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OPTIMAL INTEGRATED DESIGN OF CONTROLLED STRUCTURES

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ABSTRACT :
Traditionally, the structure and the controller have been designed separately in buildings. The structure is typically optimized to minimize weight subject to stress and stiffness constraints and the control system is optimized to minimize a quadratic performance measure accounting for deformation and control effort. However, because of interaction between the structure and the control system, simultaneous optimization design of both systems may be necessary in order to obtain required performance with minimum cost. A design procedure based on two steps has been presented that is able to achieve the goal. Step 1: design of a weaker structure and of the controller independently in order to satisfy the performance requirements; Step 2: structure redesign of building and controller by changing mass, stiffness and damping matrix by minimizing the power of the active control force. The procedure is valid both for linear and nonlinear structures. A SDOF steel portal frame is used to exemplify the method in detail. The procedure has also been applied to the buildings used in the SAC steel project. This procedure is particularly efficient for tall buildings, where the demand of structural steel (or other construction materials) increases as the buildings become taller. However by using active control systems or equivalent passive devices it is possible to achieve the same performance in term of horizontal drift without increasing the construction materials, obtaining an overall weight savings and consequentially reduction of construction costs.

KEYWORDS:
Active control, integrated design, LQR, passive control, redesign, simultaneous optimization

1. INTRODUCTION

In the last thirty years, many researches have been done on integrated design of structural/control systems. Integrated optimal structural/control system design has been acknowledged as an advanced design methodology for space structures, but not many applications can be found in civil engineering (Soong and Cimellaro, 2008). Onoda and Haftka (1987) minimized the weight of a structure and controller subject to constraints on the magnitude of structural responses. Salama *et al*. (1988) realized the simultaneous design of structure and control system for a composite objective function that is a linear combination of structural and control objective functions.

Chattopadhyay and Seeley (1994) developed an optimization procedure to address the optimal placement of controllers and structural design, where the objective functions such as those connected with the fundamental natural frequency of the structure and energy dissipated by the piezoelectric actuators were considered. Khot (1998) proposed a method to design the structure and the controller simultaneously using a multiobjective optimization approach to suppress the initial disturbance.

In this paper, an optimization procedure for achieving integrated control/structural systems is developed. A steel portal frame and a SAC project building of nine stories (Othori *et al*., 2004) are used to show the method in detail.

2. INTEGRATED DESIGN

Consider a multi-degree-of-freedom inelastic building structure subjected to a one-dimensional external excitation. The general equation of motion of the inelastic system with active control forces is given by

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)+\mathbf{T}_s\mathbf{f}_s\left[\mathbf{x}(t)\right]=\mathbf{H}\mathbf{u}(t)+\boldsymbol{\eta}w(t) \tag{2.1}$$

where $\mathbf{x}$(t) is the displacement vector, **M** and **C** are the mass and inherent damping matrices, respectively, **K** is the stiffness matrix of all linear elements, $\mathbf{u}$(t) is the active control force vector; **H** is the location matrix for the active control forces; $\boldsymbol{\eta}$ is the excitation influence matrix; $\mathbf{T}_s$ is the location matrix of the restoring forces and f_s[x(t)] is a vector of nonlinear restoring forces in the structural elements. $\mathbf{f}_s[\mathbf{x}(t)]$ in Eqn. (2.1) can be separated into two parts, one representing the elastic behavior and the second representing the nonlinear hysteretic behavior, i.e.,

$$\mathbf{f}_s\left[\mathbf{x}(t)\right]=\tilde{\mathbf{K}}\mathbf{x}(t)+\tilde{\mathbf{H}}\tilde{\mathbf{f}}_s\left[\mathbf{x}(t)\right] \tag{2.2}$$

in which $\tilde{\mathbf{K}}$ is a linear elastic stiffness matrix for the linear elastic parts of the nonlinear or hysteretic structural components; $\tilde{\mathbf{f}}_{\mathbf{s}}=[f_1\ldots f_2\ldots f_l]^T$ is a vector representing the nonlinear or hysteretic parts of the restoring forces for nonlinear structural components; and $\tilde{\mathbf{H}}$ is a location matrix for the nonlinear elements. The basic problem of the integrated design approach is to determine $\boldsymbol{\xi}$ and $\mathbf{u}$(t) such that an appropriate objective functional is minimized.

