, crack widths are so excessive to make impossible the transfer of shear forces across them. Consequently the ‘truss model with variable strut angle’ with the present limit of the maximum value of  $ctg\theta$  is not by far applicable to beams of the past. It is worth highlighting this consideration

because present codes that deal with the assessment of existing structures use the same shear strength relations adopted for new building design. However it is necessary to differentiate the approach, trying to find the values of the maximum limit of  $ctg\theta$  consistent with the test results reported in the literature regarding beams having reinforcement similar to the one used in the early 20<sup>th</sup> century.

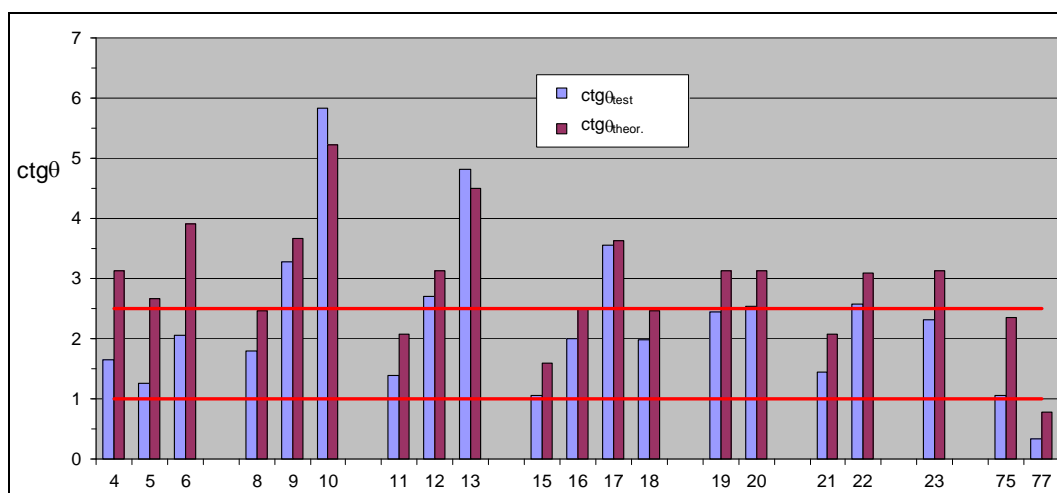


Figure 3  $ctg\theta_{theor}$  and  $ctg\theta_{test}$  (horizontal lines indicates the limits according to Eurocode 2)

In the draft version [CEN, 1991] (no more in force) of Eurocode 2, as in the previous Italian technical standards [Ministero dei Lavori Pubblici, 1996], shear capacity was evaluated using the ‘modified hyperstatic truss model’. According to this approach shear resistance is evaluated under the assumption of  $\theta=45^\circ$ , evaluating the web tension strength separately from the web compressive strength. In particular the first one ( $V_{Rd,s}$ ) is the summation of the concrete strength ( $V_{cd}$ ) and of the shear reinforcement strength ( $V_{yd}$ ).

Many codes used to assume  $V_{cd}$  equal to the value calculated for beams without shear resistance. This assumption, generally on the safe side, is difficult to be justified because it is based on the hypothesis that in the element with shear reinforcement the ‘dowel effect’ and the ‘aggregate interlocking effect’ give the same contribution that they would give if the element had no shear reinforcement. Besides, when the beam has shear reinforcement, the flexural resistance of the ‘concrete cantilever’ between two following cracks is strongly reduced by the very low spacing of the shear cracks; this means that the contribution of the ‘concrete cantilever’ resistance to shear strength in a beam with shear reinforcement is less than the one in a beam without shear reinforcement. Especially because of the difficulty to evaluate the contribution of  $V_{cd}$  to  $V_{Rd,s}$ , the present codes have adopted the ‘truss model with variable strut angle’ in which the resistance of the ‘concrete cantilever’ is not considered (the contribution of  $V_{cd}$  misses) and web tension strength is only due to shear reinforcement but, at the same time, inclined struts can have  $\theta \leq 45^\circ$ . This model seems to be more consistent with the results of tests performed on present beams but, as previously showed, seems to loose reliability for beams of the early 20<sup>th</sup> century. To compare these two models, for the beams of table 2.1, web tension shear strength  $V_{Rd3, ENV1992-1-1}$  has been calculated using the ultimate resistance values of the materials and according to the ‘modified hyperstatic truss model’ of the old draft of Eurocode 2 [CEN, 1991]. From figure 4 it is worth noting that, almost in all the examined cases, the ‘modified hyperstatic truss model’ of the old draft of Eurocode 2 [CEN, 1991], confirming what previously discussed, gives results that are more similar to those of the laboratory tests.

