A NOVEL PROCEDURE FOR DETERMINATION OF HYDRODYNAMIC PRESSURE ALONG UPSTREAM FACE OF DAMS DUE TO EARTHQUAKES

I. Gogoi[1] and D. Maity[2]

[1] *Lecturer, Dept. of Civil Engineering, Assam Engineering Institute, Guwahati. India*
[2] *Associate Professor, Dept. of Civil Engineering, Indian Institute of Technology, Kharagpur, India*
Email: gogoi.indrani@gmail.com, dmaity@civil.iitkgp.ernet.in

ABSTRACT :

The estimation of hydrodynamic pressures along the upstream face of the dam is a critical parameter for the accurate analysis and design of a dam. The accurate estimation of the hydrodynamic pressures necessitates the consideration of interaction between the dam, the reservoir and the foundation. The interaction effects of the unbounded domain of the reservoir and the absorptive materials deposited at the reservoir bottom are frequency dependent which can be incorporated in a frequency domain procedure easily. But in a time domain procedure the frequency dependent interaction effects are lost. In a frequency domain solution, the excitation frequencies are extracted from the earthquake signal using a Fourier transformation, but do not give any information about how it varies with time. At the same time, the pressures obtained in a time domain solution do not give any information as to how the pressures may vary with different frequencies at every time instant during an earthquake. A short-time Fourier transform based formulation is presented in this paper to evaluate the hydrodynamic pressures in time domain to account for the frequency dependent interaction effects of the dam-reservoir system.

KEYWORDS: Earthquake, hydrodynamic pressure, infinite reservoir, truncation boundary, absorptive reservoir bottom, Short Time Fourier Transform

1. INTRODUCTION

The seismic analysis of dam-reservoir system has intrigued researchers since 1933 (Westergaard 1933 and von Kármán 1933). The added mass approach was generally used to evaluate the hydrodynamic effects of the reservoir on the dam. Since earthquakes are random in nature, Kotsubo (1960) emphasized that evaluation of hydrodynamic pressure using the added mass approach was not accurate. Consequent research carried out by Chopra (1968), Chopra & Chakrabarti (1972), Saini et. al. (1978) and Maity & Bhattacharya (1999) has shown the importance of considering the compressibility of reservoir water as the hydrodynamic effects of the unbounded reservoir are frequency dependent. Considering the enhanced capabilities of computer processors and increased memory storage, the analysis techniques have improved.

The versatility of the finite element method has motivated researchers to apply the technique in the analysis of dam-reservoir systems. The significant advantage of finite element method is that it enables a single program to treat almost any configuration and to represent complex structures reliably for an accurate stress analysis. However, while using finite element technique in the analysis of a dam-reservoir system, difficulty arises in effectively modelling the large extent of the reservoir that is practically unbounded. The unbounded reservoir has important consequence in the analysis of the dam-reservoir system, as waves traveling to infinity are not reflected back towards the dam. This leads to the development of an energy dissipation mechanism called radiation damping, which is frequency dependent. The accurate modelling of radiation damping is of extreme importance as it effects the hydrodynamic pressure generated in the reservoir and hence the response of the dam. For efficient numerical solution of the system, the unbounded reservoir is truncated at a certain distance away from the dam. Accuracy in the results may be obtained by truncating the reservoir at a larger distance away from the dam. However, this results in an increased cost of computation. If the effect of dam-reservoir interaction is included in the analysis, the cost of computation will further increase to be prohibitive. Therefore, it is necessary to impose an efficient boundary condition at the truncated surface of the reservoir that can account for radiation damping. Various boundary conditions along the truncation surface for the analysis of dam-reservoir system have been developed and used effectively in the frequency domain by Sharan (1992). Although similar boundary conditions have been proposed by Maity & Bhattacharyya (1999) in the time domain, it does not exist for excitation frequency equal to or greater than the resonant frequency. Also, this truncation boundary condition (TBC) can not include the absorption effect of the reservoir bed. The earthquake excitation is generally recorded and used in a time domain solution as a time history, its sensitiveness to the frequency content at every time instant is generally ignored.

