SEISMIC DESIGN OF STRUCTURES WITH SUPPLEMENTAL MAXWELL MODEL-BASED BRACE-DAMPER SYSTEMS

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ABSTRACT:

In this paper, a procedure to achieve a target performance of multi-degree-of-freedom structures with Maxwell model-based brace-damper systems is proposed. A single story structure with a viscous damper installed at the top of a Chevron-type brace is first used to investigate the effect of brace stiffness on the story displacement. Closed-form solutions relating the brace stiffness and damping coefficient to the target displacement reduction are derived for this simple structure. For a given brace stiffness, the closed-form solution is minimized to give design formulae for the optimal damping coefficient and maximum performance. The model is subsequently extended to multistory buildings with viscous dampers installed at the top of Chevron-type braces. The optimal brace stiffness and damping coefficients are obtained through an iterative process where an index, defined in terms of the sum of the mean square of the interstory drift, is minimized. The reduction in response, and hence improved performance of the structure, can be achieved by a strategic combination of the brace stiffness and viscous damper coefficients, instead of relying on damper coefficients alone as conventionally been done.

KEYWORDS: Maxwell model, brace stiffness, viscous damper, optimization

1. INTRODUCTION

Passive control devices, such as viscous fluid dampers, visco-elastic dampers, friction dampers, and tuned mass dampers, are generally accepted as being effective in mitigating the hazards posed by wind and earthquake (Soong and Dargush 1997, Constantinou et al. 1998, Lee et al. 2006). These devices, when properly designed, enhance the performance of the structure by modifying their dynamic response characteristics. Although the installation of these devices invariably incurs additional cost, their strategic use is cost-effective as the extra expense is often offset by the need to increase the lateral stiffness and strength of the structure in conventional approaches and the need to enhance the ductility capacity of the structure when adapted to seismic environment. However, for economic use of these devices, careful selection of damper parameters and their tactical placement for maximum efficiency becomes important, and these considerations have been an active area of research in recent years (Gluck et al. 1996, Yang et al. 2002, Lavan and Levy 2006, Singh and Moreschi 2001).

FEMA 356 (2000) provides an approximate and easy-to-use design formula to account for the increase in structural damping from the addition of supplemental viscous dampers during seismic rehabilitation of buildings. More specifically, an effective damping ratio arising from the supplemental linear viscous dampers is added to the inherent damping ratio of the structure, where the additional damping ratio, assuming first mode response, is computed by (FEMA 2000):

$$\xi_d = \frac{T\sum_j c_j \cos^2\theta_j \phi_{rj}^2}{4\pi\sum_i (\frac{w_i}{g})\phi_i^2} \tag{1.1}$$

where θ_j is the angle of inclination of the j^{th} device from the horizontal, c_j is the damping coefficient of the j^{th} device, ϕ_{rj} is the first modal relative displacement between the ends of the j^{th} device in the horizontal direction, T is the first mode period, ϕ_i is the first modal displacement at i^{th} floor, w_i is the weight of the i^{th} floor. It is evident from Eqn. 1.1 that the effectiveness of the supplemental dampers, as measured by the additional damping ratio ξ_d, depends on the orientation of the dampers. For a given damping coefficient c_j, contribution to the effective damping ratio by the damper will be proportional to the $\cos^2\theta_j$ term, which means viscous dampers aligned horizontally, i.e. parallel to the floor, will be more efficient in increasing the effective damping. Hence, instead of using a diagonal brace, the Chevron-type brace is generally considered to be more beneficial since the viscous dampers can be installed and connected to the next story horizontally.

Although FEMA 356 (2000) permits the use of energy dissipation devices for seismic rehabilitation of buildings, it does not provide guidelines for optimal design of these devices. In the literature review conducted in this paper, several researchers (Gluck et al. 1996, Yang et al. 2002, Lavan and Levy 2006) provided procedures for determining the optimal damper parameters and configuration, and most of these procedures are efficient in arriving at the optimal parameters. For structures with viscous dampers installed at the top of Chevron-type braces, procedures proposed by Takewaki and Yoshitomi (1998) and Singh et al. (2003) are subjected to a constraint on the total equivalent damping ratio. Although their design procedures maximize the effect of the added dampers, the influence of brace stiffness was not thoroughly investigated. Singh et al. (2003) noted that increased brace flexibility tends to reduce the effectiveness of the viscous dampers. It was suggested that a brace stiffness that is ten times the story stiffness can be considered essentially rigid, and that a brace stiffness that is five times the story stiffness would be adequate without compromising the damping effectiveness significantly. Despite such recommendations, it remains unclear how significantly will the brace stiffness affect the damper performance as well as the structural response.