A possible objective function can be expressed by combining the classical quadratic performance criteria (Soong, 1990) with a cost function *W* depending on structural parameters ξ and active control force $\mathbf{u}$(t). A solution procedure is given by Soong and Manolis (1987) and Soong (1990) using variational calculus. Their solution determines a system of coupled nonlinear equations, but due to the complex nature of these equations, the optimization problem is usually nonconvex, therefore numerical solution are usually required to obtain a solution.

2.1. Redesign approach

The optimization problem becomes easier when the design procedure is divided in two steps. In fact, in control of buildings, the structure is traditionally designed first and then the controller. The proposed design method reverses the procedure by designing the structure after the controller is given. The fundamental idea of redesign was proposed by Smith *et al*. (1992). In this paper, the idea of redesign is incorporated in the integrated design of structural/control systems. The procedure is summarized in the following steps (Cimellaro *et al*. 2008a):

First step: The desired structure is chosen and it is assumed fixed while the controller is designed in order to satisfy a given performance requirement (e.g., drift, absolute acceleration, base shear, etc.) of the initial structure. The dynamic response of the initial structure in this step is called “Ideal Response”.

Second step: The structure and the controller are redesigned cooperatively to achieve a common goal (the ideal dynamic response of the first step). This structure redesign is accomplished to reduce (minimize) the amount of active control power needed to achieve the “Ideal Response”. In other words, the structure is redesigned for better controllability.

These steps can be better understood by considering relationship between spectral acceleration and spectral displacement (S_a-S_d) in structural design. In Figure 1 is shown a typical S_a-S_d spectrum for several damping levels. $S_d(T_0, \beta_0)$ and $S_a(T_0, \beta_0)$ are the spectral coordinates of the original structure with period T_0 and damping β_0. In Step 1, the structure at point 1 is made lighter by reducing its stiffness and it moves to point 2 in Figure 1. Then a controller is applied to bring back the structure to the initial ideal response at point 3. In Step 2, the structure is redesigned in order to achieve the same performance, but with less amount of active control forces or damping. During the redesign, mass, stiffness and damping are modified in order to achieve this goal, reaching finally point 4 in Figure 1. At the end of this step, the building will maintain the same performance, but with less amount of control forces and material. The integrated redesign procedure is formulated in the following, for the case when the building is assumed linear for simplicity; however the case of inelastic structures has been considered from the authors elsewhere (Cimellaro *et al*., 2008b).

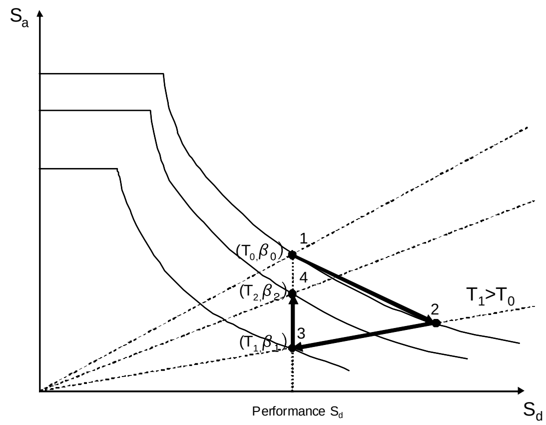

Figure 1 Redesign procedure in S_d-S_a plane

In the state space, assuming the structure is linear Eqn. (2.1) becomes

$$\dot{\mathbf{z}}(t)=\mathbf{A}\mathbf{z}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{e}(t) \tag{2.3}$$

where

$$\mathbf{z}(t)=\begin{bmatrix}\mathbf{x}(t)\\ \dot{\mathbf{x}}(t)\end{bmatrix};\ \mathbf{e}(t)=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\boldsymbol{\eta}\end{bmatrix}w(t);\ \mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix};\ \mathbf{B}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{H}\end{bmatrix}; \tag{2.4}$$

