MODAL ANALYSIS OF CONCRETE ARCH DAMS IN TIME DOMAIN INCLUDING DAM-RESERVOIR INTERACTION

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ABSTRACT :

In this article, two approaches are studied for modal analysis of concrete arch dams in time domain. The results of these investigations are compared against the direct approach which is envisaged as an exact method. The decoupled modal approach relies on independent modes of the dam and the reservoir, and coupled modal approach, which employs mode shapes of coupled system. An asymmetric eigenvalue problem is required to be solved to calculate the coupled modes, which makes the programming very complicated. However, in the decoupled modal approach, a symmetric eigenvalue problem is employed which is resulted by elimination of asymmetric parts of the initial equation. The mode shapes extracted through this problem are utilized to calculate the response. The analysis of Shahid Rajaee concrete arch dam is considered as a controlling example, and the number of required mode shapes for an accurate analysis is compared for these two modal approaches.

KEYWORDS: Dam-reservoir interaction, Coupled modal approach, Decoupled modal approach

1. INTRODUCTION

Dynamic analysis of concrete dams is a quite demanding procedure because it is a complex and time-consuming process. In this regard, governing dynamic characteristics of the coupled dam-reservoir system must be considered. To this end, various methods are proposed by investigators [1,2]. A method that can be utilized for this purpose is modal approach. This technique is very efficient due to reduction in degrees of freedom of the system. In this approach, mode shapes and natural frequencies are calculated based on an eignvalue problem. Number of unknowns equals to the number of considered modes. Therefore, the modal approach is computationally very efficient in comparison to direct method, which number of unknowns equals to number of DOFs. The turning point about this method is the eignproblem. The governing matrices of coupled dam-reservoir system are unsymmetric and it is computationally cumbersome to solve such an eigenproblem. While, decoupled modes can be obtained efficiently by using standard eigen-solution routines, due to the fact that they are the eigenvectors of the decoupled dam-reservoir system.
This paper studies two modal approaches for concrete arch dams in time domain. Decoupled modal approach deals with the mode shapes of dam alone(empty reservoir) and mode shapes of reservoir with rigid walls at the boundaries except at the free surface. Furthermore, coupled modal analysis is also utilized which uses coupled mode shapes of dam-reservoir system. A specific technique is implemented to solve unsymmetric eigenproblem. Meanwhile, direct method (pseudo–symmetric approach) is employed to evaluate the accuracy of modal methods. In order to carry out the numerical examples a computer program is developed in FORTRAN. Eventually, the efficiency and the advantages of these two modal methods are studied.

2. METHOD OF ANALYSIS

In this study, the dam is discretized by solid finite elements, while fluid elements are utilized for the reservoir region. Considering Lagrangian-Eulerian method for this case, the coupled equations of the system may be written as:

$$\begin{bmatrix} \mathbf{M} & 0 \\ \mathbf{B} & \frac{1}{\rho c^2}\mathbf{G} \end{bmatrix}\begin{bmatrix} \ddot{\mathbf{r}} \\ \ddot{\mathbf{p}} \end{bmatrix}+\begin{bmatrix} \mathbf{C} & 0 \\ 0 & \frac{1}{\rho}\mathbf{L} \end{bmatrix}\begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{p}} \end{bmatrix}+\begin{bmatrix} \mathbf{K} & -\mathbf{B}^T \\ 0 & \frac{1}{\rho}\mathbf{H} \end{bmatrix}\begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix}=\begin{bmatrix} -\mathbf{M\,J\,a}_g \\ -\mathbf{B\,J\,a}_g \end{bmatrix} \tag{2.1}$$

$\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ in this relation represent the mass, damping and stiffness matrices of the dam body. $\mathbf{G}$, $\mathbf{L}$ and $\mathbf{H}$ are assembled matrices of fluid domain. The unknown vector is comprised of two parts, $\mathbf{r}$ is the vector of nodal relative displacements and the vector $\mathbf{p}$ represents nodal pressures. Meanwhile $\mathbf{a_g}$ and $\mathbf{J}$ denote the vector of ground accelerations and the influence matrix. Furthermore, $\mathbf{B}$ is often referred to as interaction matrix. Eqn. 2.1 can also be written alternatively in a more compact form:

$$\overline{\mathbf{M}}\ddot{\overline{\mathbf{r}}} + \overline{\mathbf{C}}\dot{\overline{\mathbf{r}}} + \overline{\mathbf{K}}\overline{\mathbf{r}} = -\overline{\mathbf{M}}\overline{\mathbf{J}}\mathbf{a_g} \tag{2.2}$$

