SEISMIC STABILITY OF DETACHED CONCRETE BLOCK OF CONCRETE GRAVITY DAM

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ABSTRACT :

Ultimate state of a dam can be defined by its function of keeping the ponded water under control. An existing crack can penetrates the dam before subsequent earthquakes may form a detached concrete block. It is indisputable that the seismic stability of the block is very important for keeping the ponded water under control. Analytical procedures are developed to determine the motion of a free-standing block subjected to horizontal ground acceleration, slide, rock, rock-slide as well as impact when it rocks back and forth. The results reveal that the block begins to slide before the first impact even though it starts in a pure rocking mode. Numerical method FEM has been used to solve this large displacement contact problem of free-standing block. Analytical and numerical results consist with each other well. In consequence, the dynamic responses of a concrete gravity dam with crack at upper position forming a detached block are studied with FEM procedure. Its stability is studied for two types of cracks and the results indicate that the detached block of a dam of Type B crack is more vulnerable to be overturned by horizontal ground motion.

KEYWORDS: Concrete gravity dam, Dynamic stability, Detached concrete block, Earthquake, Free-standing block

1. INTRODUCTION

An existing crack can penetrates the dam before subsequent earthquakes, which may be caused by temperature, shrinkage or strong earthquake, etc. There are only a few cases of damaged concrete dams in earthquakes, the cracks observed were at the upper part of dam for all these dams, Xin Feng Jiang of China, Koyna of India and Sefidrud of Iran. Ultimate state of a dam can be defined by its function of keeping the ponded water under control. It is indisputable that the seismic stability of upper detached concrete block is very important for keeping the ponded water under control. The seismic stability of cracked dam is commonly checked for its permanent sliding displacements using Newmark's rigid block model, based on certain assumptions that ignore the true dynamic response behavior of the sliding [FEMA 65, 2005]. However, seismic forces vary with time and alternate between upstream and downstream directions and ground motion may be amplified by the structure, upper detached concrete blocks may suffer very strong shaking. There is a need to investigate the dynamic sliding, rocking and compacting of the detached concrete block during earthquake with nonlinear dynamic procedure. Only a few studies refer to the dynamic stability of dams. Chopra and Zhang (1991) studied the base sliding of concrete gravity dams due to earthquakes. Since the stocky shape and large size of dam make the rocking response at the base small, only earthquake-induced sliding motion of a rigid dam was considered in their study. Malla and Wieland (2006) studied the dynamic stability of detached concrete blocks in an arch dam. There are many studies on the dynamic response of free-standing rigid blocks to earthquakes, but most of them addressed rocking motion only for slender structures [Yim, Chopra and Penzien, 1980, Zhang and Makris, 2001]. Uematsu (2000) compared the results of analysis and experiment of rocking motion to verify the criteria for overturning of the rigid bodies considering the period characteristics of the input motions from the overturning simulation to the sinusoidal motions.
Comparing with stocky triangular shape of dam, the shape of upper detached concrete block of dam is close to rectangle block and its rocking motion will not die out quick enough to be ignored in the analysis. Sliding, rocking and compacting should be formulated properly to predict the realistic response of the upper detached concrete block of a dam during earthquakes to judge its stability.

2. NONLINEAR FORMULATION OF FREE-STANDING BLOCK

Consider the free-standing rigid rectangular block shown in Figure 1, which may translate with the ground, slide, rock, or slide-rock depending on the level of the ground. The mode of the block motion depends on static friction, width-to-height ratio of the block and the magnitude of the horizontal ground acceleration. If the block moves back and forth, it will impact on the ground and loses a part of its kinetic energy.

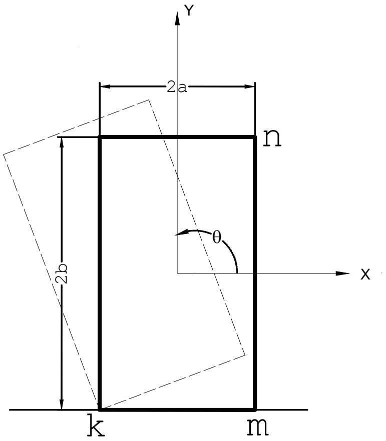

Figure 1 Schematic of rigid rectangular block

The motion of the rigid block is governed by following equations:

$$\begin{aligned} -\ddot{x}M + F + \lambda Mg &= 0 \\ -\ddot{y}M + V - Mg &= 0 \\ -\ddot{\theta}I - Vr_x + Fr_y &= 0 \end{aligned} \tag{2.1}$$

in which, M is the mass of the block, I is the mass moment of inertia of the block about its center of gravity, g is the acceleration of gravity, λMg is the force due to horizontal acceleration of the ground $-\lambda g$. V and F are vertical and horizontal forces, and $\ddot{x}, \ddot{y}, \ddot{\theta}$ are the horizontal acceleration, vertical acceleration of the center of mass and angular acceleration of the block. F is friction between the block and the ground, therefore

$$|F| \leq \mu V \tag{2.2}$$

μ is the coefficient of friction. r_x and r_y are the coordinate of the center of mass to the pivot point i.

$$\begin{aligned} r_x &= x_i \cos\theta - y_i \sin\theta \\ r_y &= x_i \sin\theta + y_i \cos\theta \end{aligned} \tag{2.3}$$

if $\ddot{\theta} \neq 0$ or $\theta \neq 0$. i=m, k or n depending on the point in contact. $(x_{m,}\ y_m) = (-a, b)$ and $(x_{k,}\ y_k) = (a, b)$ from Fig.1. Rocking will not start if $\mu \leq a/b$. Then $\ddot{y}$ equals zero too and the block can only start in slide mode. Consequently, $|r_x| < a$. As the cracked surface of the concrete dam is rough, the coefficient of friction for such a rough surface would be higher than 0.65 for a planar interface. The shaking table test by Niwa and Clough (1980) showed extreme rocking of the cracked upper section of a model dam. Therefore, it is reasonable to use the coefficient of friction from 0.8 to 1.05 so that rocking mode can start.
At the first time that the block starts rocking, the pivot point i of the block contacts with the ground, that is

$$\begin{aligned} &\dot{y}_i = \dot{y} - \dot{\theta} r_x = 0 \\ &V > 0 \end{aligned} \tag{2.4}$$

As the friction is big enough that the rocking starts without sliding, then we have

$$\begin{aligned} &|F| \le \mu V \\ &\dot{x}_i = \dot{x} + \dot{\theta} r_y = 0 \end{aligned} \tag{2.5}$$

Differentiating Eqn.2.4 and 2.5 with respect to time and substituting the results into 2.3 and reorganizing, we obtain the equation of motion of the block

$$\ddot{\theta} = \frac{-(r_x + \lambda r_y) g}{I / M + r_x^2 + r_y^2} \tag{2.6}$$

and expressions of the contact forces

$$\begin{aligned} &F = -\left(\ddot{\theta} r_y + \dot{\theta}^2 r_x\right) M - \lambda M g \\ &V = \left(\ddot{\theta} r_x - \dot{\theta}^2 r_y\right) M + M g \end{aligned} \tag{2.7}$$

But the restriction in Eqn.2.2 must be met. The block transits from rock mode to rock-slide mode when friction is unable to provide enough horizontal force. The horizontal force

$$F = -\mathrm{sign}(\dot{x}_i) \mu V \tag{2.8}$$

where sign is the signum function. And equations of rocking and sliding motion of the block become

$$\begin{aligned} &\ddot{\theta} = \frac{-\left(r_x + \mathrm{sign}(\dot{x}_i) \mu\, r_y\right) g + \left(r_x r_y + \mathrm{sign}(\dot{x}_i) \mu\, r_y^2\right) \dot{\theta}^2}{I / M + r_x^2 + \mathrm{sign}(\dot{x}_i) \mu\, r_x r_y} \\ &\ddot{x}_i = -\mathrm{sign}(\dot{x}_i) \mu V / M + \lambda g \end{aligned} \tag{2.9}$$

The vertical force V can be calculated with the same formula of Eqn.2.7.
When the block rocks back and forth or rotates to ±90°, it impacts on the ground. The velocities just after an impact can be calculated with the momentum impulse principle. If the relative velocity at the impacting point in tangential direction is not zero, inclined impact will happen, and then the sliding mode during the impacting depends on the coefficient of friction, the position of a block and the relative velocity itself, in general. The differential equations of motion at the impacting point are

$$\begin{aligned} &d\dot{x}_i = a_1 dP_x - a_2 dP_y \\ &d\dot{y}_i = -a_2 dP_x + a_3 dP_y \end{aligned} \tag{2.10}$$

where P_x, P_y are the total momentum in x and y direction, respectively. And

$$a_1 = \frac{1}{M} + \frac{r_y^2}{I}, \quad a_2 = \frac{r_x r_y}{I}, \quad a_3 = \frac{1}{M} + \frac{r_x^2}{I} \tag{2.11}$$