For seismic analysis of a dam-reservoir system, the time variation of ground acceleration (a_g) is the most useful way of defining the shaking of the ground during an earthquake. In an accelerogram, the ground motion is defined by numerical values at discrete time instants. Typically the time interval is chosen to be 1/100 to 1/50 of a second (Chopra 1998). A seismic analysis can be carried out either in the time-domain, using time-histories of acceleration or in the frequency domain, calculating frequency content using Fast Fourier Transforms (FFT). Each of these techniques can reveal important information about the behavior of the situation investigated; however, each technique loses some information. Earthquakes are transient and non-stationary events. This means that either the input acceleration or the behavior of the system at different moments in time is not identical. When an accelerogram is converted into the frequency domain using an FFT, the time-dependent behavior is lost. However, when the ground acceleration input is used as a time history, the frequency dependent response of a system can not be determined.

It is evident from Haigh et al.(2002) and Safak (2006) that it is important to account for the frequency content of an earthquake excitation, when the dynamic response of the system analyzed is frequency dependent. The Short Time Fourier Transform (STFT) algorithm used by Nagarajaiah & Varadarajan (2005) is an effective technique that may be used to determine the time-frequency distribution of an earthquake excitation. The basic idea of STFT is to break up the non-stationary signal into small time segments and obtain the FFT of each time segment to ascertain the frequencies that exist in it.

Since, it is observed that the effect of the boundary conditions imposed at the far end of the reservoir is frequency dependent (Gogoi & Maity 2005), the hydrodynamic pressure obtained in a time domain solution due to an earthquake may not be sensitive to the frequency content of the earthquake at every time instant. Therefore, in this paper, a unique method of evaluation of hydrodynamic pressure developed on the upstream face of a concrete dam due to seismic excitation is presented.

2. TIME HISTORY ANALYSIS PROCEDURE WITH FREQUENCY DEPENDENT BOUNDARIES

The water in the reservoir is considered as non-viscous, linearly compressible and is of small amplitudes of motion. The reservoir water is modelled by finite element technique considering pressure as nodal degrees of freedom. The Helmholtz wave equation may be used to obtain an equation to determine the magnitude of hydrodynamic pressure generated due to small amplitude vibration of compressible but non-viscous water, which can be expressed as

$$\nabla^2 p(x,y,t) = \frac{1}{c^2}\ddot{p}(x,y,t) \tag{2.1}$$

Considering the boundary conditions of the reservoir (Fig. 1), the hydrodynamic pressure $p(x,y,t)$ at any point (x,y) at a time instant t can be given,

$$p(x,y,t) = -2\rho\, a\, H_f \sum_{m=1}^{\infty} \frac{\lambda_m^2 I_m}{\beta_m k_m} e^{(-k_m x)} (\Psi_m) e^{i\omega t} \tag{2.2}$$

The frequency dependent boundary conditions along the reservoir bed and the truncated surface are given in details in Gogoi & Maity (2005).