The exact form of different matrices in Eqn. 2.2 can be obtained by comparing it to Eqn. 2.1. It is obvious that $\overline{\mathbf{M}}$ and $\overline{\mathbf{K}}$ are unsymmetric matrices. In order to solve Eqn. 2.2, one would use unsymmetric approaches which is very time consuming and difficult from programming point of view. The necessity for unsymmetric solver will be obviated by employing pseudo-symmetric technique. In this approach, the lower partition of Eqn. 2.1 is multiplied by the factor of $-\rho a_0^{-1}$, in which a_0 is one of the coefficients in Newmark algorithm. In this way, the effective stiffness matrix would be symmetric and it can be solved by the usual Newmark's algorithm [4].

2.1. Decoupled Modal Technique

This technique utilizes independent modes of dam-reservoir system. The sub-space method that is efficient for symmetric matrices can be used to solve the eigenproblem. The main advantage of the proposed method is that one can avoid solving an unsymmetric eigen-problem. Decoupled modes are to be calculated by solving the following eigen-problem:

$$\mathbf{K_S X} = \mathbf{M_S X}\Lambda \tag{2.3}$$

where $\mathbf{K_S}$ and $\mathbf{M_S}$ are the symmetric parts of $\overline{\mathbf{M}}$ and $\overline{\mathbf{K}}$. These latter matrices include some unsymmetric parts and they can be written totally as follows:

$$\overline{\mathbf{M}} = \mathbf{M_S} + \mathbf{M_U} = \begin{bmatrix} \mathbf{M} & 0 \\ 0 & \frac{1}{\rho c^2}\mathbf{G} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \mathbf{B} & 0 \end{bmatrix} \tag{2.4a}$$

$$\overline{\mathbf{K}} = \mathbf{K_S} + \mathbf{K_U} = \begin{bmatrix} \mathbf{K} & 0 \\ 0 & \frac{1}{\rho}\mathbf{H} \end{bmatrix} + \begin{bmatrix} 0 & -\mathbf{B^T} \\ 0 & 0 \end{bmatrix} \tag{2.4b}$$

Although the eigenvectors obtained through the above relation, are not the true mode shapes of the coupled system, these can be presumed as Ritz' vectors which can be similarly combined to estimate the true solution. Having the orthogonal condition and normalizing the modal matrix with respect to mass matrix, one would have:

$$\mathbf{X^T M_S X} = \mathbf{I} \tag{2.5a}$$

$$\mathbf{X^T K_S X} = \Lambda \tag{2.5b}$$

Also, having in mind that $\mathbf{K_U} = -\mathbf{M_U^T}$, following relations can be derived based on Eqns. 2.3, 2.4 and 2.5:

$$\mathbf{X^T}\overline{\mathbf{M}}\mathbf{X} = \mathbf{I} + \mathbf{X^T M_U X} \tag{2.6a}$$

$$\mathbf{X^T}\overline{\mathbf{K}}\mathbf{X} = \Lambda - \mathbf{X^T M_U^T X} \tag{2.6b}$$

As usual in modal techniques, the solution is written as a combination of different modes ($\overline{\mathbf{r}} = \mathbf{XY}$) where the vector $\mathbf{Y}$ contains the participation factors of the modes. Substituting this relation into Eqn. 2.2 and multiplying both sides of that equation by $\mathbf{X^T}$, it yields:

$$\left(\mathbf{X}^{\mathrm{T}}\overline{\mathbf{M}}\,\mathbf{X}\right)\ddot{\mathbf{Y}}+\mathbf{X}^{\mathrm{T}}\overline{\mathbf{C}}\,\mathbf{X}\dot{\mathbf{Y}}+\left(\mathbf{X}^{\mathrm{T}}\overline{\mathbf{K}}\,\mathbf{X}\right)\mathbf{Y}=-\mathbf{X}^{\mathrm{T}}\overline{\mathbf{M}}\,\overline{\mathbf{J}}\,\mathbf{a}_g(\mathbf{t}) \tag{2.7}$$

Substituting Eqn. 2.6 into Eqn. 2.7 results in

$$\left(\mathbf{I}+\mathbf{X}^{\mathrm{T}}\mathbf{M}_{\mathrm{U}}\,\mathbf{X}\right)\ddot{\mathbf{Y}}+\mathbf{X}^{\mathrm{T}}\overline{\mathbf{C}}\,\mathbf{X}\dot{\mathbf{Y}}+\left(\Lambda-\mathbf{X}^{\mathrm{T}}\mathbf{M}_{\mathrm{U}}^{\mathrm{T}}\,\mathbf{X}\right)\mathbf{Y}=-\mathbf{X}^{\mathrm{T}}\overline{\mathbf{M}}\,\overline{\mathbf{J}}\,\mathbf{a}_g(\mathbf{t}) \tag{2.8}$$