Then, the first step is to design a linear control law such that the structural system has acceptable performance such as satisfaction of certain constraints on the dynamic response. For example

$$\mathbf{u}(t)=\mathbf{G}\mathbf{z}(t) \tag{2.5}$$

where $\mathbf{G}$ is a gain matrix. Then, the redesign concept is to change the mass, stiffness, damping matrices, respectively by $\mathbf{\Delta M}$, $\mathbf{\Delta K}$ and $\mathbf{\Delta C}$, and to determine the control force $\mathbf{u}$ so that the new system becomes

$$(\mathbf{M}+\mathbf{\Delta M})\ddot{\mathbf{x}}(t)+(\mathbf{C}+\mathbf{\Delta C})\dot{\mathbf{x}}(t)+(\mathbf{K}+\mathbf{\Delta K})\mathbf{x}(t)=\mathbf{H}\mathbf{u}_a(t)+\boldsymbol{\eta}w(t) \tag{2.6}$$

where

$$\mathbf{u}_a(t)=\mathbf{G}_a\mathbf{z}(t) \tag{2.7}$$

where $\mathbf{G_a}$ is the active part of the controller after redesign. The main idea is to separate the control law, Eqn. (2.5), into a passive part which is implemented into the physical system by redesign, and an active part which constitutes the remaining active control law required after structure redesign. Therefore, the control law is written in the following form

$$\mathbf{Hu}(t) = \mathbf{HGz}(t) = \mathbf{HG}_a \begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} - \Delta\mathbf{M}\ddot{\mathbf{x}}(t) - \Delta\mathbf{C}\dot{\mathbf{x}}(t) - \Delta\mathbf{K}\mathbf{x}(t) \tag{2.8}$$

and the closed-loop system after redesign is

$$(\mathbf{M}+\Delta\mathbf{M})\ddot{\mathbf{x}}(t) + (\mathbf{C}+\Delta\mathbf{C})\dot{\mathbf{x}}(t) + (\mathbf{K}+\Delta\mathbf{K})\mathbf{x}(t) = \mathbf{HG}_a\mathbf{z}(t) + \boldsymbol{\eta}w(t) \tag{2.9}$$

where $\mathbf{u_a}$(t), which is given by the Eqn. (2.7), is the active part of the controller and $\Delta\mathbf{M}\ddot{\mathbf{x}}(t)+\Delta\mathbf{C}\dot{\mathbf{x}}(t)+\Delta\mathbf{K}\mathbf{x}(t)$ is the passive part. The objective of the redesign is to find the passive control (**ΔM**, **ΔK**, **ΔC**) in order to minimize the control power needed to satisfy Eqn. (2.8) for any given G. Note that the closed-loop system response before and after redesign remains unchanged; therefore, all the designed closed-loop system properties remain unchanged. Let $\mathbf{B_k}$, $\mathbf{B_c}$ and $\mathbf{B_m}$ be the stiffness, damping and mass connectivity matrices of the structural system. The changes in the structural parameters can be expressed in the form:

$$\begin{aligned} \Delta\mathbf{K} &= \mathbf{B_k G_k B_k^T} & & \mathbf{G_k} = diag(\ldots,\Delta k_i,\ldots) \\ \Delta\mathbf{C} &= \mathbf{B_c G_c B_c^T} & \text{with} \quad & \mathbf{G_c} = diag(\ldots,\Delta c_i,\ldots) \\ \Delta\mathbf{M} &= \mathbf{B_m G_m B_m^T} & & \mathbf{G_m} = diag(\ldots,\Delta m_i,\ldots) \end{aligned} \tag{2.10}$$