The Coulomb friction is used for the tangential impact. That is

$$
\begin{aligned}
dP_x &= -sign(\dot{x}_i)\mu dP_y && \dot{x}_i \neq 0 \\
dP_x &= a_2/a_1 dP_y && \dot{x}_i = 0
\end{aligned}
\tag{2.12}
$$

We use superscript – and + to denote the velocity before and after impact, respectively. If $\dot{x}_i^- \neq 0$, we can assume that at time t_1 during impacting $\dot{x}_i(t_1) = 0$, and integrate Eqn.2.10 from 0 to t_1.

$$
\begin{aligned}
\int_0^{t_1} d\dot{x}_i dt &= -(a_1 sign(\dot{x}_i^-)\mu + a_2)\int_0^{t_1} dP_y dt \\
\int_0^{t_1} d\dot{y}_i dt &= (a_2 sign(\dot{x}_i^-)\mu + a_3)\int_0^{t_1} dP_y dt
\end{aligned}
\tag{2.13}
$$

$$
\begin{aligned}
\int_0^{t_1} dP_y dt &= \frac{\dot{x}_i^-}{(a_1 sign(\dot{x}_i^-)\mu + a_2)} \geq 0 \\
\dot{y}_i(t_1) &= \dot{y}_i^- + \frac{(a_2 sign(\dot{x}_i^-)\mu + a_3)}{(a_1 sign(\dot{x}_i^-)\mu + a_2)}\dot{x}_i^-
\end{aligned}
\tag{2.14}
$$

The first condition that the momentum in y direction is always positive requires that

$$
\begin{aligned}
&\mu > a_2/a_1, && \text{for } \dot{x}_i^- < 0 \\
&\mu > -a_2/a_1, && \text{for } \dot{x}_i^- > 0
\end{aligned}
\tag{2.15}
$$

These imply the friction should be big enough, otherwise, tangential velocity will keep in the same direction. In that case, Eqn.2.13 becomes

$$
\begin{aligned}
\dot{x}_i^+ - \dot{x}_i^- &= -(a_1 sign(\dot{x}_i^-)\mu + a_2)\int_0^t dP_y dt \\
\dot{y}_i^+ - \dot{y}_i^- &= -(1+e)\dot{y}_i^- = (a_2 sign(\dot{x}_i^-)\mu + a_3)\int_0^t dP_y dt
\end{aligned}
\tag{2.16}
$$

$$
\dot{x}_i^+ = \dot{x}_i^- + \frac{a_1 sign(\dot{x}_i^-)\mu + a_2}{a_2 sign(\dot{x}_i^-)\mu + a_3}(1+e)\dot{y}_{i0}
\tag{2.17}
$$

where $0 \leq e \leq 1$ is the coefficient of restitution for normal impact. From second equation of Eqn. 2.14, the requirement that t_1 exists is

$$
\begin{aligned}
&\dot{y}_i(t_1) = \dot{y}_i^- + \frac{(a_2 sign(\dot{x}_i^-)\mu + a_3)}{(a_1 sign(\dot{x}_i^-)\mu + a_2)}\dot{x}_i^- \leq -e\dot{y}_i^- \\
&-(1+e)\dot{y}_i^- \geq \frac{(a_2 sign(\dot{x}_i^-)\mu + a_3)}{(a_1 sign(\dot{x}_i^-)\mu + a_2)}\dot{x}_i^- \\
&\frac{a_1 sign(\dot{x}_i^-)\mu + a_2}{a_2 sign(\dot{x}_i^-)\mu + a_3} \geq -\frac{\dot{x}_i^-}{(1+e)\dot{y}_i^-}, \quad \dot{x}_i^- > 0 \\
&\frac{a_1 sign(\dot{x}_i^-)\mu + a_2}{a_2 sign(\dot{x}_i^-)\mu + a_3} \leq -\frac{\dot{x}_i^-}{(1+e)\dot{y}_i^-}, \quad \dot{x}_i^- < 0
\end{aligned}
\tag{2.18}
$$

It is easy to verify that if Eqn. 2.15 is false, Eqn. 2.18 must be false. If both Eqn. 2.18 and 2.15 are met, the impact point will stick in tangential direction. While if Eqn.2.18 is met but Eqn.2.15 is false, $\dot{x}_i$ will change in direction after t_1. For $\dot{x}_i^- = 0$, the motion after impact will depends on the relation between μ and $|a_2 / a_1|$ too.
The momentums for all cases are summarized in Table 2.1, it is easy to calculate the velocities at the center of mass and angular velocity of the block after impact according to the momentum impulse principle.
Because the $\dot{y}_i^- < 0$ and $\dot{y}_i^+ \geq 0$, the block will jump unless the coefficient of restitution e is zero.