It must be mentioned that the generalized effective stiffness matrix obtained from Newmark method for this case is unsymmetrical. However, it may be easily transformed to a symmetric matrix. Toward this end, without loss of generality, the mode shapes for the dam are ordered first and the ones for the finite reservoir is considered subsequently in the modal matrix:

$$\mathbf{X}=\begin{bmatrix}\mathbf{X}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{X}_2\end{bmatrix} \quad , \quad \Lambda=\begin{bmatrix}\Lambda_1 & \mathbf{0}\\ \mathbf{0} & \Lambda_2\end{bmatrix} \tag{2.9}$$

In order to have a symmetric effective stiffness matrix, one could multiply the lower partition of Eqn.2.8 by a factor of $-\mathbf{a}_0^{-1}$. This yields the following relations for the effective stiffness matrix and force vector:

$$\hat{\mathbf{K}}=\begin{bmatrix}\mathrm{a}_0\mathbf{I}_1 & \mathbf{0}\\ -\mathbf{X}_2^{\mathrm{T}}\mathbf{B}\mathbf{X}_1 & -\mathbf{I}_2\end{bmatrix}+\mathrm{a}_1\begin{bmatrix}2\beta_{\mathbf{d}}\Lambda_1^{1/2} & \mathbf{0}\\ 0 & -\dfrac{1}{\rho\mathbf{a}_0}\mathbf{X}_2^{\mathrm{T}}\mathbf{L}\mathbf{X}_2\end{bmatrix}+\begin{bmatrix}\Lambda_1 & -\mathbf{X}_1^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{X}_2\\ 0 & -\dfrac{1}{\mathbf{a}_0}\Lambda_2\end{bmatrix} \tag{2.10}$$

$$\hat{\mathbf{F}}_{\mathbf{n}+1}=\begin{bmatrix}-\mathbf{X}_1^{\mathrm{T}}\mathbf{M}\mathbf{J}\mathbf{a}_{\mathbf{g}}\\ -\dfrac{1}{\mathbf{a}_0}\mathbf{X}_2^{\mathrm{T}}\mathbf{B}\mathbf{J}\mathbf{a}\end{bmatrix}+\begin{bmatrix}\mathbf{I}_1 & 0\\ -\dfrac{1}{\mathbf{a}_0}\mathbf{X}_2^{\mathrm{T}}\mathbf{B}\mathbf{X}_1 & -\dfrac{1}{\mathbf{a}_0}\mathbf{I}_2\end{bmatrix}\left(\mathbf{a}_0\mathbf{Y}_{\mathbf{n}}+\mathbf{a}_2\dot{\mathbf{Y}}_{\mathbf{n}}+\mathbf{a}_3\ddot{\mathbf{Y}}_{\mathbf{n}}\right)+\begin{bmatrix}2\beta_{\mathbf{d}}\Lambda_1^{1/2} & 0\\ 0 & -\dfrac{1}{\rho\mathbf{a}_0}\mathbf{X}_2^{\mathrm{T}}\mathbf{L}\mathbf{X}_2\end{bmatrix}\left(\mathbf{a}_1\mathbf{Y}_{\mathbf{n}}+\mathbf{a}_4\dot{\mathbf{Y}}_{\mathbf{n}}+\mathbf{a}_5\ddot{\mathbf{Y}}_{\mathbf{n}}\right) \tag{2.11}$$

It is usually assumed that the damping matrix of the dam is of viscous type for the analysis carried out by modal approach in time domain. Therefore, the equivalent damping factor (i.e., $\beta_{\mathbf{d}}$) is assumed constant for all modes in this approach. Finally, the symmetric form of the equation can be rewritten as follows:

$$\begin{bmatrix}\mathrm{a}_0\mathbf{I}_1+\mathbf{a}_1\mathbf{C}_1^{*}+\Lambda_1 & -\mathbf{X}_1^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{X}_2\\ -\mathbf{X}_2^{\mathrm{T}}\mathbf{B}\mathbf{X}_1 & -\mathbf{I}_2-\dfrac{\mathbf{a}_1}{\rho\mathbf{a}_0}\mathbf{X}_2^{\mathrm{T}}\mathbf{L}\mathbf{X}_2-\dfrac{1}{\mathbf{a}_0}\Lambda_2\end{bmatrix}\begin{bmatrix}\mathbf{Y}_1\\ \mathbf{Y}_2\end{bmatrix}_{\mathbf{n}+1}=\begin{bmatrix}\hat{\mathbf{F}}_1\\ \hat{\mathbf{F}}_2\end{bmatrix}_{\mathbf{n}+1} \tag{2.12}$$

It is also noticed that, the vector of participation factors is assumed to be partitioned into two parts in this relation, and as before the indices 1, 2 correspond to dam and reservoir modes, respectively. Eqn. 2.12 can be solved for the vector of participation factors at each time step [5].