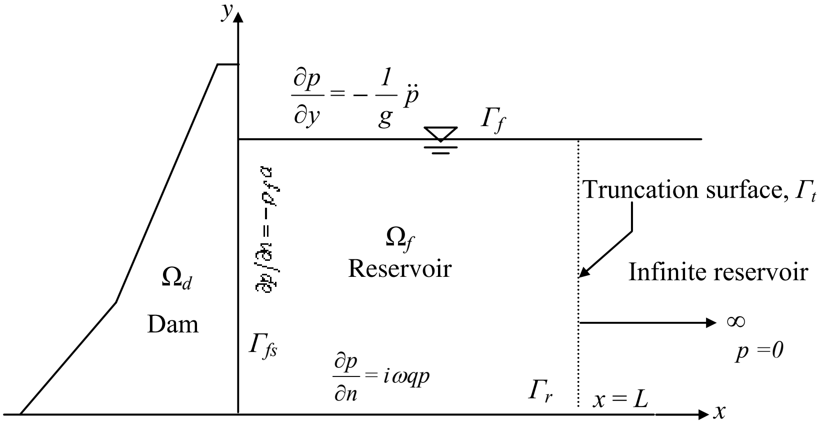

Figure 1 Reservoir and its boundary conditions

The effect of reservoir bottom absorption is an important parameter that influences the response of a dam-reservoir system during an earthquake. The dynamic problem of a dam-reservoir system has been rigorously analyzed in the frequency domain considering the effects of radiation damping and reservoir bottom absorption (Hall & Chopra 1982, Sharan 1992 and Hatami 1997). Many time-domain models consider radiation damping with standard viscous dampers (Vargas-Loli & Fenves, 1989, El-Aidi & Hall, 1989 and Tsai et al.1990). The reservoir bottom absorption effect is frequency dependent, which makes it difficult to incorporate in a time-history analysis as the solution procedure cannot account for the frequency content of the seismic excitation at every time instant. In an endeavor to understand the behavior of structural systems due to earthquake excitation various techniques are being developed to account for the frequency dependent parameters (Haigh et al. 2002, Safak 2006) in a time-history analysis. The boundary conditions at the reservoir bottom and truncation surface respectively are sensitive to excitation frequency. Hence, it is important to estimate the frequency content of an earthquake signal at every time step to effectively account for radiation damping and reservoir bottom absorption effect.

A Fourier transformation generally breaks down a signal into constituent sinusoids of different frequencies. While transforming a time-based signal to frequency-domain, the time information is lost and it is difficult to obtain the frequency information at a particular time instant. To overcome this deficiency, Gabor (1946) adapted the Fourier transform to analyze only a small section of the signal at a time - a technique called *windowing* the signal. Gabor's adaptation, called the *Short-Time Fourier Transform* (STFT), maps a signal into a two-dimensional function of time and frequency. However, the information obtained by STFT has limited precision, and that precision is determined by the size of the window.

Here, the technique commonly used in digital signal processing called Short Time Fourier Transformation (STFT) has been adopted to determine the frequencies at a time instant. To capture the time variation of the frequency contents of the signal, the signal $S(\tau)$ is multiplied by a sliding window $h(\tau\text{-}t)$, centered at time t, and taking the Fourier transform of the weighted signal. Mathematically,

$$S_t(\omega) = \int S(\tau) h(\tau - t) e^{-i\omega\tau} d\tau \tag{2.3}$$

gives the short time Fourier transform at every window. The spectral density of the modified signal at every time instant, t can be obtained by

$$P(t,\omega) = \left|S_t(\omega)\right|^2 \tag{2.4}$$

The instantaneous frequency at time t can be located from the center of the $S_t(\omega)$ by taking its first moment as

$$\omega_t = \frac{\int \omega \left|S_t(\omega)\right|^2 d\omega}{\int \left|S_t(\omega)\right|^2 d\omega} \tag{2.5}$$

This instantaneous excitation frequency of the earthquake signal so obtained can be incorporated in the boundary conditions at the reservoir-reservoir bed interface and truncation boundary.

3. SEISMIC ANALYSIS OF DAM-RESERVOIR SYSTEM USING SHORT TIME FOURIER TRANSFORM

A dynamic analysis procedure in time domain cannot account for the spectral contents of the seismic excitation when the input is in the form of time history. The present boundary conditions at the reservoir bottom and the truncated surface are frequency dependent. Therefore, to increase the efficiency and accuracy of present algorithm for seismic analysis in time domain, the frequency dependent reservoir bottom absorption effect can be accounted for by using the excitation frequency (ω) of the earthquake at every time step. The spectral content of the seismic excitation is extracted and incorporated in the present analysis as explained.