Most approaches in the literature ensure an improved performance of the building by increasing the damper dissipative capacity, which translates into an increase in the overall damping of the building. The increase in overall damping of the building is typically assessed in terms of the amount of damping associated with the first mode. However, in the context of optimization, improved performance should be examined in terms of response reduction via response parameters such as displacement, velocity, acceleration, inter-story drift or base shear force. For example, a performance index can be specified in terms of the story displacement, and the target reduction in the performance index can thus be regarded as the goal of the optimization. In this paper, a gradient-based approach for optimizing the structures with multiple Maxwell models is proposed. For a given target performance, it is found that there exists an optimal brace stiffness for both single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) structures.

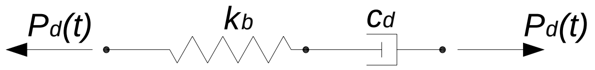


Figure 1 Maxwell model - serial arrangement of linear spring and viscous dashpot

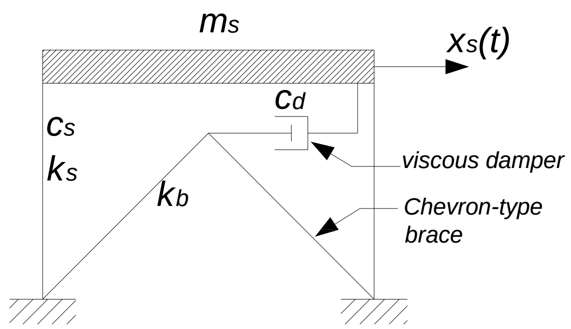


Figure 2 One-story structure with a viscous damper installed at the top of a Chevron-type brace

2. ANALYTIC MODEL FOR A SDOF STRUCTURE WITH A MAXWELL MODEL

A SDOF structure with a viscous damper installed at the top of a Chevron-type brace is used as the basic model for the first phase of the study. The serial arrangement of a viscous damper and brace assembly can be represented by a Maxwell model, as shown in Figure 1, which can be described by the following first order differential equation

$$P_d(t)+\frac{c_d}{k_b}\dot{P}_d(t)=c_d\dot{\Delta}_d(t) \tag{2.1}$$

where $P_d(t)$ is the damper force, c_d is the damping coefficient of the viscous damper, k_b is the horizontal stiffness of the Chevron-type brace, and $\Delta_d(t)$ is the sum of the deformation of the spring and dashpot elements in the Maxwell model. When the brace and viscous damper assembly, as represented by the Maxwell model, is used in conjunction with a SDOF structure, which is shown in Figure 2, the equations of motion of this structure-brace-damper system become

$$m_s\ddot{x}_s(t)+c_s\dot{x}_s(t)+k_s x_s(t)+P_d(t)=-m_s\ddot{x}_g(t) \tag{2.2}$$

$$P_d(t)+\frac{c_d}{k_b}\dot{P}_d(t)=c_d\dot{x}_s(t) \tag{2.3}$$

where m_s, c_s, and k_s are the mass, damping, and stiffness of the SDOF structure, respectively, $P_d(t)$ is the force exerted by the viscous damper, $x_s(t)$ is the story displacement, and $\ddot{x}_g(t)$ is the ground acceleration.

2.1. Minimization of the mean square displacement response

In order to analyze the system for response reduction, a displacement performance index, defined as the mean square of the structure's displacement in the frequency domain, is used as the objective function

$$J_s=\sigma_x^2=\int_{-\infty}^{\infty}\left|x_s(\omega)\right|^2 d\omega \tag{2.4}$$