2.2. Coupled Modal Technique

It was mentioned that $\overline{\mathbf{M}}$ and $\overline{\mathbf{K}}$ are unsymmetrical matrices. In order to use advantages of orthogonality conditions, two sets of eigenvectors need to be calculated by the following eigen-problems:

$$\overline{\mathbf{K}}\,\mathbf{X}^{\mathbf{R}}=\overline{\mathbf{M}}\,\mathbf{X}^{\mathbf{R}}\overline{\Lambda} \tag{2.13a}$$
$$\overline{\mathbf{K}}^{\mathrm{T}}\,\mathbf{X}^{\mathrm{L}}=\overline{\mathbf{M}}^{\mathrm{T}}\,\mathbf{X}^{\mathrm{L}}\overline{\Lambda} \tag{2.13b}$$

In these equations, $\mathbf{X}^{\mathbf{R}}$ and $\mathbf{X}^{\mathrm{L}}$ are denoted as matrices of right and left eigenvectors for the coupled system, respectively. Coefficient matrices of Eqn. 2.13b are the transpose of those of Eqn. 2.13a. Thus, the eigenvalues of a specific mode are the same; However, corrosponding eigenvectors are different. Eqns. 2.13 are solved by inverse iteration method owing to its capability of solving unsymmetric eigenproblem. Basically, inverse iteration method is not efficient for finding eigenvector. However, the convergency rate can be improved by shifting technique[6]. This method always converges to the eigenvector corresponding to the smallest eigenvalue. Meanwhile, there might be a deviation from convergence to smallest eigenvalue and its corresponding eigenvector. Therefore, it is recommended that one use inverse iteration technique to achieve an appropriate approximation; Thereafter, with each cycle of inverse iteration, impose the Rayleigh quotient shift to reach the desired precision. However, in every shift it is required to check the deviation to other modes. Subsequently, the orthogonality conditions in this case are written in the following form:

$$\left(\mathbf{X}^{L}\right)^{T}\overline{\mathbf{M}}\,\mathbf{X}^{R}=\mathbf{I} \tag{2.14a}$$

$$\left(\mathbf{X}^{L}\right)^{T}\overline{\mathbf{K}}\,\mathbf{X}^{R}=\overline{\Lambda} \tag{2.14b}$$

The response of the system can be obtained by linear combination of right eigen-vectors ($\overline{\mathbf{r}}=\mathbf{X}^{R}\,\overline{\mathbf{Y}}$). Finally, the coupled equation of dam-reservoir system can be written as:

$$\left(\mathbf{X}^{L}\right)^{T}\overline{\mathbf{M}}\mathbf{X}^{R}\ddot{\overline{\mathbf{Y}}}+\left(\mathbf{X}^{L}\right)^{T}\overline{\mathbf{C}}\mathbf{X}^{R}\dot{\overline{\mathbf{Y}}}+\left(\mathbf{X}^{L}\right)^{T}\overline{\mathbf{K}}\mathbf{X}^{R}\overline{\mathbf{Y}}=-\left(\mathbf{X}^{L}\right)^{T}\overline{\mathbf{MJ}}\mathbf{a}_{g} \tag{2.15}$$

The damping matrix $\mathbf{C}$ which is part of matrix $\overline{\mathbf{C}}$ (see Eqn. 2.1) is assumed to be of Rayleigh type in this approach. Substituting Eqn. 2.14 into Eqn. 2.15 yields the following equation:

$$\mathbf{I}\ddot{\overline{\mathbf{Y}}}+\left(\mathbf{X}^{L}\right)^{T}\overline{\mathbf{C}}\,\mathbf{X}^{R}\,\dot{\overline{\mathbf{Y}}}+\Lambda\,\overline{\mathbf{Y}}=-\left(\mathbf{X}^{L}\right)^{T}\overline{\mathbf{M}}\,\overline{\mathbf{J}}\,\mathbf{a}_{g}(\mathbf{t}) \tag{2.16}$$

Applying Newmark's method on Eqn. 2.16, one can write $\hat{\overline{\mathbf{K}}}\mathbf{Y}_{n+1}=\hat{\overline{\mathbf{F}}}_{n+1}$ where $\hat{\overline{\mathbf{K}}}$ and $\hat{\overline{\mathbf{F}}}$ are the effective stiffness matrix and effective force vector.