Table 2.1

conditions	Momentum
$\dot{x}_i^- = 0$, $\mu \geq \left\lvert \dfrac{a_2}{a_1} \right\rvert$	$P_y = -\dfrac{I + Mr_y^2}{I/M + r_x^2 + r_y^2}(1+e)\dot{y}_i^-$ $P_x = -\dfrac{Mr_x r_y}{I/M + r_x^2 + r_y^2}(1+e)\dot{y}_i^-$
$\dot{x}_i^- = 0$, $\mu < \left\lvert \dfrac{a_2}{a_1} \right\rvert$	$P_y = \dfrac{(1+e)\dot{y}_i^-}{a_2 sign(a_2)\mu - a_3}$ $P_x = \dfrac{sign(a_2)\mu}{a_2 sign(a_2)\mu - a_3}(1+e)\dot{y}_i^-$
$\dot{x}_i^- \neq 0$ $\dfrac{a_1 sign(\dot{x}_{i0})\mu + a_2}{a_2 sign(\dot{x}_{i0})\mu + a_3} < -\dfrac{\lvert \dot{x}_{i0} \rvert}{(1+e)\dot{y}_i^-}$	$P_y = \dfrac{-(1+e)\dot{y}_i^-}{a_2 sign(\dot{x}_i^-)\mu + a_3}$ $P_x = \dfrac{sign(\dot{x}_i^-)\mu}{a_2 sign(\dot{x}_i^-)\mu + a_3}(1+e)\dot{y}_i^-$
$\dot{x}_i^- \neq 0$ $\dfrac{a_1 sign(\dot{x}_{i0})\mu + a_2}{a_2 sign(\dot{x}_{i0})\mu + a_3} \geq -\dfrac{\lvert \dot{x}_{i0} \rvert}{(1+e)\dot{y}_i^-}$ $\mu \geq \left\lvert \dfrac{a_2}{a_1} \right\rvert$	$P_y = \dfrac{\dot{x}_i^-}{a_1 sign(\dot{x}_i^-)\mu + a_2} + \left((1+e)\dot{y}_i^- + \dfrac{a_2 sign(\dot{x}_i^-)\mu + a_3}{a_1 sign(\dot{x}_i^-)\mu + a_2}\dot{x}_i^- \right)\dfrac{a_1}{a_2^2 - a_1 a_3}$ $P_x = \dfrac{-sign(\dot{x}_i^-)\mu}{a_1 sign(\dot{x}_i^-)\mu + a_2}\dot{x}_i^- + \left((1+e)\dot{y}_i^- + \dfrac{a_2 sign(\dot{x}_i^-)\mu + a_3}{a_1 sign(\dot{x}_i^-)\mu + a_2}\dot{x}_i^- \right)\dfrac{a_2}{a_2^2 - a_1 a_3}$
$\dot{x}_i^- \neq 0$ $\dfrac{a_1 sign(\dot{x}_i^-)\mu + a_2}{a_2 sign(\dot{x}_i^-)\mu + a_3} \geq -\dfrac{\lvert \dot{x}_i^- \rvert}{(1+e)\dot{y}_i^-}$ $\mu < \left\lvert \dfrac{a_2}{a_1} \right\rvert$	$P_y = \dfrac{\dot{x}_i^-}{a_1 sign(\dot{x}_i^-)\mu + a_2} + \left((1+e)\dot{y}_i^- + \dfrac{a_2 \lvert \dot{x}_i^- \rvert \mu + a_3 \dot{x}_i^-}{a_1 sign(\dot{x}_i^-)\mu + a_2} \right)\dfrac{1}{a_2 sign(\dot{x}_i^-)\mu - a_3}$ $P_x = \dfrac{\lvert \dot{x}_i^- \rvert \mu}{a_1 sign(\dot{x}_i^-)\mu + a_2} + \left((1+e)\dot{y}_i^- + \dfrac{a_2 \lvert \dot{x}_i^- \rvert \mu + a_3 \dot{x}_i^-}{a_1 sign(\dot{x}_i^-)\mu + a_2} \right)\dfrac{a_1 sign(\dot{x}_i^-)\mu - a_2}{a_2 sign(\dot{x}_i^-)\mu - a_3}$