3.1 Extraction of Frequencies from Time History Data of Recorded Earthquake

The earthquake data represented by accelerograms as shown in Figure 2 do not indicate the frequency components of the earthquake signal. A Fast Fourier Transform (FFT) converts the signal to frequency domain and can be used to obtain the power spectrum, which is a measurement of power at various frequencies. The accelerations due to Koyna earthquake (1967) are recorded at a time step, $\Delta t = 0.01$. Hence, this set of data is sampled at a frequency, $f_n = 100$ Hz. The Short Time Fourier Transform (STFT) of the Koyna earthquake (Figure 2) signal is evaluated at at $\Delta t = 0.005$ seconds, where the frequencies of the Koyna earthquake reach a peak value of 80 rad/sec (Figure 3). The frequency inputs in the proposed algorithm at each time step are the frequencies corresponding to the maximum spectral density in the moving window. These dominant frequencies are adopted as excitation frequency at each time step.

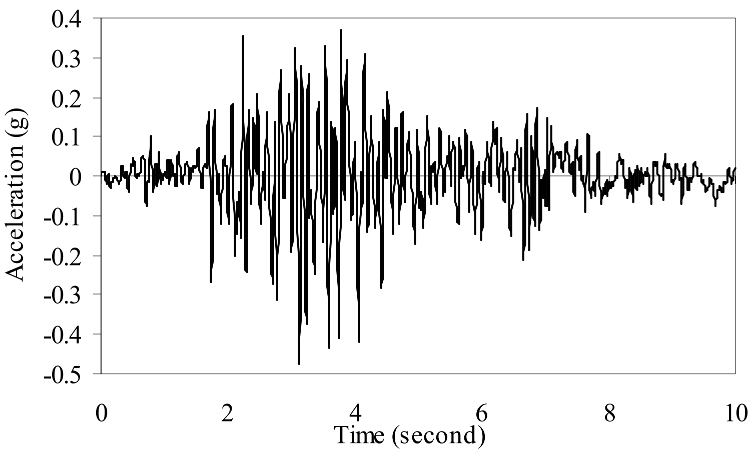

Figure 2 Horizontal accelerogram of Koyna earthquake, December 11, 1967

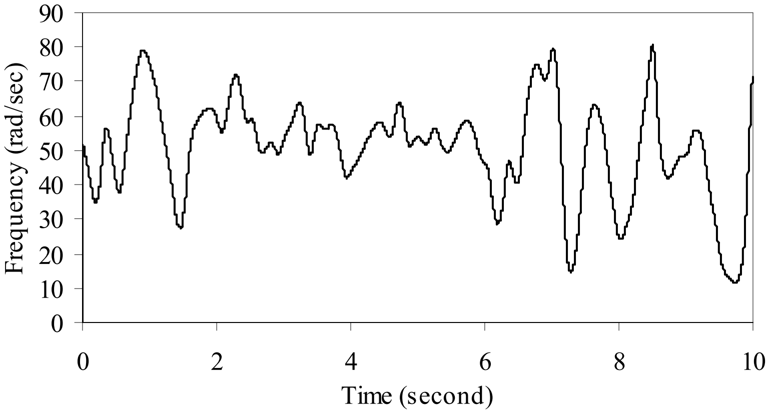

Figure 3 STFT of Koyna earthquake, 1967 ($\Delta t = 0.005$ sec)