3. MODELING AND BASIC DATA

The Shahid Rajaee concrete arch dam is considered over a rigid foundation The dam is 138 m height with the crest length of 420 m. Finite element mesh of dam and reservoir are shown in Fig. 1. The 20-node isoparametric elements are used to discretize dam and reservoir domains. Two layers of elements are used along the thickness of the dam in order to capture the stress variation more accurately. The concrete is assumed to be homogeneous with isotropic linearly viscoelastic behavior and the following main characteristics: E_c=30 GPa, ν = 0.18, γ_c =24 kN/m^3. The water is taken as compressible, inviscid fluid, with weight density of 9.81 kN/m^3, and pressure wave velocity of 1440 m/s.

Figure 1 Finite element mesh of Shahid Rajaee dam-reservoir system

Furthermore, Sommerfeld boundary condition is imposed at the upstream end of the impounded water domain. Reservoir bottom is considered completely reflective. Viscous damping is assumed for decoupled modal approach and damping coefficients are considered constant in all modes ($\beta_d = 0.10$). Also, Rayleigh damping matrix is utilized for coupled modal and direct approaches and corresponding coefficients are determined such that equivalent damping for frequencies close to the first and sixth modes of vibration would be 10% of the critical damping. The dynamic excitation considered, is two components of Friuli-Tolmezzo earthquake records. It comprises the horizontal component inserted in stream direction and the vertical components. The cross-canyon component is neglected to maintain a symmetric condition. The peak ground accelerations are 0.56g and 0.45g for horizontal and vertical components, respectively [7].

4. RESULTS

The results for three mentioned methods are presented in this section. The results are presented in two tables for each part. The first table represents maximum displacements in three directions which are cross-stream (x), downstream (y) and vertical (z) for each analysis. To this end, three nodes are considered at the dam crest. These nodes are located in the middle, left and right quarter points of the dam. All of them are located in the upstream edge. The dam body is symmetric with respect to the yz plane and the cross-canyon component of the earthquake is neglected as previously mentioned. Thus, symmetric results are expected inevitably and this can be utilized for additional control of results. The other table, presents maximum tensile and compressive principal stresses for upstream, downstream edge of the dam body and the percentage of error with respect to exact results. Moreover, envelopes of principal stresses are illustrated in each case.

4.1. Pseudo-symmetric Technique

Maximum displacements for the crest nodes and principal stresses are presented in Tables 4.1 and 4.2 based on this technique. These results are considered as the exact values and will be used to compare with the modal results. Envelopes of compressive stress are plotted for upstream and downstream faces .in Fig. 2. It is reminded that foundation was assumed as rigid. Therefore, high tensile stresses are inevitable especially near the foundation.

Table 4.1 Maximum displacements of the dam crest (mm)

Displacement component	Max. displacement	
	Center point	Right 1/4 point
Cross-canyon	0.00	-24.14
Stream direction	125.30	52.04
Vertical	-10.05	4.83

Table 4.2 Maximum principal stresses (MPa)

Max. compressive stress		Max. tensile stress	
Downstream	Upstream	Downstream	Upstream
-12.02	-13.94	7.83	20.47

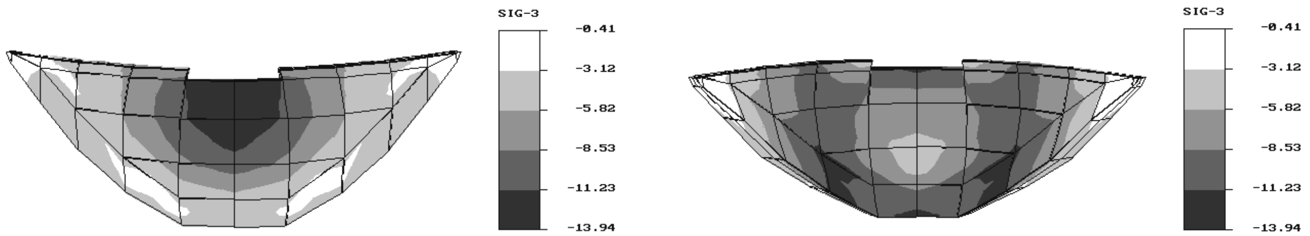

Figure 2. Envelope of maximum principal compressive stress (MPa)

4.2. Decoupled Modal Technique

The first mode shape of the dam and reservoir with rigid boundaries are displayed in Fig. 3. Moreover, the first five natural frequencies of each domain are listed in Table 3. It is noticed that the natural frequencies of the dam are wider spread in comparison to the ones related to the reservoir domain. This means that a much higher number of modes are required for the reservoir in comparison with the dam for an accurate solution. Moreover, the 40th natural frequency of dam is close in value to the 80th natural frequency of the reservoir. Modal analysis is carried out for different number of modes employed and the results are summarized in Tables 4.4 and 4.5.