3. NUMERICAL SOLUTION OF THE E□UATION OF MOTION

The solution of Eqn.2.6 and 2.9 in conjunction with the constraint imposed by Eqn.2.2 is obtained numerically with fourth-order Runge-Kutta Method. An example is given here with following conditions: a=0.5, b=1.0, μ=0.6, e=0.6, and

$$\lambda = \cos(\pi t) \tag{3.1}$$

$0 \le t \le 2.0$. This is similar to B-type pulse of ground acceleration in near fault field. The time step used for the integration is 0.00025sec. As a comparison, an elastic block of the same size and same load condition was calculated with a general-purpose FEM computer program MARC, 195 elements was used in a 3D discrete model.

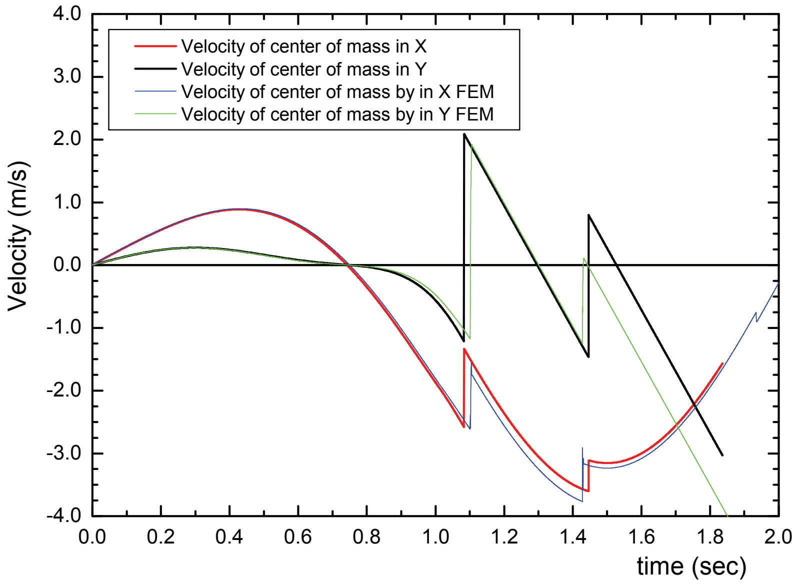

Figure 2 Velocity at the center of mass of the block

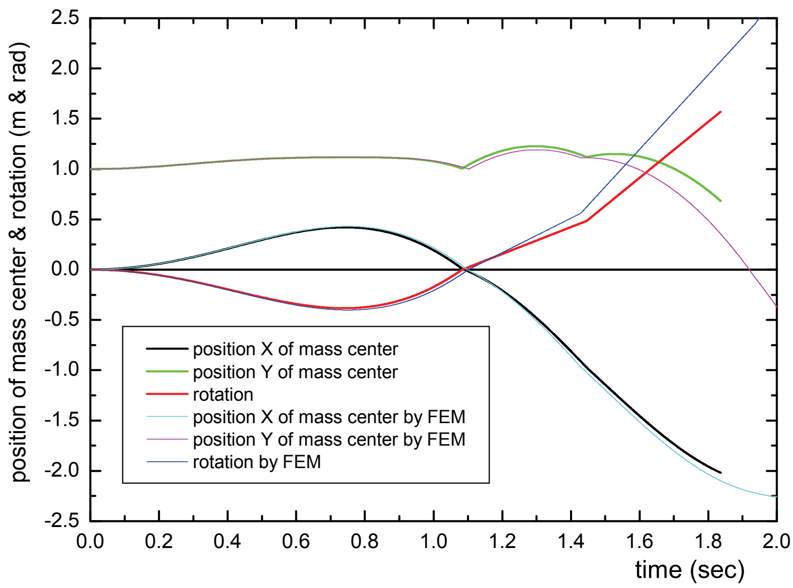

Figure 3 Position of the center of mass & rotation of the block

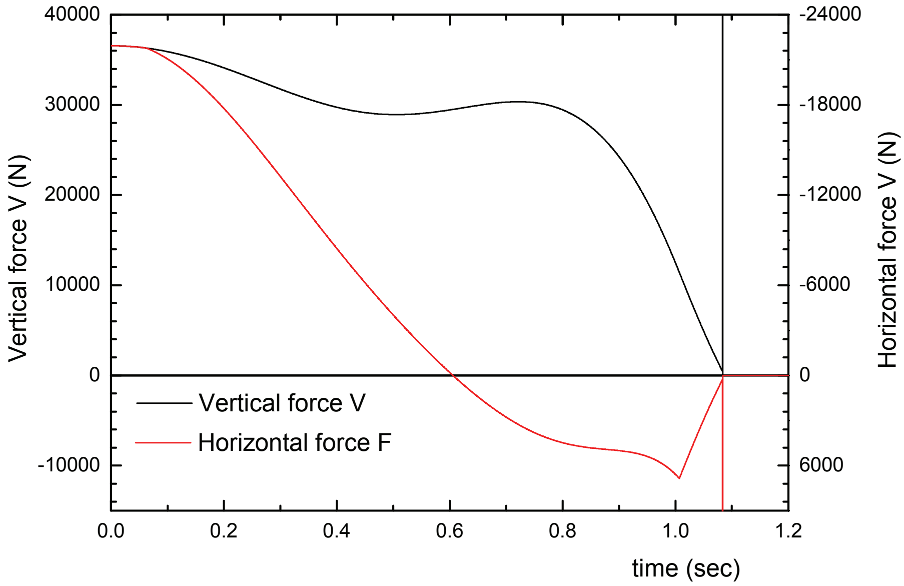

Figure 4 Forces at contact point

In Figure 2 and 3, are shown the velocity and position of the center of mass as well as the rotations obtained by two methods. Two results agree well until second impact. Difference in the mass moment of inertia between analytical model and discrete model of FEM cause some discrepancy in the angular motion of the block then the time of impact. Figure 4 gives the reactions at the contact point. Vertical force V and horizontal force F fluctuate as the block moves, and only in a portion of time near the start time and before impact, friction force F reached the maximum. Before the impact, vertical force V decreases very quickly so that the block begins sliding. The block jumps after impact as expected. The maximum rotation clockwise is about 23°, less than the critical value of 26.56°, then the potential energy accumulated converts back to kinetic one as the block rocks back. In consequence, the block turns back faster.

4. STABILITY OF DETACHED CONCRETE BLOCK OF DAM

A typical gravity dam is considered, 100 meters in height, 10 meters in top width and 0.75 downstream slope, as shown in Fig.5. Two different crack types are assumed, one is a horizontal line 16.7m from the top(left, type A), another is a fold line perpendicular to both downstream and upstream face(right, type B), points on the downstream face and upstream face are 16.7m and 21.7m below the top, respectively.