For closed-form expression of the integral, response of the structure in the frequency domain needs to be determined. Assuming an elastic response, Fourier transform of Eqn. 2.2 and 2.3 permit the displacement response of the structure to be written as

$$x_s(\omega)=H(\omega)\ddot{x}_g(\omega) \tag{2.5}$$

where $H(\omega)$ is the transfer function between structure's displacement and ground acceleration. Although a more realistic power spectral density (PSD) function can be used for earthquake excitations, closed-form expression for the mean square of the response displacement in Eqn. 2.4 generally becomes difficult. In this paper, the PSD function of the ground excitation $S_{gg}(\omega)$ is taken to be that of white noise excitation having a constant spectral density S_o i.e. $S_{gg}(\omega)=S_o$, so that a closed-form solution can be obtained. By defining $\omega_s=\sqrt{k_s/m_s}$ and $\xi_s=c_s/2m_s\omega_s$, which are the natural circular frequency and the inherent damping ratio of the structure, respectively, and α and β as

$$\alpha\equiv\frac{k_b}{k_s},\quad \beta\equiv\frac{c_d}{2m_s\omega_s} \tag{2.6}$$

which are the stiffness of the brace and the damping ratio of the viscous damper relative to that of the structure, the performance index in Eqn. 2.4 can be integrated analytically for the white noise excitation (Crandall and Mark 1973) to give

$$J_s=\pi S_o\frac{4\xi_s\alpha\beta+\alpha^2+4\beta^2}{\omega_s^3\{8\alpha\beta\xi_s^2+2[\alpha^2+4(\alpha+1)\beta^2]\xi_s+2\alpha^2\beta\}} \tag{2.7}$$

The performance index is in a suitable form that optimal values for α and β that minimize the structure response can be determined. However, by solving $\partial J_s/\partial\alpha=0$ and $\partial J_s/\partial\beta=0$ simultaneously, one would find a set of optimal parameters that lead to infinite brace stiffness and infinite damping coefficient, which are unrealistic. Since the brace has finite stiffness in practical applications, the main design parameter strictly becomes the damping coefficient as a constant stiffness can be assigned to the brace. Thus, by setting $\partial J_s/\partial\beta=0$, the optimal damping ratio of the damper and the resultant displacement performance index are

$$\beta_{s,opt}=\frac{\alpha}{2(1-2\xi_s)},\quad J_{s,opt}=\pi S_o\frac{2(1-\xi_s)}{\omega_s^3[\alpha+4(\xi_s-\xi_s^2)]} \tag{2.8a and b}$$

It can be seen from Eqn. 2.8 that the brace stiffness, as characterized by α, affects the efficiency of the damper. On purpose to quantify the influence of the brace stiffness, the displacement performance index of the original system, i.e. without added dampers, is first considered, which is given by

$$J_{s,org}=\pi S_o\frac{1}{2\omega_s^3\xi_s} \tag{2.9}$$

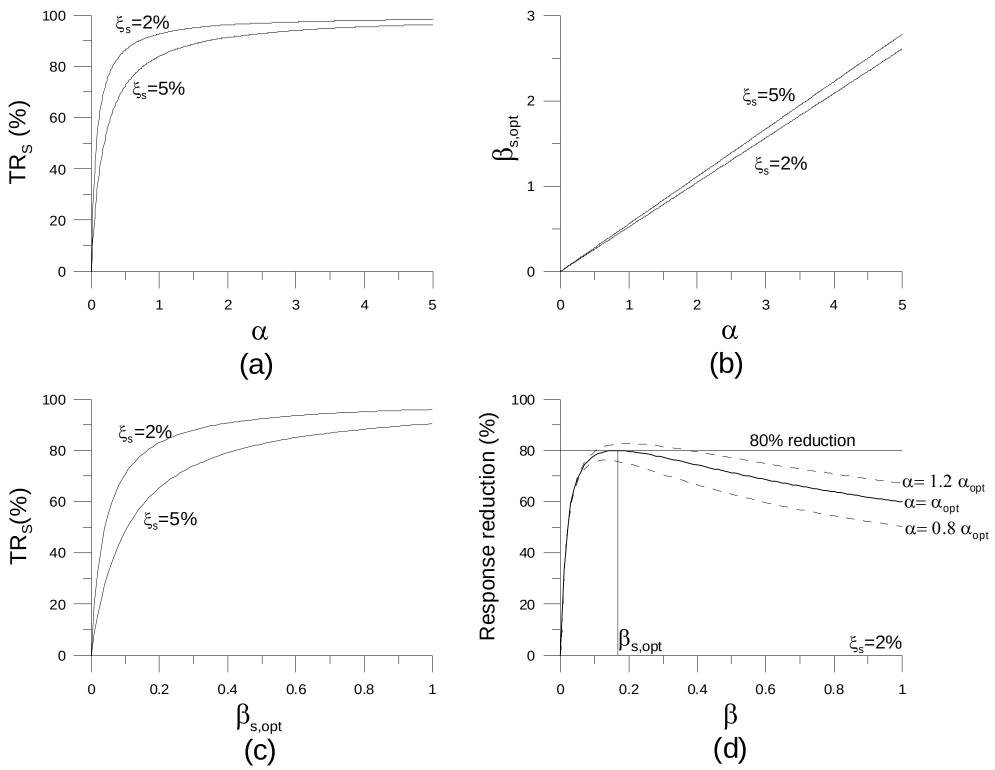


Figure 3 Optimal design parameters for a SDOF structure with a Maxwell model under white noise excitation