This gives the following presentation for the desired control law

$$\mathbf{HG}\begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} = \mathbf{HG}_a\begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} - [\Delta\mathbf{K} \quad \Delta\mathbf{C}]\begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} - \Delta\mathbf{M}\ddot{\mathbf{x}}(t) \tag{2.11}$$

Substituting the solution of $\ddot{\mathbf{x}}(t)$ from Eqn. (2.1), it yields:

$$\mathbf{HGz}(t) = (\mathbf{G}_{active} + \mathbf{G}_{passive})\mathbf{z}(t) \tag{2.12}$$

where

$$\mathbf{G}_{active} = \mathbf{HG}_a, \qquad \mathbf{G}_{passive} = -\mathbf{I_0 B_p G_p B_p^T L} \tag{2.13}$$

with

$$\mathbf{L} = \begin{bmatrix} I \\ \mathbf{M}^{-1}(\mathbf{HG} - [\mathbf{K} \quad \mathbf{C}]) \end{bmatrix}, \mathbf{B_p} = \begin{bmatrix} B_k & 0 & 0 \\ 0 & B_c & 0 \\ 0 & 0 & B_m \end{bmatrix}, \mathbf{G_p} = \begin{bmatrix} G_k & 0 & 0 \\ 0 & G_c & 0 \\ 0 & 0 & G_m \end{bmatrix}, \mathbf{I_0} = [I \quad I \quad I] \tag{2.14}$$

For the passive control law to be physically implemented, it must satisfy certain inequality constraints due to the physics of the problem. For example, the stiffness and the damping of any element of the system after redesign cannot be negative while the weight of any element cannot decrease lower than a specified bound. Then if **C** = diag[k_i,..c_i,…m_i,…] is a matrix with diagonal elements containing the specified lower bound values of the structural elements after redesign and **S** = diag[k_{0i},..c_{0i},…m_{0i},…] is the matrix of the initial parameters, then these constraints can be presented as

$$\mathbf{G}_p + \mathbf{S} \geq \mathbf{C} \tag{2.15}$$

The objective function being minimized can be the power of the active part of the control law given by

$$F\left(\mathbf{G_p}\right)=\int \mathbf{u}_a^T\left(t\right)\mathbf{R}\mathbf{u}_a\left(t\right)dt=trace\left(\mathbf{G_a}\mathbf{R_{XX}}\mathbf{G_a^T}\mathbf{R}\right) \tag{2.16}$$

An approach to solve numerically the constraint optimization problem is to use the *exterior penalty function method* (Vanderplaats, 2005). Further details can be found in Cimellaro *et al*. (2008a-c).

3. NUMERICAL EXAMPLES

3.1. SDOF steel portal frame

Consider a 2-D moment resisting one-story and one-bay steel frame, where the dimensions are given in Figure 2. The frame consists of two columns (W14×257 and W14×311) and one beam (W33×118). The columns are 345 MPA (50ksi) steel and the beam is 248 MPA (36ksi). The frame is subjected to a zero-mean white noise stationary horizontal base acceleration with peak ground acceleration of 0.25 g. The mass is M=159.450 kN sec^2/m, the stiffness is K=76987.117 kN/m and the damping coefficient is C= 140.147 kN sec/m that is determined assuming Rayleigh damping equal to 2%. The period of the uncontrolled frame is T_0 =0.28 sec. The required lateral stiffness K_s necessary for supporting the gravity loads is K_s=0.18K. The frame has been designed in order to limit the drift to 0.5% (x_{lim}=1.98cm). Following Step 1, consider now a possible reduction of K by introducing a diagonal active brace member while maintaining the original performance level (0.5% drift). Mass will be changed accordingly while damping reduces according to Rayleigh damping constraint. In this example, using the proposed methodology, a reduction of stiffness of 60% is selected in order to satisfy the same performance level of 0.5% drift with a maximum active control force of 94.86 kN. Many combinations are possible in determining the section properties of the columns and the beam for which it is possible to obtain a stiffness reduction of about 60%. In this example, the two columns are substituted by two W14×99. The initial steel mass of the frame is 4959.5 kg, while with active brace the structural steel mass is 2775.7kg obtaining a structural steel weight reduction of 44% (Figure 3b). This simple example shows that a substantially lighter structure can be designed to achieve a specified performance objective when an active brace is integrated into the structure in an optimal fashion.