Because of the complexity in geometry, a discrete model of FEM is used to solve the problem, and linear elastic material and fixed bottom are assumed in the computation. There is no doubt about the importance of the water in the reservoir influencing the stability of the detached concrete block of a dam during earthquake, but it needs future study to simulate in FEM the gushing water in the crack when it opens and closes as well as the interaction between water and the structure for large displacement problem. For that reason, no water is considered in present stage of study.

The ground acceleration is the same as defined in Eqn. 3.1. The coefficient of friction μ=1.05. The size of the time step is the same as above.

Under horizontal ground acceleration of 1.0g, the detached blocks of both types are not overturned. When the acceleration increases to 1.5g, the type B block has been overturned. Figure 6 that plots the rotation and the angular velocity of the detached blocks demonstrates the responses of two blocks. Jumping and high frequency vibration of the block after impact are examined from the angular velocity. Maximum rocking downward is about 1.44° at 0.589s for type A block, about 1.42° at 0.484s for type B block. Maximum rocking upward is about 17.546° at 1.9675s and the angular velocity is negative at 2.0s for it, so that the type A block stands on the top of the dam after the earthquake (Fig.8 left). It is not so luck for type B block, rocking upward is about 24.542° at 2.0s with a positive angular velocity, its rotation continues to 36.77° at 3.0s (Fig.8 right) and it falls down finally.

If the same ground motion applies in reverse direction to the dam of the type B crack, maximum rocking downward is about 10.13° at 2.147s, maximum rocking upward is about 6.205° at 0.847s (Fig.7). The detached block stands on the top of the dam after the earthquake with a horizontal dislocation of 1.64m to upstream at 3.0s.