The improved performance $J_{s,opt}$ can be examined relative to its original displacement performance index $J_{s,org}$ using the ratio TR_s, referred to as the target reduction for the response displacement, which is defined as

$$TR_s = \frac{J_{s,org} - J_{s,opt}}{J_{s,org}} = \frac{\alpha}{\alpha + 4(\xi_s - \xi_s^2)} \tag{2.10}$$

2.2. Observations from the SDOF system

The brace-damper system can be designed on the basis of a target reduction in the response displacement. Note that the target reduction in Eqn. 2.10 does not depend on the damping ratio β of the damper but depends on the brace stiffness ratio α and the inherent damping ratio ξ_s of the structure. Once the target reduction in mean square displacement is selected, Eqn. 2.10 can be inverted to give the brace stiffness, which is given by $k_b = \alpha\, k_s$. Upon substituting the brace stiffness ratio and structural damping ratio into Eqn. 2.8a, the damping coefficient can be determined from $c_d = 2 m_s\, \omega_s\, \beta$. Figures 3(a) and (b) show the target reduction TR_s and the optimal damping ratio $\beta_{s,opt}$ as a function of the brace stiffness ratio α for structural damping ratios $\xi_s = 2\%$ and 5%. It can be seen from Figure 3(a) that *TRs* increases rapidly with α in the small brace stiffness range. This means that the system is characterized by a fast reduction in response displacement as the brace stiffness increases when the brace stiffness is small. The target reduction shows a diminishing return in the large brace stiffness range, say $\alpha > 1$ for both damping ratios. The required optimal damping ratio $\beta_{s,opt}$ can be found from Figure 3(b) upon the determination of the brace stiffness ratio α, or directly from Figure 3(c), given the target reduction. It can also be seen from Figure 3(c) that, for a given target displacement reduction, larger damping coefficient is needed for the viscous damper if the inherent structural damping ratio $\xi_s = 5\%$.

Figure 3(d) shows the response displacement reduction, defined by $(J_{s,org} - J_s)/J_{s,org}$ where J_s and $J_{s,org}$ are given in Eqns. 2.7 and 2.9, versus the damping ratio β of the damper for inherent damping ratio $\xi_s = 2\%$ .The solid line corresponds to the case of $\alpha = \alpha_{opt}$ for 80% target reduction in response displacement. It can be seen that, when $\beta = \beta_{s,opt}$, the response reaches maximum reduction, which is 80% in this case, and when $\beta \neq \beta_{s,opt}$, the response reduction will be less than 80%. Since the brace stiffness can potentially be treated as a design parameter, it is instructive to examine the influence of non-optimal brace stiffness on the response reduction. The upper dashed line in Figure 3(d) corresponds to the case when the brace stiffness is 20% greater than the optimal brace stiffness for 80% reduction, i.e. $\alpha = 1.2\alpha_{opt}$. In this case, there is a range of damping ratio β where the response reduction will be greater than 80%. The lower dashed line in Figure 3(d) corresponds to the case when the brace stiffness is only 80% of the optimal brace stiffness i.e. $\alpha = 0.8\alpha_{opt}$. In this case, the response reduction will always be less than 80% irrespective of the damping ratio β. It may thus be concluded that if a certain level of response reduction is targeted, then there exists a minimum brace stiffness below which the targeted response reduction cannot be achieved.