3.2 Hydrodynamic Pressure neglecting Dam Flexibility due to Seismic Excitation

The hydrodynamic pressure developed at the bottom of the dam-reservoir interface due to seismic excitation is evaluated. Due to lack of analytical technique to evaluate the hydrodynamic response of the reservoir due to seismic excitation, the accuracy of the proposed technique is verified by convergence study. The effectiveness of the proposed boundary condition using Short Time Fourier Transform (STFT) is examined for different reservoir depths and earthquake excitations. The hydrodynamic pressure coefficient is evaluated for a reservoir depth of (i) 30m and (ii) 150m considering the frequency content of the Koyna earthquake. A convergence study was carried out by Gogoi & Maity (2005) to study the effectiveness of the present boundary condition. It was observed from the results that a reservoir length of $L = 0.5H_f$ is effective for all ranges of excitation frequencies to determine the hydrodynamic pressure in the reservoir due to harmonic excitation. Therefore, the hydrodynamic pressure coefficients are evaluated at the vertical upstream face of the dam considering a length of the reservoir, $L = 0.5\ H_f$ for the first 6 seconds and compared with that obtained at $L = 3.0H_f$. The reflection coefficient is considered as 0.5 for the seismic analysis. It is seen from the Figures 4 and 5, that the hydrodynamic response due to Koyna earthquake can be obtained accurately by imposing the proposed truncation boundary at a distance of $L = 0.5H_f$. This is because the fundamental frequencies of the reservoir having depth of 30m and 150m are less than the peak frequency content of Koyna earthquake, which is approximately 80 rad/sec.

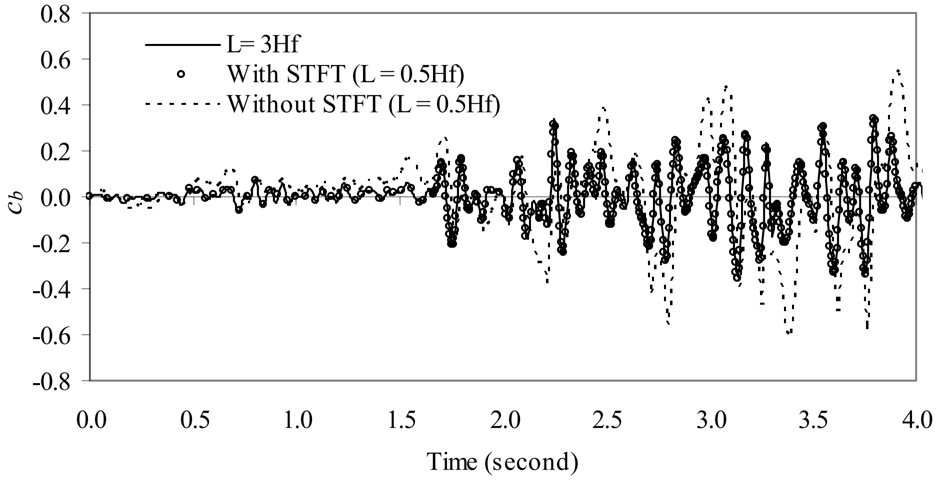

Figure 4 Maximum hydrodynamic pressure coefficient at the bottom of dam-reservoir interface due to Koyna earthquake ($H_f = 30$ m)

