DAM-RESERVOIR INTERACTION ANALYSIS USING FINITE ELEMENT MODEL

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ABSTRACT :

In this paper, dam-reservoir interaction is analyzed using finite element approach. The reservoir fluid domain is assumed to be incompressible and inviscid. In the derivation of boundary conditions, it is assumed that the interface of dam and reservoir is vertical. Moreover, bottom of reservoir is assumed to be rigid and horizontal. The governing equation with related boundary conditions is implemented in the finite element code considering horizontal and vertical earthquake components. The weighted residual standard galerkin method with 8-node elements is used for developing finite element model. Both sommerfeld boundary condition and perfect damping boundary condition are developed for truncating surface of unbounded fluid domain and the results of two boundary conditions are compared with analytical results.

KEYWORDS: Interaction, finite element, hydrodynamic pressure, earthquake, dam

1. INTRODUCTION

There are a large number of concrete dams in the world. Some of them are in seismically active areas. The analysis of dams is a complex problem due to the dam-reservoir interaction. An important factor in the design of dams in seismic regions is the effect of hydrodynamic pressure exerted on the face of dam as a result of earthquake ground motions. The seismic response of a gravity dam is influenced by its interaction with reservoir.

The hydrodynamic pressure acting on dam faces during earthquakes has been recognized as a main loading in the design of dams. The first analysis of hydrodynamic force on dam faces during earthquakes was reported by Westergaard (1933). The results were checked by a simplified analysis. In the following years, many researchers have extensively studied hydrodynamic analysis of dam using various methods.

For an accurate analysis of hydrodynamic pressure on the dam having irregular geometries, the reservoir is generally treated as an assemblage of finite elements. The finite element method is becoming more popular in reservoir simulation, partly due to its flexibility in dealing with boundaries. The element shape is not required to be square so that the element mesh can handle a very complex geometry. Zienkiewicz et al. (1965) studied the dynamic response of submerged structures in incompressible water using finite element method. Chopra (1970) used the finite element method as a numerical technique for dam-reservoir analysis. He studied the response of the hydrodynamic force on a dam impounding reservoir under horizontal excitation.

In the finite element analysis of dam-reservoir interaction problems arise due to unbounded reservoir domain. This problem is solved by truncating the infinite reservoir domain at a certain distance from dam-reservoir interface. For an accurate analysis, the behavior of outgoing pressure waves at the truncation surface must be truly represented. The applied truncated boundary at reservoir farfield depends on geometrical configuration. For a finite reservoir, the reflected waves from the truncated farfield are not negligible and may result in significant increase in induced hydrodynamic pressure in the reservoir. For the case of an infinite reservoir, the location of truncated boundary condition for the outgoing pressure waves in a numerical model with limited length is very important in hydrodynamic analysis. The proper boundary condition at truncated boundary of reservoir has been the subject of many studies in dynamic analysis of structures.

Zienkiewicz et al. (1977) examined the formulation of infinity conditions in the solution of pressure wave equation in the reservoir. They concluded that sommerfeld boundary condition is appropriate for large reservoir model and can be easily incorporated in the finite element discretization of the reservoir domain. Hall and Chopra (1982) studied the hydrodynamic effects of the impounded reservoir on the seismic response of gravity dams using one-dimensional boundary conditions for the radiation of waves in truncated boundary. Sharan (1985) proposed a radiation boundary condition for the truncated boundary of the incompressible reservoir model. His proposed boundary condition was based on analytical solution for the pressure wave equation in the reservoir under horizontal earthquake component in frequency domain. He extended the proposed truncated boundary condition for a compressible model.

The objective of this paper is present a formulation for dam-reservoir system analysis using finite element model considering horizontal and vertical components of earthquake. . In the derivation of boundary conditions, it is assumed that the reservoir fluid domain is incompressible. The interface of dam and reservoir is considered vertical and bottom of reservoir is assumed to be rigid and horizontal.

2. FORMULATIN OF UNBOUNDED RESERVOIR DOMAIN

For incompressible and inviscid fluid, the hydrodynamic pressure resulting from the ground motion of a rigid dam satisfies the Laplace equation in the following form:

$$\nabla^2 P = 0 \tag{4.1}$$

Following boundary conditions are defined by assuming effects of surface waves and viscosity of the fluid are neglected:

$$\frac{\partial P}{\partial x} = -\rho a_x \quad \text{on} \quad S_1 \tag{4.2}$$

$$\frac{\partial P}{\partial y} = -\rho a_y \quad \text{on} \quad S_2 \tag{4.3}$$

$$P = 0 \quad \text{on} \quad S_3 \tag{4.4}$$

$$P = 0 \quad \text{on} \quad S_4 \tag{4.5}$$

In the above equations, a_x and a_y are the earthquake acceleration subjected on dam face and reservoir bottom in the horizontal and vertical direction, respectively. The reservoir domain has been shown in figure 1. S_1, S_2, S_3 and S_4 are dam-reservoir interface, reservoir bottom, truncated boundary and reservoir free surface, respectively.