### 2.3 The transversal shear behaviour

In this paragraph the interpretation of transversal shear behaviour is showed using Load Path Method [Schlaich & Schafer, 1996; Palmisano et al., 2003; Palmisano et al., 2007].

A simplified model of the diagonal compressive flux in an element subjected to shear and bending is showed in figure 5 [Mezzina et al., 2007]. The flux starts from the longitudinal compression zone (on the top of the beam in figure 5) and, in the descending path, it keeps itself spread in all the web width in order to save strain energy.

However, it is obliged to concentrate on the longitudinal bars that are the only one able to carry the horizontal longitudinal thrusts due to the deviation of the shear path. This concentration can happen thanks to the formation of a transversal arch; in this model, transversal thrusts arise and they can find equilibrium because of the transversal horizontal link of the stirrup. In the absence of this link the only way, for these thrusts, to find equilibrium is to use the concrete tension strength. Detail (II) of figure 5 shows that the presence of a floor slab, giving a compression path to the transversal top thrusts, makes possible the adoption of top open stirrups.

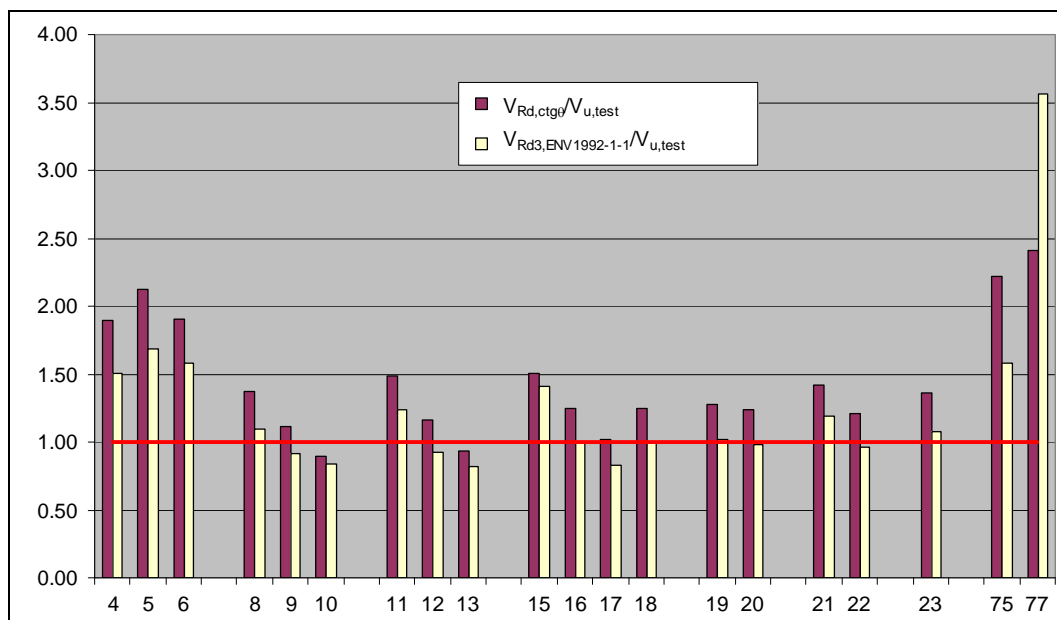


Figure 4 Ratio of the shear capacity ( $V_{Rd,ctg\theta}$ ) according to Eurocode 2 [CEN, 2004] and of the web tension shear strength ( $V_{Rd3,ENV1992-1-1}$ ) according to the old draft of Eurocode 2 [CEN, 1991] to the test ultimate shear strength ( $V_{u,test}$ )

Beams of the early 20<sup>th</sup> century often had “U” shaped links as stirrups (figure 6); this means that, for the transversal bottom thrust, equilibrium can be maintained only thanks to concrete tension strength.

Figure 6 shows a model to analyse transversal behaviour for beams with “U” shaped links. The most critical condition is where bottom lateral transversal thrust ( $H_{end}$ ) is applied. The ultimate shear for the transversal behaviour ( $V_{Rd,lat}$ ) is given by the capacity of concrete cover to carry the transversal thrusts using its tensile strength and avoiding the ejection of the longitudinal bars.