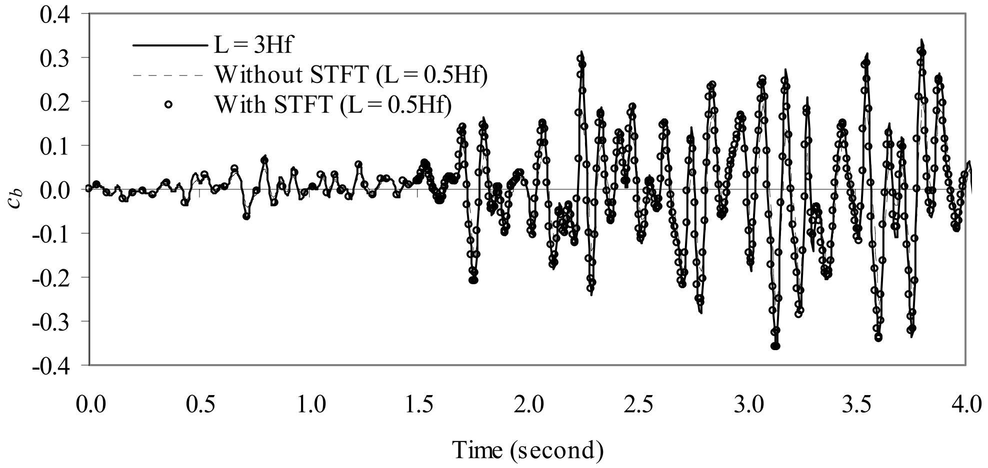

Figure 5 Maximum hydrodynamic pressure coefficient at the bottom of dam-reservoir interface due to Koyna earthquake ($H_f = 150$m)

3.3 Hydrodynamic Pressure considering Dam Reservoir Interaction due to Seismic Excitation

To evaluate the effectiveness of the developed truncation boundary condition (TBC) with the incorporation of Short Time Fourier Transform (STFT) in the proposed algorithm for dam-reservoir interaction, a typical dam-reservoir system is considered having geometry as shown in Figure 6. The dimension and the material properties of the dam in the present case are: height of the dam = 103 m; width at the top of the dam is 14.8 m and at the base is 70.0 m, modulus of elasticity = 31500 MPa; Poisson's ratio = 0.235 and mass density = 2415.816 kg/m^3. Structural damping is considered as 3%. The dam is discretized with 8-noded quadratic elements and is analyzed using plain strain formulation. In the present investigation, the dynamic magnifications of strength and stiffness parameters due to rapid application of seismic strains are not considered. The depth of the water is considered to be equal to the height of the dam. The acoustic wave speed in water and the mass density of water are considered to be 1438.7m/sec and 999.8 kg/m^3 respectively. Due to lack of classical solution, the response of the coupled dam-reservoir system is compared with those obtained at $L = 3.0H_f$. The crest displacement of the dam and the hydrodynamic pressure coefficient (c_b) at the bottom of the upstream face of the dam due to the horizontal component of Koyna earthquake are presented herein. It is evident from Figs. 18 and 19 that at $L = 0.5H_f$, the algorithms using STFT gives crest displacements and pressure coefficients without much loss in accuracy.