Results show that an increase in the utilized number of modes for dam and reservoir domain, leads to convergence to the exact results. Utilizing 40 modes for the dam and 80 modes for the reservoir domain produces quite good results. For these numbers of modes, maximum compressive and tensile stresses are 0.58% and 9.32% less than those for direct method. Even though stress is considered as a design scale, displacements are also acceptable(less than 10% erre for the case).

Table 4.3 Natural frequencies of decoupled approach

Mode number	Natural frequency (Hz)	
	Dam	Reservoir
1	2.40539	3.46680
2	3.01180	4.38139
3	3.48274	5.26924
4	4.82462	5.80770
5	5.47359	6.25512

Table 4.4 Maximum displacements of the dam crest (mm)

Mode combination (Dam-reservoir)		Cross-canyon		Stream		Vertical	
		Max. dis.	Error %	Max. dis.	Error %	Max. dis.	Error %
20-40	Center point	0.00	0.00	112.04	10.39	-10.11	0.65
	Right 1/4 point	-21.33	11.64	55.27	6.19	-4.54	194.0
40-80	Center point	0.00	0.00	112.77	9.81	-10.39	0.37
	Right 1/4 point	-22.96	4.89	53.91	3.59	4.46	7.66
50-100	Center point	0.00	0.00	112.59	9.95	-10.12	0.70
	Right 1/4 point	-23.41	3.01	51.92	0.24	4.38	9.37
80-160	Center point	0.00	0.00	117.81	5.77	-10.28	2.28
	Right 1/4 point	-24.17	0.14	50.01	3.91	4.42	8.39

Table 4.5 Maximum principal stresses (MPa) (for the decoupled modal technique)

Mode combination (Dam-reservoir)	Max. compressive stress				Max. tensile stress			
	Downstream		Upstream		Downstream		Upstream	
	Stress	Error %	Stress	Error %	Stress	Error %	Stress	Error %
20-40	-12.55	4.41	-13.95	0.13	5.66	27.72	17.43	14.86
40-80	-12.59	4.73	-14.02	0.58	6.17	21.19	18.56	9.32
50-100	-12.19	1.44	-13.89	0.36	6.15	21.41	18.87	7.82
80-160	-12.15	1.13	-13.58	2.52	6.51	16.82	19.58	4.33

Considering 80 modes for the dam and 160 modes for the reservoir, cause the more exact results. The envelope of principal compressive stress is depicted in Fig. 4. It is observed that the distributions of maximum stresses are quite similar for the modal and direct approaches (Fig. 2vs. Fig.4). For better evaluation of the results, time histories of the horizontal and vertical components of displacement at dam crest are presented in Fig. 5 (for 40 and 80 modes). In each graph, the result corresponding to direct method is also shown for comparison purposes. Comparison of these results is another criteria for obtaining the appropriate number of modes.