In this model the following assumptions have been made:

- $ctg\theta'$  should not be less than an assumed value  $ctg\theta'_{min}$ ;
- the deviation of the inclined struts in the bottom of the beam web starts from a distance  $h/2$  (Fig. 6) from the top of the beam if  $ctg\theta' > ctg\theta'_{min}$ ;
- the stress in the inclined strut is constant;
- the forces in every “U” shaped link are equal;
- $H_{end}$  divides itself into two identical parts (Fig. 7): one crosses  $a_{sup}$  and the other one crosses  $a_{inf}$ ;
- $\gamma$  is the angle of the transversal (Fig. 7) and longitudinal (Fig. 8) diffusion of  $H_{end}$  within the concrete.

The reason of the assumption a) is that, if the overall depth of the beam is very large with respect to its width, the inclined strut, in order to save strain energy, tends to deviate in the bottom of the beam; a limit on the minimum value of  $ctg\theta'$  is needed to take account of this consideration. This limit could be taken equal to that ( $ctg\theta'_{min}=0.5$ ) usually assumed in the design of deep beams and column footings.

As a consequence of assumptions c) and d), the inclined strut should be divided into a number of parts equal to the number of “U” shaped links.

The resistance of the concrete cover is governed by the minimum value ( $a_{transv,min}$ ) between  $a_{sup}$  and  $a_{inf}$  (Fig. 7). Because of the abovementioned assumptions the ultimate shear of the transversal behaviour is

$$V_{Rd,lat} = \left\{ \left[ f_{ctd} \cdot (2 \cdot a_{transv,min}) \cdot a_{long,min} \right] \left[ \frac{n_w}{2} \frac{1}{ctg\theta'} \right] \right\} \frac{z}{s} ctg\theta_d \quad (2.4)$$

where:

- $f_{ctd}$  is the design value of the axial tensile strength of concrete;
- $a_{long,min} = \min[2 \cdot \delta_{lat} \cdot tg\gamma; (s - \phi_w)]$  is the longitudinal width of the diffusion area of  $H_{end}$ ;  $s$  is the longitudinal spacing of shear reinforcement;  $\phi_w$  is the shear reinforcement diameter;
- $n_w$  is the total number of links of a stirrup.

An application of this model is showed in figures 9 and 10. The following data have been used:

- design values of the material strength;
- $R_{ck} = 20\text{MPa}$  (characteristic compressive cubic strength of concrete);  $f_{ck} = 16.60\text{MPa}$  (design compressive strength of concrete);  $f_{yk} = 215.00\text{MPa}$  (characteristic yield strength of reinforcement);
- $\phi_w = 6\text{mm}$ ;  $n_w = 4$ ;  $s = 15\text{cm}$ ;  $\phi_l = 16\text{mm}$ ;  $c_{lat} = 2.0\text{cm}$ ;  $c_{inf} = 2.0\text{cm}$ ;
- $ctg\theta'_{min} = 0.5$ ;  $\gamma = 45^\circ$ ;  $z = 0.90 \cdot d$ .

In the example of figure 9  $b_w = 30\text{cm}$  and  $h = var.$ , whereas  $b_w = var.$  and  $h = 50\text{cm}$  in the case of figure 10. In these figures the comparison between the shear capacity ( $V_{Rd,EC2}$ ) evaluated according to Eurocode 2 [CEN, 2004] (which neglects the transversal behaviour) and the one ( $V_{Rd,lat}$ ) calculated taking account only of the transversal behaviour is showed. According to this approach, the shear capacity of the section is the minimum value between the two calculated resistances. It is worth noting that in both cases the shear capacity is governed by the transversal behaviour.

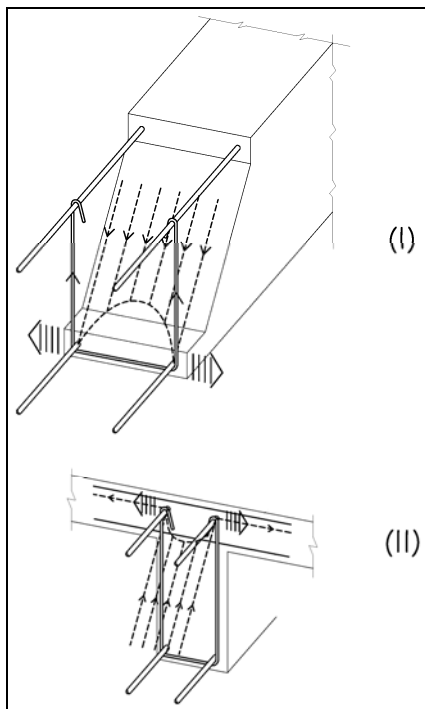


Figure 5 Shear transversal behaviour of a beam (axonomic view)