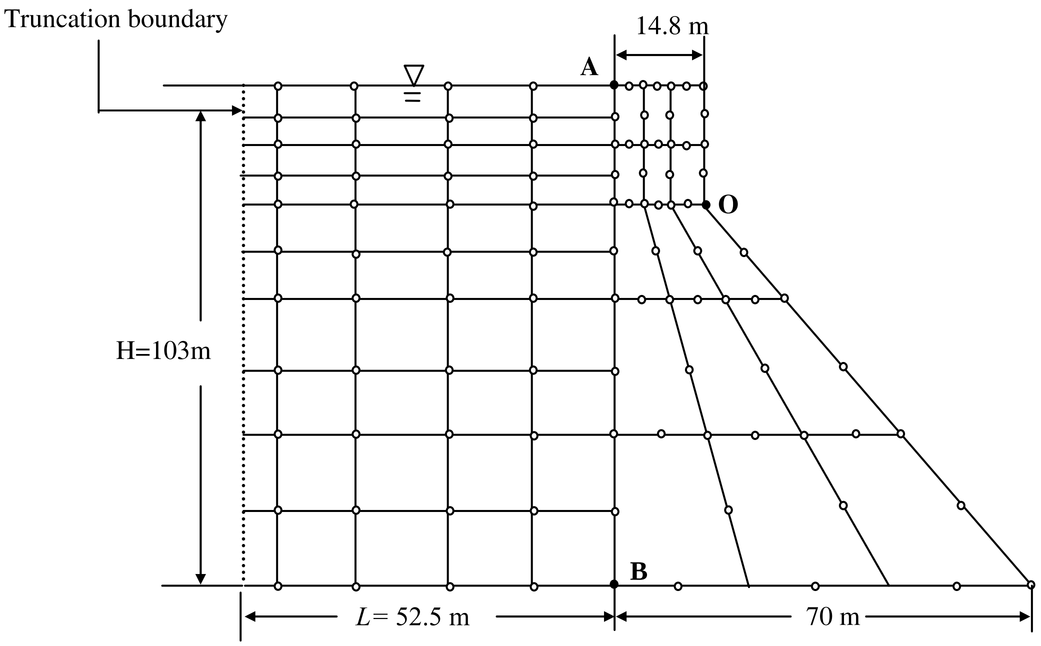

Figure 6 Finite element mesh of dam-reservoir system

The random vibration caused by an earthquake generally consists of many frequencies. Hence, it is important to evaluate the effectiveness of the proposed algorithm for seismic analysis of dam-reservoir system in time domain. To observe the effect of the reservoir bottom absorption due to seismic excitation, the dam with the same material and geometrical properties as above is considered. The variation of normalized crest displacement due to Koyna earthquake (Fig. 2) is plotted to observe the effect of absorption at the reservoir bottom. It is observed from Figures 7 and 8 that the crest displacement and hydrodynamic pressure coefficient (c_b) is reduced considerably due to absorptive reservoir bottom with reflection coefficient of $\alpha = 0.5$. The hydrodynamic pressure at the upstream face of the dam depends on the acceleration of the dam-reservoir interface. The acceleration of the flexible dam thus affects the hydrodynamic response due to seismic excitation.

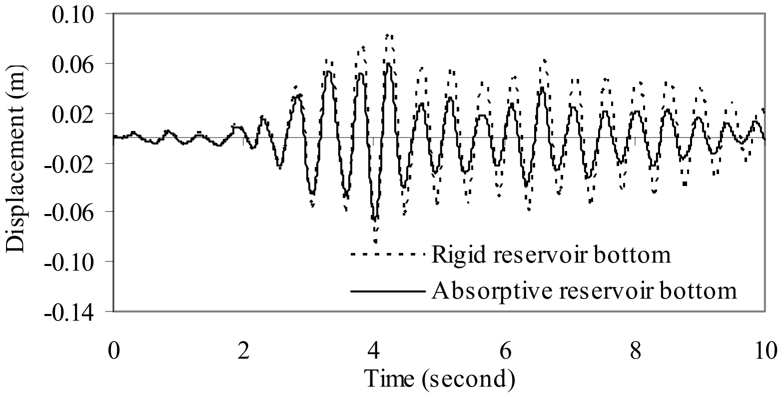

Figure 7 Effect of reservoir bottom absorption on crest displacement of the dam due to Koyna earthquake

The principal stresses σ_{p1} (maximum tensile and minimum compressive) and σ_{p2} (maximum compressive and minimum tensile) at point O (Figure 6) are plotted in Figures 9 and 10 respectively. It is observed from these graphs that the principal stresses in the dam reduce significantly in the presence of the absorptive reservoir bottom.

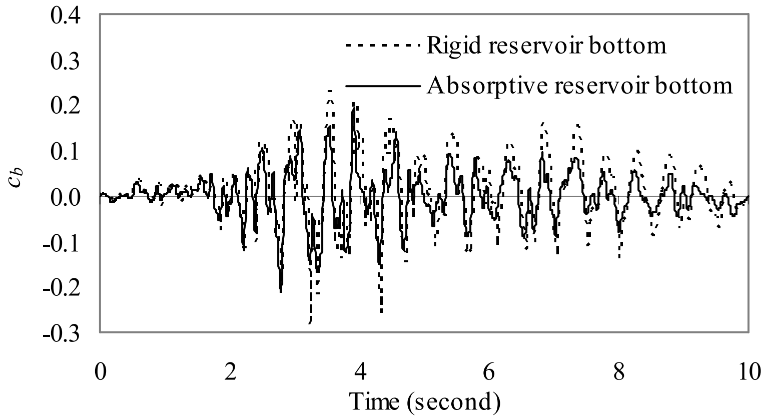

Figure 8 Effect of reservoir bottom absorption on hydrodynamic pressure coefficient at point B due to Koyna earthquake