3. ANALYTIC MODEL FOR MDOF STRUCTURES WITH MULTIPLE MAXWELL MODELS

The dynamic response of a multistory building with n degrees of freedom installed with viscous dampers at the top of Chevron-type braces is governed by the following two equations

$$\mathbf{M}_s\ddot{\mathbf{x}}_s(t) + \mathbf{C}_s\dot{\mathbf{x}}_s(t) + \mathbf{K}_s\mathbf{x}_s(t) + \sum_{i=1}^{p}\mathbf{r}_{di}P_{di}(t) = \mathbf{E}_s\ddot{x}_g(t) \tag{3.1}$$

$$\mathbf{P}_d(t) + \Gamma\dot{\mathbf{P}}_d(t) = \Lambda\dot{\mathbf{x}}_s(t) \tag{3.2}$$

where

$$\mathbf{P}_d(t) = \begin{bmatrix} P_{d1}(t) \\ \vdots \\ P_{dp}(t) \end{bmatrix}; \Gamma = \begin{bmatrix} \frac{c_{d1}}{k_{b1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{c_{dp}}{k_{bp}} \end{bmatrix}; \Lambda = \begin{bmatrix} c_{d1}\mathbf{r}_{d1}^T \\ \vdots \\ c_{dp}\mathbf{r}_{dp}^T \end{bmatrix} \tag{3.3}$$

$\mathbf{M}_s$, $\mathbf{C}_s$, $\mathbf{K}_s$ are the $n \times n$ mass, damping, and stiffness matrix of the structure, respectively, p is the number of stories installed with viscous dampers, $\mathbf{r}_{di}$ is the $n \times 1$ location vector of the viscous dampers in i^{th} story, P_{di} is the resisting force from the viscous damper at i^{th} story, $\mathbf{E}_s = -\mathbf{M}_s\mathbf{e}$ is the $n \times 1$ location vector for the ground excitation, $\mathbf{x}_s(t)$ is the $n \times 1$ displacement vector of the structure, $\ddot{x}_g(t)$ is the ground acceleration. To facilitate the process of optimal design, an auxiliary degree of freedom is introduced at the connection between the brace and viscous damper in order to define the deformation of the brace and viscous damper separately. The use of an auxiliary degree of freedom also enables the two equations of motion to be compactly combined into one equation, which can be re-written in the following form

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{E}\ddot{x}_g(t) \tag{3.4}$$

where

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}; \mathbf{C} = \begin{bmatrix} \mathbf{C}_s + \mathbf{C}_{ss} & -\mathbf{C}_{sa} \\ -\mathbf{C}_{as} & \mathbf{C}_{aa} \end{bmatrix}; \mathbf{K} = \begin{bmatrix} \mathbf{K}_s + \mathbf{K}_{ss} & -\mathbf{K}_{sa} \\ -\mathbf{K}_{as} & \mathbf{K}_{aa} \end{bmatrix}; \mathbf{E} = \begin{bmatrix} \mathbf{E}_s \\ \mathbf{0} \end{bmatrix}; \mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_s(t) \\ \mathbf{x}_d(t) \end{bmatrix} \tag{3.5}$$

and

$$\mathbf{C}_{ss} = \sum_{i=1}^{p} c_{di}(\mathbf{u}_i\mathbf{u}_i^T); \mathbf{C}_{sa} = \sum_{i=1}^{p} c_{di}(\mathbf{u}_i\mathbf{r}_i^T); \mathbf{C}_{as} = \sum_{i=1}^{p} c_{di}(\mathbf{r}_i\mathbf{u}_i^T); \mathbf{C}_{aa} = \sum_{i=1}^{p} c_{di}(\mathbf{r}_i\mathbf{r}_i^T) \tag{3.6}$$

$$\mathbf{K}_{ss} = \sum_{i=1}^{p} k_{bi}(\mathbf{v}_i\mathbf{v}_i^T); \mathbf{K}_{sa} = \sum_{i=1}^{p} k_{bi}(\mathbf{v}_i\mathbf{r}_i^T); \mathbf{K}_{as} = \sum_{i=1}^{p} k_{bi}(\mathbf{r}_i\mathbf{v}_i^T); \mathbf{K}_{aa} = \sum_{i=1}^{p} k_{bi}(\mathbf{r}_i\mathbf{r}_i^T) \tag{3.7}$$

the superscript ^{T} denotes the conjugate of the transpose of a vector or matrix, $\mathbf{u}_i$ is a $n \times 1$ influence vector containing 1 at the structural degree of freedom where the damper is connected i.e. i^{th} story, and zero elsewhere, and $\mathbf{v}_i$ is a $n \times 1$ influence vector containing 1 at the structural degree of freedom where the brace is connected i.e. $(i-1)^{th}$ story and zero elsewhere, $\mathbf{r}_i$ is a $p \times 1$ location vector containing 1 at the position associated with the i^{th} damper in vector $\mathbf{x}_s$ and zero elsewhere.