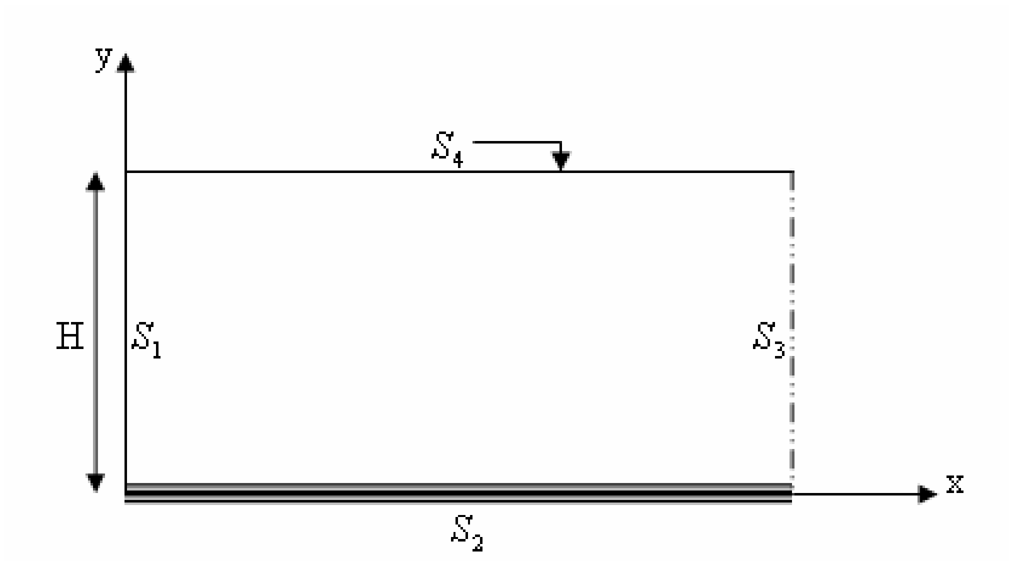

Figure 1:Reservoir domain and boundary conditions

Analytical solution of Eqn. (4.1) due to given boundary conditions is:

$$P = P_a + P_v = 2\rho a_x H \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\lambda_n^{2}} \exp(-\lambda_n \frac{x}{H}) \cos(\lambda_n \frac{y}{H})$$
$$+ \frac{\rho a_y}{\lambda_n \cot \lambda_n H} (\cos \lambda_n y - \cot \lambda_n H \sin \lambda_n y) \tag{4.6}$$

Where $\lambda_n = \frac{2n-1}{2}\pi$ and $n = 1,2,3,\dots$.

3. FINITE ELEMNT FORMULATION

Assuming hydrodynamic pressure to be unknown, the pressure at any point inside an element can be written by

$$P^{(e)}(x,y) = [N(x,y)]\vec{P}^{(e)} = \sum_{i=1}^{m} N_i(x,y) P_i^{(e)} \tag{4.7}$$

Where $\vec{P}^{(e)}$ is the vector of pressure at the element nodes and $[N(x,y)]$ is the matrix of interpolation functions.

To solve with finite element method, reservoir domain was divided to E element with m node. Using standard galerkin method, we can write the Eqn. (4.1) in the following form :

$$\iint_{S^{(e)}} N_i \left[\frac{\partial^2 P^{(e)}}{\partial x^2} + \frac{\partial^2 P^{(e)}}{\partial y^2} \right] dS = 0 \qquad i = 1,2,\ldots,m \tag{4.8}$$

In which N_i is the interpolation function.