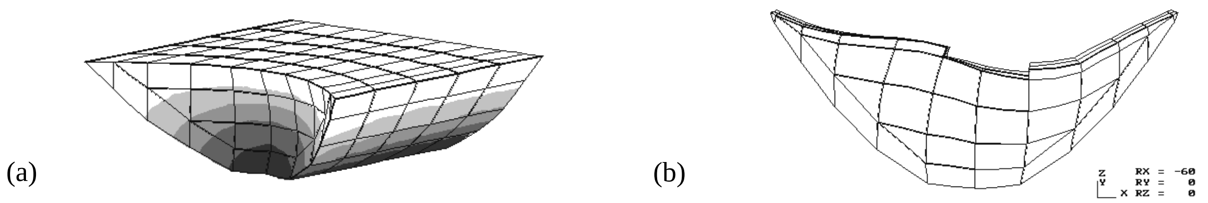

Figure 3 The first mode shape (a) reservoir (b) dam body

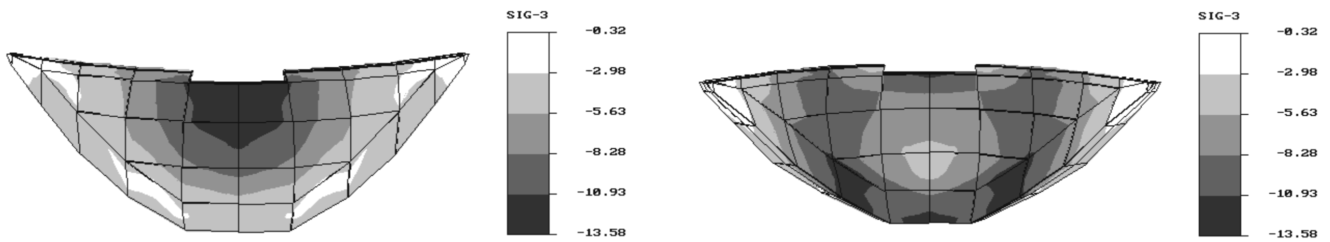

Figure 4. Envelope of maximum principal compressive stress (MPa)

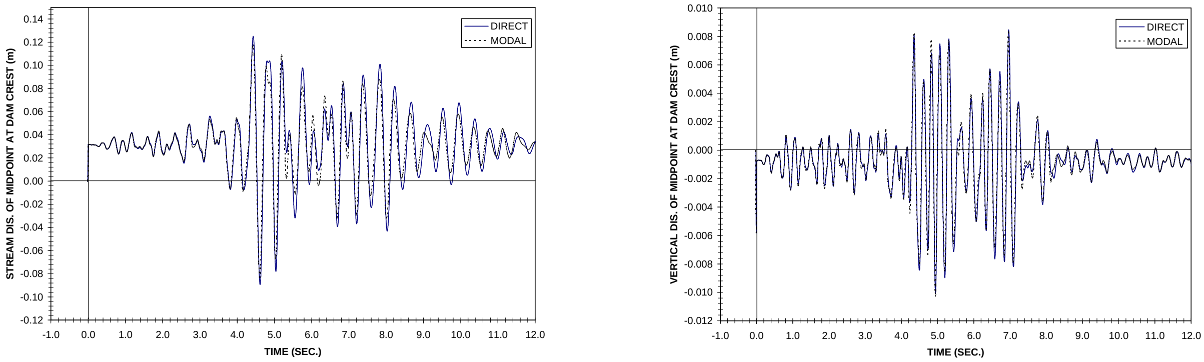

Figure 5. Horizontal and vertical displacements of dam crest

4.3. Coupled Modal Technique

The first three natural frequencies of the coupled system are listed in Table 4.6. Also, the first mode shape is illustrated in Fig. 6. Modal analysis is carried out for different mode numbers and the results are summarized in Tables 4.7 and 4.8. It is noticed that the accuracy of the results is improving in comparison with the direct method, as the number of modes increases. It is observed that negligible errors are obtained by considering 15 modes(less than 5% for displacements and stresses). Moreover, for practical purposes, considering 10 modes for coupled system leads to acceptable responses. It must be mentioned that number of modes in this technique is noticeably less than decoupled method. The envelopes of maximum compressive stress is presented in Fig. 7. There is good agreement between these envelopes and exact ones(Figs. 2). Time histories of the horizontal and vertical components of displacement at dam crest are depicted in Fig. 8 for 15 modes in comparison with direct method.

Table 4.6 Natural frequencies of coupled dam-reservoir system

Mode number	Natural frequency (Hz)
1	2.02228
2	2.35887
3	3.0020

Table 4.7 Maximum displacements of the dam crest (mm)

Mode combination (Dam-reservoir)		Cross-canyon		Stream		Vertical	
		Max. dis.	Error %	Max. dis.	Error %	Max. dis.	Error %
5	Center point	0.00	0.00	127.70	2.14	-9.72	3.29
	Right 1/4 point	-22.91	5.09	56.33	8.24	-4.65	196.31
10	Center point	0.00	0.00	128.38	2.68	-11.28	12.29
	Right 1/4 point	-22.76	5.70	52.80	1.45	4.92	1.97
15	Center point	0.00	0.00	125.70	0.54	-9.64	4.08
	Right 1/4 point	-23.36	3.20	50.98	2.05	4.51	6.66

Table 4.8 Maximum principal stresses (MPa) (coupled modal technique)

Mode combination (Dam-reservoir)	Max. compressive stress				Max. tensile stress			
	Downstream		Upstream		Downstream		Upstream	
	Stress	Error %	Stress	Error %	Stress	Error %	Stress	Error %
5	-12.47	3.75	-14.71	5.59	6.41	18.12	17.45	14.74
10	-12.47	3.74	-14.51	4.09	7.92	1.16	19.00	7.19
15	-12.31	2.39	-13.92	0.12	7.19	8.13	19.72	3.63