3.1. Minimization of the mean square displacement response

Although a conventionally designed structure may be expected to respond inelastically under a design level earthquake, potential plastic deformation and hence structural damage can be avoided by judicious addition of viscous dampers, introduced as part of the hazard mitigation strategy in order to ensure an elastic response of the structure. Similar to the SDOF system presented in Section 2.1, a displacement performance index can be defined as the sum of the mean square of the interstory drift. To that end, Fourier transform of Eqn. 3.4 gives

$$\mathbf{X}(\omega) = \mathbf{H}(\omega)\mathbf{E}\ddot{x}_g(\omega) \tag{3.8}$$

where $\mathbf{H}(\omega) = [-\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K}]^{-1}$ is the $(n+p)\times(n+p)$ frequency response function, and $\mathbf{X}(\omega)$ and $\ddot{x}_g(\omega)$ are the Fourier transforms of $\mathbf{x}(t)$ and $\ddot{x}_g(t)$, respectively, j is the imaginary number $\sqrt{-1}$. In this paper, the story drift is converted to an interstory drift vector using a transformation matrix $\mathbf{D}$ defined by

$$\mathbf{y}(t) = \mathbf{D}\mathbf{x}(t) \tag{3.9}$$

where $\mathbf{y}(t)$ is the $m\times 1$ response vector of interstory drift and $\mathbf{D}$ is the $m\times(n+p)$ transformation. The interstory drift vector $\mathbf{y}(t)$ is subsequently used as the performance criterion for optimization. Note that Eqn. 3.9 selects partially the response of the structure for the performance index since $m \le n$. In equation form, the performance index can be written as the sum of the mean square of the 'selected' structural interstory drift as

$$J_s = \int_{-\infty}^{\infty} \left\|\mathbf{Y}(\omega)\right\|^2 d\omega = \int_{-\infty}^{\infty} tr\{\mathbf{S}_{\mathbf{YY}}(\omega)\} d\omega \tag{3.10}$$

where $tr\{\bullet\}$ is the trace of a square matrix, and $\mathbf{S}_{\mathbf{YY}}(\omega)$ is the PSD function matrix of the interstory drift vector. Since the transfer function $\mathbf{H}(\omega)$ in Eqn. 3.8 contains a damping matrix $\mathbf{C}$, the damping coefficients of the dampers are assembled into the damping matrix $\mathbf{C}$ using a location vector $\mathbf{b}$ such that

$$\mathbf{C} = \mathbf{C}_s + \sum_{i=1}^{p} c_{di}\mathbf{b}_i\mathbf{b}_i^T \tag{3.11}$$

where $\mathbf{b}_i = [\mathbf{u}_i \quad \mathbf{r}_i]^T$ is a $(n+p)\times 1$ location vector for the viscous damper at i^{th} story. For optimization of the damper parameters, the following partial derivatives of the transfer function are needed

$$\frac{\partial \mathbf{H}(\omega)}{\partial c_{di}} = -j\omega\mathbf{H}(\omega)\mathbf{b}_i\mathbf{b}_i^T\mathbf{H}(\omega) \tag{3.12}$$

$$\frac{\partial \mathbf{H}^T(\omega)}{\partial c_{di}} = j\omega\mathbf{H}^T(\omega)\mathbf{b}_i\mathbf{b}_i^T\mathbf{H}^T(\omega) \tag{3.13}$$

The partial derivative of the displacement performance index J_s with respect to the damping coefficient c_{di} can be shown to be

$$\frac{\partial J_s}{\partial c_{di}} = \int_{-\infty}^{\infty} tr\{\mathbf{P}(\omega) + \mathbf{P}^T(\omega)\} d\omega, \quad i = 1 \sim p \tag{3.14}$$

where $\mathbf{P}(\omega) = -j\omega\mathbf{D}\mathbf{H}(\omega)\mathbf{b}_i\mathbf{b}_i^T\mathbf{H}(\omega)\mathbf{E}S_{gg}(\omega)\mathbf{E}^T\mathbf{H}^T(\omega)\mathbf{D}^T$.