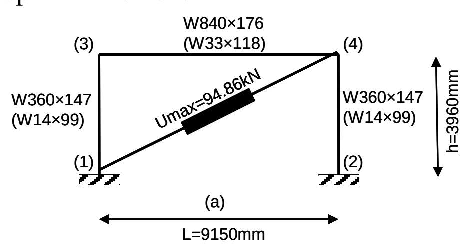

Figure 2 Steel portal frame with active brace

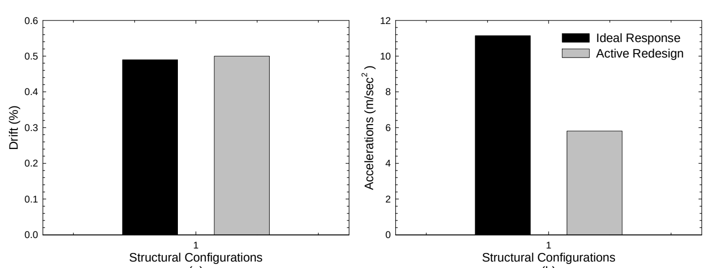

Figure 3 Response comparison with different design configurations

3.2. 9-story shear-type building

The nine-story benchmark structure considered in this example is 45.73 m (150 ft) by 45.73 m (150 ft) in plan, and 37.19 m (122 ft) in elevation. Typical floor-to-floor heights (measured from center-of-beam to center-of-beam for analysis purposes) are 3.96 m (13 ft). The floor-to-floor height of the basement level is 3.65 m (12 ft) and for the first floor is 5.49 m (18 ft).

Table 1. Drift and acceleration response for the redesign structure

Story level	Uncontrolled		Redesign approach		
(1)	(2)	(3)	(4)	(5)	(6)
No.	Drift [%]	$\ddot{x}_a$ [m/sec^2]	Drift [%]	$\ddot{x}_a$ [m/sec^2]	U_{max} [kN]
9	0.31	3.61	0.23	1.87	134.97
8	0.18	2.97	0.10	1.67	107.11
7	0.94	2.71	0.67	1.89	110.81
6	0.27	2.71	0.19	1.70	114.80
5	0.90	2.64	0.73	1.80	125.26
4	0.42	3.74	0.34	1.90	113.00
3	0.38	3.50	0.31	1.98	113.42
2	0.38	2.95	0.31	1.96	108.17
1	0.79	2.79	0.71	1.93	57.02