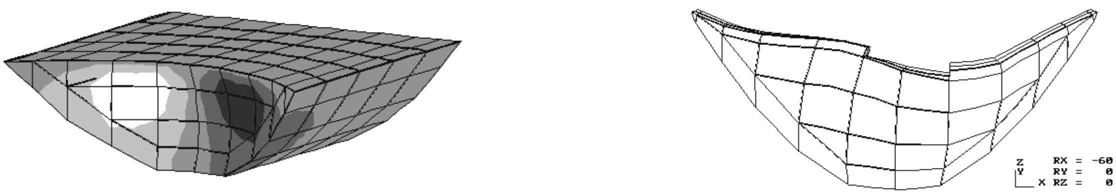

Figure 6 The first right mode shape of coupled dam-reservoir system

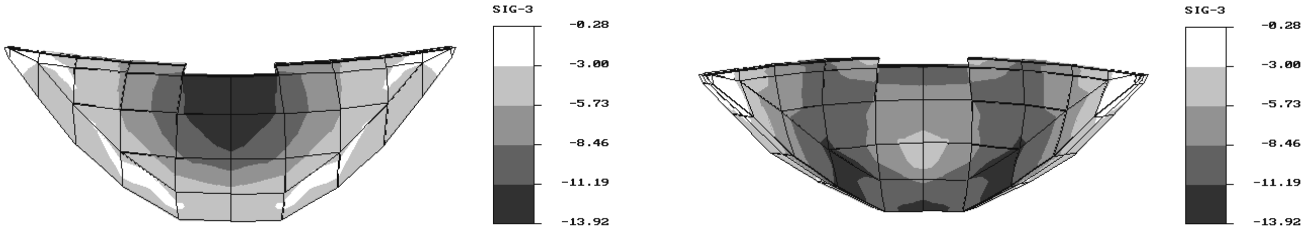

Figure 7 Envelope of maximum principal compressive stress (MPa)

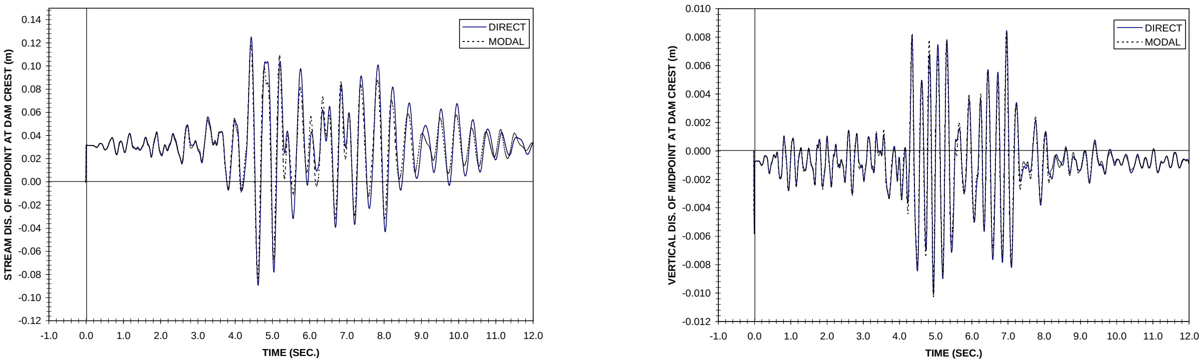

Figure 8. Horizontal and vertical displacements of dam crest

5. CONCLUSION

This paper deals with two modal techniques for analysis of dam-reservoir system in time domain which are referred to as coupled and decuopled modal techniques. Moreover, the results of direct approach are used for comparison purposes. The analysis of Shahid Rajee dam is carried out as a numerical example and for verification purposes. The following conclusions are obtained from this study:

- Decoupled modal technique accuracy is improving slowly as number of utilized modes increases. Since, there is no need to solve unsymmetric eigenproblems, this technique is convenient from programming point of view.
- Coupled modal technique yields very good approximations, even with a relatively low number of combined modes. It is concluded that this is an efficient method for analysis of dam-reservoir systems. The main disadvantage of this technique is due to the unsymmetric eigenproblem. This study devised an efficient solution to calculate the eigenproblem based on inverse iteration and bi-section method. The efficiency of coupled modal technique has been increased with the aid of this technique.
- From comparison point of view, both methods can be carried out for time domain analysis whereas coupled modal technique is significantly more efficient than the decoupled approach.

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