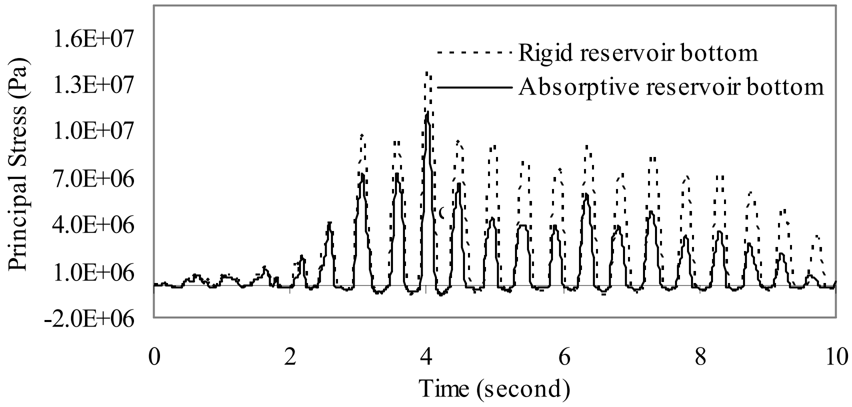

Figure 9 Effect of reflection coefficient on principal stress σ_{p1} at point O of the dam due to Koyna earthquake

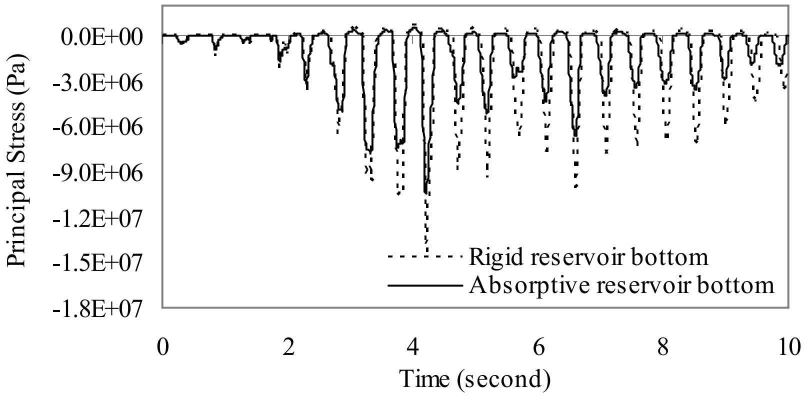

Figure 10 Effect of reflection coefficient on principal stress σ_{p2} at point O of the dam due to Koyna earthquake

4. CONCLUSIONS

A novel procedure of seismic analysis in the time domain analysis is presented in this paper that accounts for the frequency dependent boundary conditions at the reservoir bottom and truncation surface. The algorithm proposed considers the frequency content of earthquake excitation so that the damping parameters at the reservoir bottom and truncation surface can be estimated accurately. The non-stationary earthquake signal is divided into small time segments and the FFT of each time segment is obtained to ascertain the frequencies that exist in it. The dominant frequency at every time step is extracted and is used as an input in the seismic analysis. The effectiveness of the developed far-boundary condition has been increased with the incorporation of Short Time Fourier Transform (STFT) for the analysis of dam-reservoir system under seismic excitation. The proposed time-frequency hybrid method is advantageous as the frequency dependent responses of the dam-reservoir system can be obtained in a time domain procedure. The implementation of this technique is simple as the time domain procedure remains the same and the algorithm can be modified to account for the dominant frequencies at every time step. As the procedure is in time domain and can efficiently account for different excitation frequencies the transformation of frequency dependent dynamic stiffness matrix as used in various time-frequency hybrid methods can be avoided.

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