According to mentioned boundary conditions and using of gauss-green theorem the last equation is written as following:

$$-\iint_{S^{(e)}} \left[\frac{\partial N_i}{\partial x} \sum_{i=1}^{m} \frac{\partial}{\partial x}(N_i(x,y)P_i^{(e)}) + \frac{\partial N_i}{\partial y} \sum_{i=1}^{m} \frac{\partial}{\partial y}(N_i(x,y)P_i^{(e)}) \right] dS + \int_{C^{(e)}} N_i \rho a_n dC = 0 \tag{4.9}$$

or

$$-\iint_{S^{(e)}} \left(\frac{\partial N_i}{\partial x} \left[\frac{\partial N_1}{\partial x} P_1^{(e)} + \frac{\partial N_2}{\partial x} P_2^{(e)} + \ldots + \frac{\partial N_m}{\partial x} P_m^{(e)} \right] dS + \frac{\partial N_i}{\partial y} \left[\frac{\partial N_1}{\partial y} P_1^{(e)} + \frac{\partial N_2}{\partial y} P_2^{(e)} + \ldots + \frac{\partial N_m}{\partial y} P_m^{(e)} \right] dS \right) + \int_{C^{(e)}} N_i \rho a_n dC = 0 \tag{4.10}$$

At last the above equation is written as following:

$$\iint_{S^{(e)}} \left[\frac{\partial N_i}{\partial x}\frac{\partial N_1}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_1}{\partial y} \quad \frac{\partial N_i}{\partial x}\frac{\partial N_2}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_2}{\partial y} \quad \cdots \quad \frac{\partial N_i}{\partial x}\frac{\partial N_m}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_m}{\partial y} \right] \begin{Bmatrix} P_1^{(e)} \\ P_2^{(e)} \\ \vdots \\ P_m^{(e)} \end{Bmatrix} dS - \int_{C^{(e)}} N_i \rho a_n dC = 0 \tag{4.11}$$

For $i = 1,2,\ldots,m$ Eqn. (4.11) is written in the below form:

$$\iint_{S^{(e)}} \begin{bmatrix} \frac{\partial N_1}{\partial x}\frac{\partial N_1}{\partial x} + \frac{\partial N_1}{\partial y}\frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x}\frac{\partial N_2}{\partial x} + \frac{\partial N_1}{\partial y}\frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_1}{\partial x}\frac{\partial N_m}{\partial x} + \frac{\partial N_1}{\partial y}\frac{\partial N_m}{\partial y} \\ \vdots & & & \\ \frac{\partial N_m}{\partial x}\frac{\partial N_1}{\partial x} + \frac{\partial N_m}{\partial y}\frac{\partial N_1}{\partial y} & \frac{\partial N_m}{\partial x}\frac{\partial N_2}{\partial x} + \frac{\partial N_m}{\partial y}\frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_m}{\partial x}\frac{\partial N_m}{\partial x} + \frac{\partial N_m}{\partial y}\frac{\partial N_m}{\partial y} \end{bmatrix} \begin{Bmatrix} P_1^{(e)} \\ P_2^{(e)} \\ \vdots \\ P_m^{(e)} \end{Bmatrix} dS - \int \rho a_n \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_m \end{bmatrix} dC = 0 \tag{4.12}$$

The above equation is considered as following:

$$K^{(e)}\,\vec{P}^{(e)} = \vec{F}^{(e)} \tag{4.13}$$

where

$$K^{(e)} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} \\ \frac{\partial N_2}{\partial x} & \frac{\partial N_2}{\partial y} \\ \vdots & \vdots \\ \frac{\partial N_P}{\partial x} & \frac{\partial N_P}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_P}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_P}{\partial y} \end{bmatrix} = [B]^T[B] \tag{4.14}$$

or

$$K_{i,j}{}^{(e)} = \frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} \tag{4.15}$$

and

$$\vec{F}^{e} = -\int_{C^{(e)}} \rho a_n \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_m \end{bmatrix} dC^{(e)} \tag{4.16}$$

In the above equation, $C^{(e)}$ is contained the boundaries of dam-reservoir and reservoir-foundation interface and S is reservoir domain.

Obtained Eqn. (4.13) is placed in total matrix regarding to element location. Then we can write for whole domain:

$$[K]\vec{P} = \vec{F} \tag{4.17}$$

Eqn. (4.17) is solved to find the reservoir response with so-called boundary conditions.

4. CASE STUSY

We have assumed steady state condition and incompressibility characteristic for water to describe the finite element model used in present study. Due to availability of analytical solution of prescribed problem we used simple boundary assumptions to compare efficiency and accuracy of finite element model with applying simple boundaries.

To solve the problem and evaluate the produced finite element model, reservoir of Sefidrud dam in Iran has been considered as a case study. Its reservoir height is 106 meter. Density of water assumed 1000 kg/m^3 and dam excitation acceleration considered the maximum of Manjil earthquake accelerator that was exerted on Sefidrud dam in 1993. Standard Galerkin method with 8-node elements was used to model the reservoir. Figure 2 shows the meshing of Sefidrud reservoir domain.