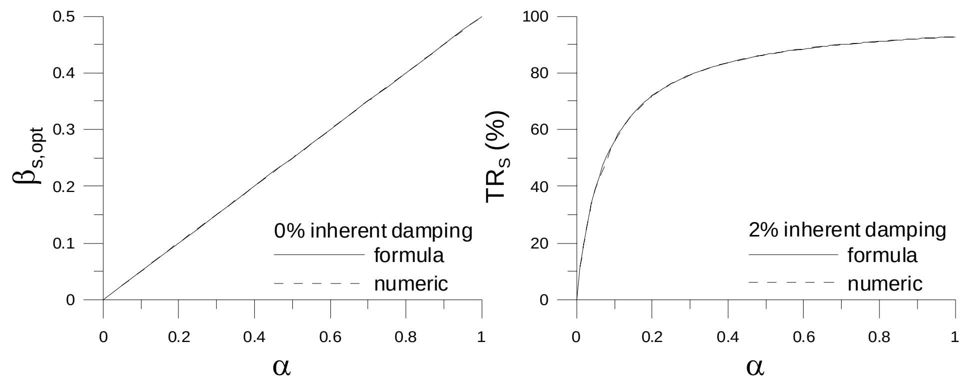
3.2. Numerical procedure and verification

The optimization of the design parameters results in a set of highly nonlinear simultaneous equations, closed form solution of which is difficult to obtain except for simple cases. The simultaneous equations are however readily amenable to numerical methods where the optimal design parameters can be readily determined for the dampers in a systematic fashion. In that regard, the conjugate gradient method (Rao 1996) is used to solve this unconstrained optimization problem. The method requires the gradients of the objective function to be determined, which have been derived in Eqn. 3.14. The conjugate gradient method also requires the calculation of an optimum step length, which has been determined using the golden section search method in this paper, as outlined in Rao (1996). Iterations are carried out such that if the certain level of target reduction is not satisfied, the brace stiffness is increased until the target reduction is reached.

A verification of the proposed procedure is shown in Figure 4(a) and 4(b), where numerical solutions are compared to closed-form solutions using SDOF structure with a single Maxwell model. Although results in Figure 4(a) and 4(b) are identical to that presented earlier in Figures 3(a) and 3(b), these figures nonetheless show that the numerical procedure developed for MDOF structures 'converges' to that of closed-form solution for the simple case of SDOF structures.

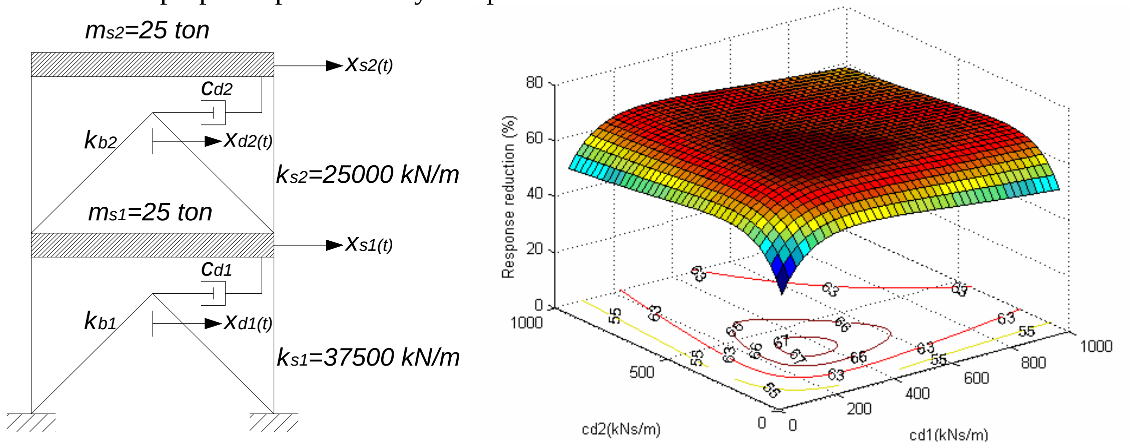
4. Numerical example

The proposed optimization procedure will now be verified using a two story shear building with two Maxwell models, as shown in Figure 5(a). The structure is the same as that studied by Lavan and Levy (2006). A 3%



(a) (b)

Figure 4 Verification of the proposed procedure by comparison of numeric results with closed-form solutions