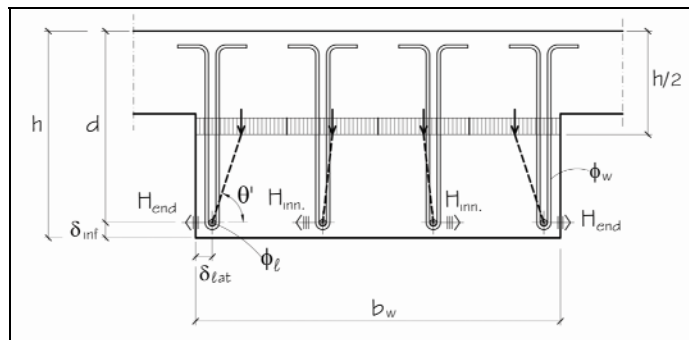


Figure 6 Analysis model of shear transversal behaviour of a beam with "U" shaped links

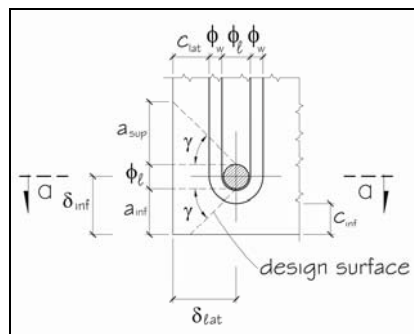


Figure 7 Left bottom node detail of the beam in figure 6

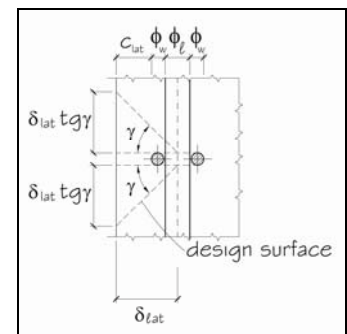


Figure 8 Transversal section of the detail in figure 7

### 3. CONCLUDING REMARKS

The results of the tests performed on the early 20<sup>th</sup> century beams show that relations of the present codes used

to evaluate shear resistance cannot be used without any modification for ‘historical’ R.C. beams. Firstly, because of great ductility of steel reinforcement used in the past, the old beams can not reach the values of  $ctg\theta$  recommended by the present codes. Secondly, the type of shear reinforcement (i.e. open “U” shaped links) makes transversal behaviour collapse anticipate the one provided by standard formulations calibrated on beams with stirrups.

It follows that particular attention has to be paid especially because, at present, the economical and social weight of existing building assessment is becoming particularly relevant.

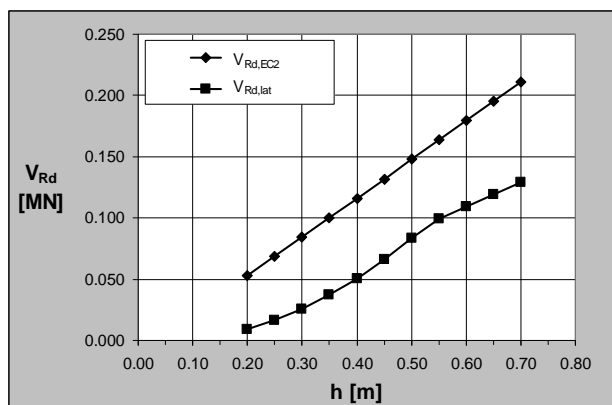


Figure 9  $V_{Rd,EC2}$  and  $V_{Rd,lat}$  for a beam with  $b_w = 30\text{cm}$  and  $h = \text{var.}$

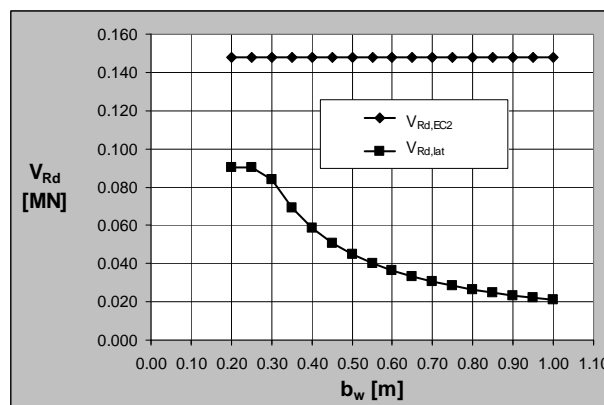


Figure 10  $V_{Rd,EC2}$  and  $V_{Rd,lat}$  for a beam with  $b_w = \text{var.}$  and  $h = 50\text{cm}$

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