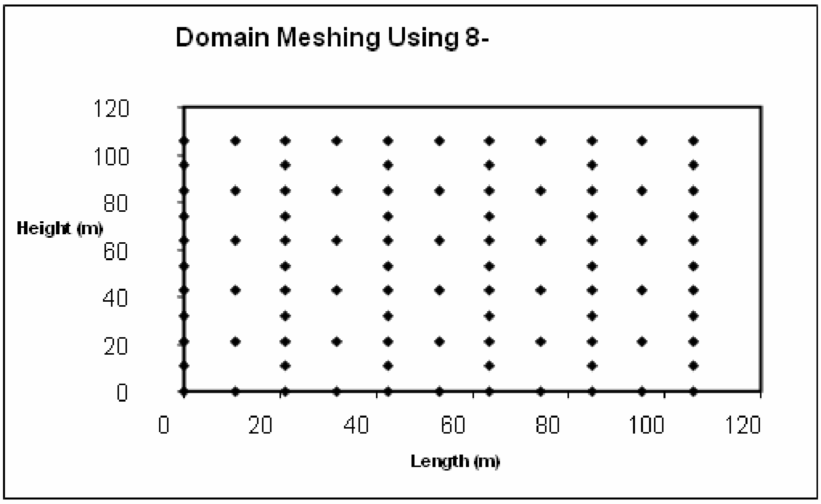

Figure 2: Meshing of Sefidrud reservoir domain

Induced hydrodynamic pressure on dam-reservoir interface was achieved with before mentioned boundary conditions. Diagrams 3 and 4 depict results for different length to height ratio of dam. Analytical solution shows agreement with finite element results.

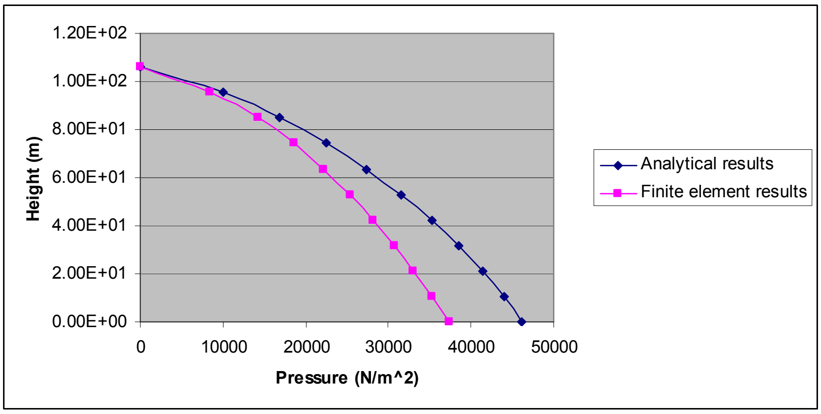

Figure 3: Hydrodynamic pressure distribution curve at dam height for $L/H = 1.0$

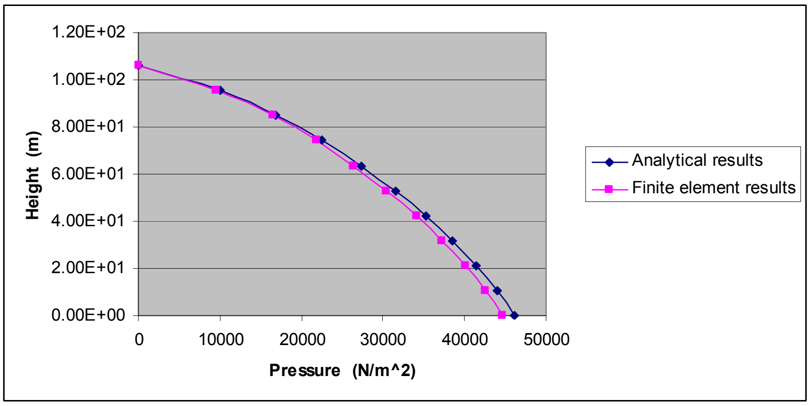

Figure 4: Hydrodynamic pressure distribution curve at dam height for $L/H = 2.0$

Diagrams show that the deviation of present model from analytical solution is negligible in $L/H=2$ ratio while for $L/H=1$ ratio there is error. This enables us to use 8-node finite element model when truncation boundary distance is twice the height of dam.

It is important to describe efficient farfield boundary condition for an effective finite element model. So-called distant boundary condition implies complete dissipation of pressure wave when they pass the truncated boundary. The error resource is because of implementation of previously assumption which in analytical solution full dissipation occurs in an infinite distance. Considering the truncated boundary more far from dam will minimize the error at the expense of increasing the calculation effort.