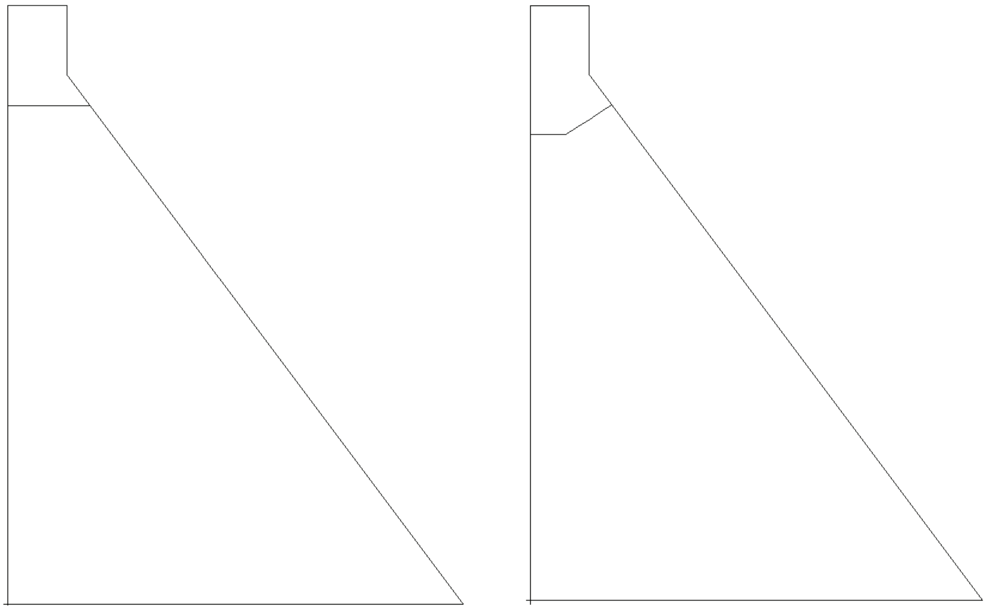

Figure 5 Dam section with crack near the top

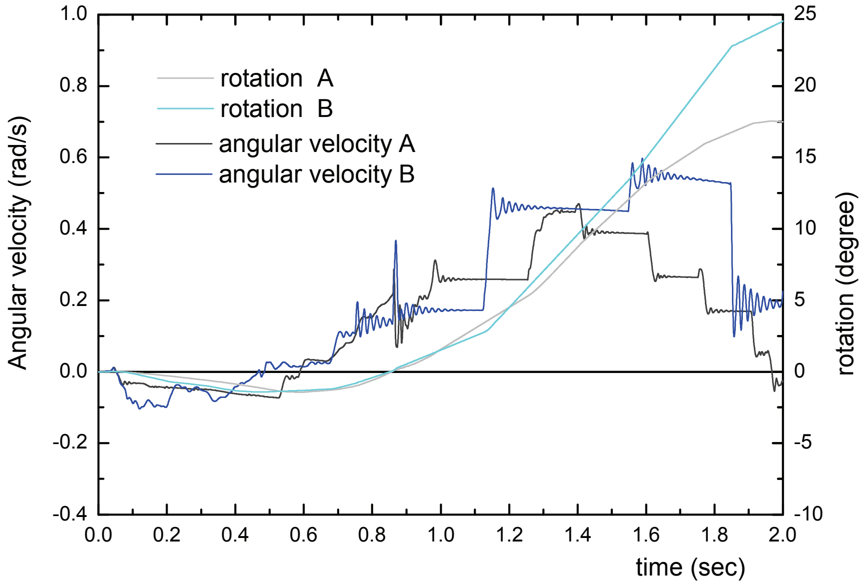

Figure 6 Rotation and angular velocity of the detached block

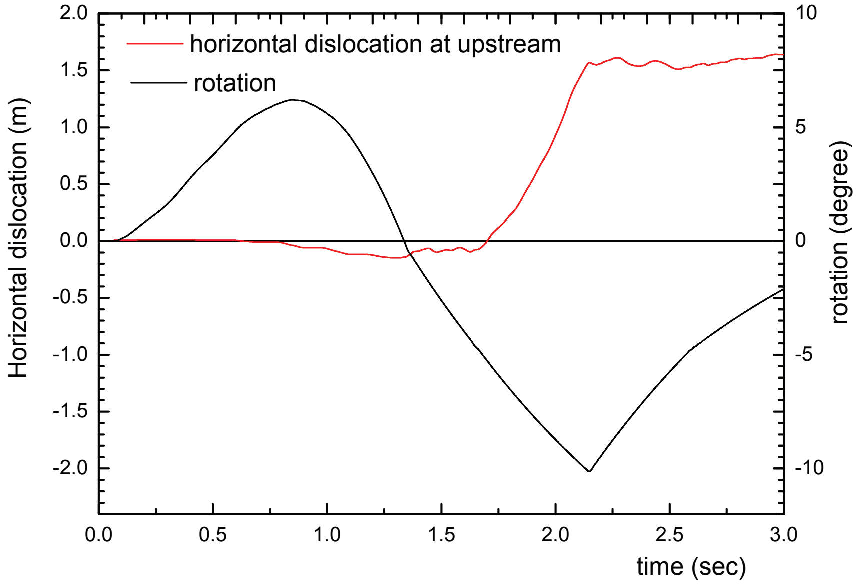

Figure 7 Rotation and horizontal dislocation of the detached block

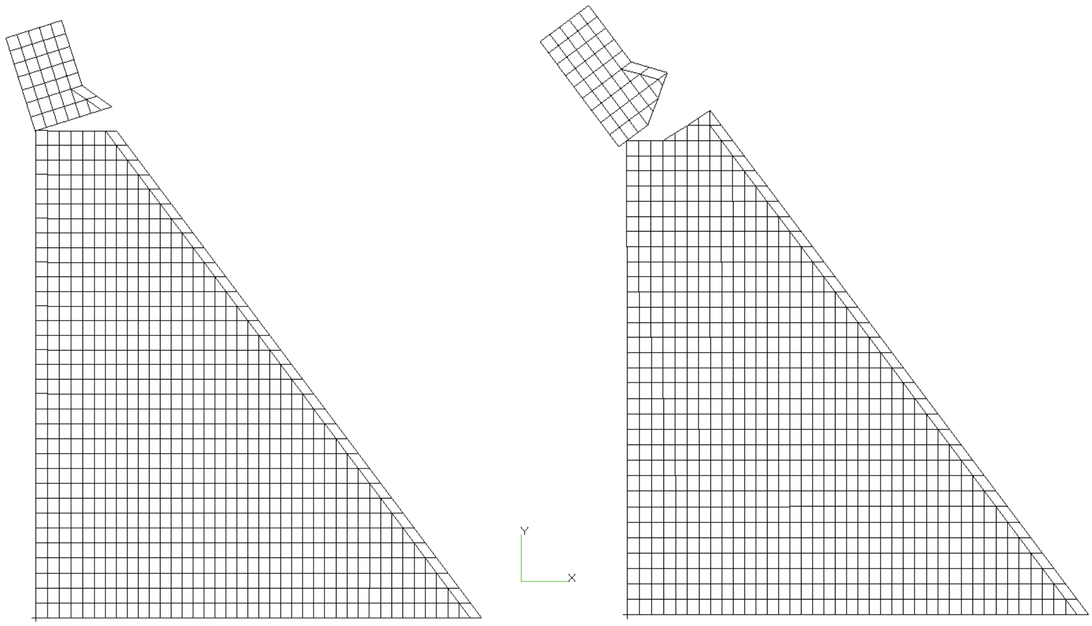

Figure 8 Response of the dam with crack near the top

5. CONCLUSIONS

Analytical procedures have been developed to determine the motion of a free-standing block subjected to horizontal ground acceleration, slide, rock, rock-slide as well as impact when it rocks back and forth. Numerical method FEM has been used to solve this large displacement contact problem of free-standing block. The results reveal that the block begins to slide before the first impact even though it starts in a pure rock mode, a constant coefficient of friction is used in the computation. Both analytical and numerical results consist with each other well. Then the dynamic responses of a concrete gravity dam with crack at upper position forming a detached block have been calculated by FEM procedure. Two types of crack have been assumed for the computation.

The results indicate that the detached block of a dam of Type B crack is more vulnerable to be overturned by horizontal ground motion. It falls down from the top of the dam to the reservoir when the maximum ground acceleration is about 1.5g. But it remains at the top with a horizontal dislocation of 1.64m towards the upstream when the direction of the ground acceleration is reversed.

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