(a) Two story shear building (b) Exhaustive search result

Figure 5 Numerical verification for multi-degree-of-freedom structures

structural damping ratio is assumed for the first two modes of the original structure. The mass, stiffness of the original structure, and the transformation matrix for the horizontal displacement coordinate are

$$\mathbf{M}_s = \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix} (ton); \mathbf{K}_s = \begin{bmatrix} 62500 & -25000 \\ -25000 & 25000 \end{bmatrix} (kN/m); \mathbf{D} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \quad (4.1)$$

For installation of the viscous dampers, the brace stiffness is assumed to be 20% of the first story stiffness i.e. $k_{b1} = k_{b2} = 7500 kN/m$. The optimal damper coefficients determined from the proposed gradient-based search are $c_{d1} = 357.48\text{kN}\cdot\text{s/m}$ and $c_{d2} = 332.64\text{kN}\cdot\text{s/m}$ for the first and second story dampers, respectively. The improved performance, as calculated by the ratio $(J_{s,org} - J_s)/J_{s,org} = 67.33\%$. This implies a reduction in the interstory drift of 67.33%. Since the proposed procedure involves rather complex numerical searches in the frequency domain, its validity needs to be verified separately. In this case, verification is confirmed by an independent time history analysis. In that regard, ground acceleration time history is first generated from the same white noise PSD function used in the optimization procedure. The average reduction in interstory drift from five time-history runs is 67%, and is the same as the frequency domain results.

An exhaustive search is further conducted on the two story shear building using the preset brace stiffness that is equals to 20% of first story stiffness, with the primary objectives of verifying (i) the accuracy of the proposed procedure, and (ii) the existence of a global maximum response reduction in terms of interstory drift. Results of the exhaustive search are shown in Figure 5(b) where the response reduction in interstory drift is plotted against the damper coefficients c_{d1} and c_{d2}. It can be seen from the figure that the response reduction increases to a maximum of 67.22% at $c_{d1} = 350\text{kN}\cdot\text{s/m}$ and $c_{d2} = 325\text{kN}\cdot\text{s/m}$ based on a grid size of 25 $\text{kN}\cdot\text{s/m}$ and 25 $\text{kN}\cdot\text{s/m}$ for c_{d1} and c_{d2}, respectively. Results from exhaustive search agree reasonable well with that of the gradient-based search ($350\text{kN}\cdot\text{s/m}$ versus $357.48\text{kN}\cdot\text{s/m}$ for c_{d1} and $325\text{kN}\cdot\text{s/m}$ versus $332.46\text{kN}\cdot\text{s/m}$ for

c_{d1}). A finer grid size in the exhaustive search is expected to yield better agreement between results of the two search methods. Although not explicitly shown here, the damping ratio of the structure increases from 3% to 8.43% for the first mode. It is also worth noting that, the serial arrangement of the brace and damper may result in an increase in the frequency of the structure, which is counter intuitive for increasing damping. In this two-story example, the frequency of the first mode increases from 8.72 Hz to 9.7 Hz.

5. Conclusions

In this paper, effects of the brace stiffness are first investigated using a simple single-story structure with a viscous damper installed at the top of a Chevron-type (inverted V) brace. Closed-form solutions, in terms of story displacement, are derived for the simple model. Results from the simple model show that the brace stiffness needs not be excessively large compared to the story stiffness, unlike the common assumption in previous studies which suggested that the brace stiffness should be at least five times the story stiffness. For a given target reduction in response, there exists a minimum brace stiffness, which may be smaller than the story stiffness depending on the target reduction and the structure's inherent damping ratio. Results also show that the reduction in response can be improved by a suitable combination of brace stiffness and damping coefficient.

A new optimization procedure is proposed for multistory buildings with viscous dampers installed at the top of Chevron-type braces. Since the procedure is numerical in nature, its accuracy is first verified against the closed-form solution from the simple model. The procedure is subsequently applied to a two story shear building, which is shown to give good results compared to that of an exhaustive search. Numerical results from the two-story shear building also indicate that, for a given brace stiffness, there exists a global maximum in response reduction. Although not explicitly shown, the proposed procedure is capable of accounting for the higher modes of vibration as well as dealing with different power spectral density functions such as that from wind or earthquake excitations. Even though the procedure is demonstrated using the interstory drift as the performance index, it can be easily extended to other structural response parameters such as floor accelerations or base shear force. By providing an initial guess of the brace stiffness, the proposed procedure will iteratively search for the optimum brace stiffness and damper coefficients until the target reduction in response is achieved.

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