In some researches, sommerfeld boundary condition is used instead of aforementioned boundary condition with Eqn. (4.4). It is described for incompressible fluid as follow:

$$\frac{\partial P}{\partial x}=0 \tag{4.18}$$

Aforementioned example has been analyzed with farfield boundary condition described with Eqn. (4.18) using finite element model. Diagrams 5 and 6 depict results for different length to height ratio of dam for this case. Results were compared with analytical solution.

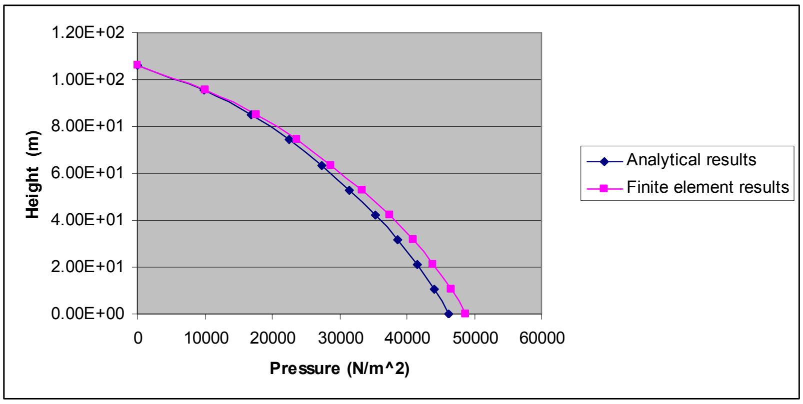

Figure 5: Hydrodynamic pressure distribution curve at dam height for $L/H=1.0$

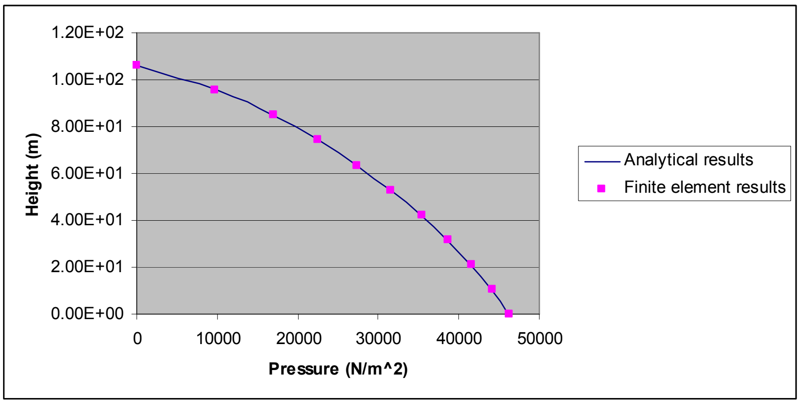

Figure 6: Hydrodynamic pressure distribution curve at dam height for $L/H=2.0$

Finally, maximum hydrodynamic pressure results have been shown in table (4.1) for both farfield boundary conditions in different ratio of length to height and comparison is made with analytical solution.

Table 4.1: Maximum of hydrodynamic pressure on dam (N/m^2)

$\frac{L}{H}$	Maximum pressure with $\partial P/\partial x = 0$ at farfield	Maximum pressure with $P = 0$ at farfield	Maximum pressure with analytical solution	Error with $\partial P/\partial x = 0$ at farfield (%)	Error with $P = 0$ at farfield
1	48756.09	37335.09	46163.13	5.620	19.12
2	46269.67	44666.12	46163.13	0.230	3.243
3	46166.77	45868.34	46163.13	0.007	0.638
4	46162.35	46101.82	46163.13	0.002	0.133

5. CONCLUTION

Considering results from provided finite element model regardless supposed radiation condition, indicates efficiency and accuracy of 8-node finite element method.
Comparing responses related to two states of farfield boundary condition and analytical solution one can conclude that on condition of avoiding extra computational effort, if farfield truncated boundary is selected near the dam, complete damping boundary condition at farfield will possess better results in comparison to sommerfeld boundary condition. In this case the sommerfeld boundary condition is not the good choice to represent behavior of wave radiation at farfield and proper radiation should be searched. If the truncated boundary is located rather far from the dam, obtained results for two considered states of farfield boundary condition are approximately the same.
According to provided results it can be conclude for incompressible fluid, if the distance of truncated location of reservoir domain is twice or more than height of reservoir, considering sommerfeld boundary condition or complete damping condition is not so influential on hydrodynamic response. It can be generalized to the other radiation boundary conditions for incompressible fluid.

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