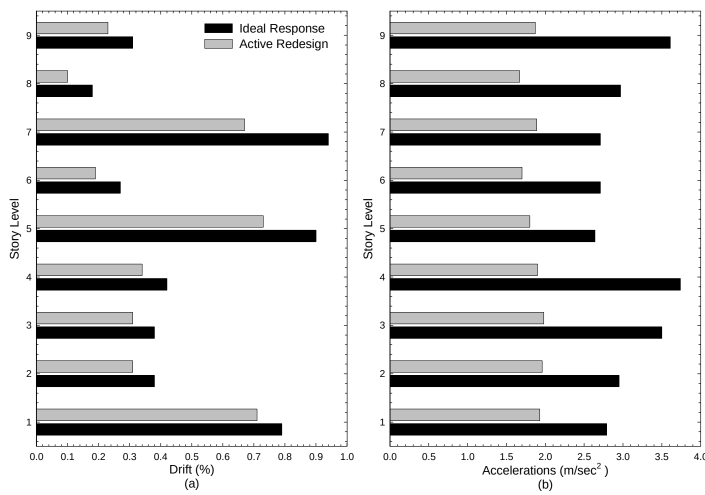

Figure 4 Response comparison with Redesign approach

The first three natural frequencies are 0.44, 1.18 and 2.05 Hz. Rayleigh proportional damping is considered, including 2% of damping ratio for the first two modes. More details about the model can be found in Othori *et al*. (2004). The structure is subjected to the first 30 sec of white noise with amplitude of 0.15g and with a sampling frequency of 0.02 sec. Initially, the story lateral stiffness is reduced proportionally to 30% of the initial stiffness value in order to obtain a first natural period increment of 83%. Then, an active brace is applied at each story level in order to achieve the same performance in term of drift of the uncontrolled initial structure. After the structure and controller are designed independently in Step 1, the controller and the building are redesigned together in Step 2 in order to achieve the same performance (Ideal Response) by reducing the amount of active control power. The initial total energy transferred to the structure from the controller is equal to 2623.0 N·m·sec and, after redesign, is equal to 1972.1 N·m·sec, so the percentage of reduction of the total energy transferred is 24.81% in Step 2 of the procedure. Response of the redesign structure are shown in columns 4, 5 and 6 of Table 1, while comparisons between the Ideal Response and the active redesign response are shown in Figure 4. The optimal percentage variations of the structural parameters

(**M**, **K** and **C**) with respect to the initial structure after active redesign are shown in Table 2. Finally, the story mass distributions before and after redesign are shown in Figure 5a, while the total mass reduction is shown in Figure 5b.

Table 2. Percentage increment or reduction of structural parameters

Story level	ΔM	ΔC	ΔK	U_{max}
	(%)	(%)	(%)	[kN]
(1)	(2)	(3)	(4)	(5)
9	-34.4	228.8	-81.8	134.97
8	-30.8	406.9	-78.4	107.11
7	-32.1	11.3	-78.1	110.81
6	-29.6	112.0	-77.4	114.80
5	-27.0	-40.5	-77.5	125.26
4	-25.2	-32.0	-76.5	113.00
3	-14.4	-12.1	-76.0	113.42
2	-4.2	41.5	-74.9	108.17
1	-12.3	-20.2	-74.8	57.02

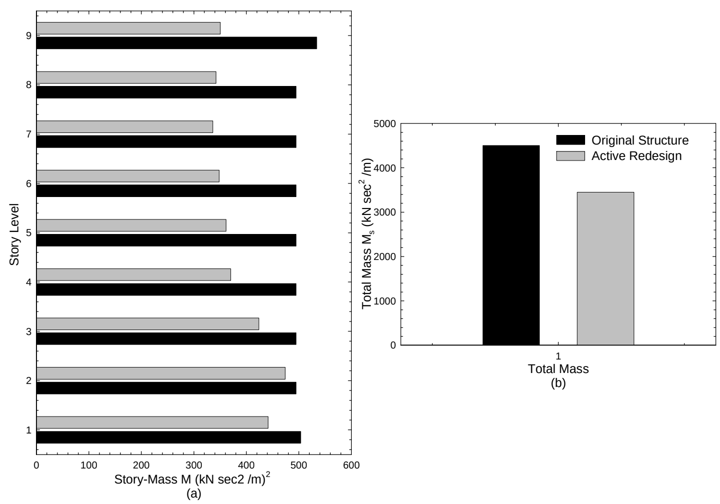

Figure 5 Structural Mass M_s before and after Redesign of the MRF

4. CONCLUSIONS

In this paper is outlined an approach to determine the optimal structural/control system such that an optimal structural configuration can be achieved while satisfying a specified performance objective. It is shown that, using the two-step redesign approach, an efficient solution procedure can be developed. The redesign approach is shown in detail for a SDOF one-story one-bay steel portal frame. A nine-story shear-type building is also used as a numerical example to show that the redesign approach is equally efficient in dealing with multi-degree-of-freedom structural systems. It can be shown that the procedure is equally applicable to nonlinear systems (Cimellaro *et al*. 2008b).

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