Structural Frequency Topology Optimization in Seismic Design Based on FEM Method

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ABSTRACT :
Owing to problems such as iterative sensitivity and complex degree of structural shape, the application of the topology optimization method in structural seismic design is limited. Furthermore, it is very difficult to choose appropriate constraint functions to meet optimization objective when commercial finite element software is used. However, if earthquake loads have been obtained, FEM (finite element model) method is an efficient way to apply the topology optimization to structural seismic design. ESO (evolutionary structural optimization) method and improved ESO method are both used to implement the single object optimization process for simple structures such as rectangular and circular pool. In the optimization process, the element connectivity parameterization formulation is used to solve an alternative to the element density formulation. As a result, some conclusion about frequency topology optimization in different structures is drawn. In addition, the application of multi-objective optimization to seismic design of structures is also discussed. The method proposed in this paper is benefit to realize structural frequency control in seismic design.

KEYWORDS:
Frequency topology optimization；structural seismic design；FEM method；ESO

1. INTRODUCTION

The aim of structural seismic design is to determine the proper vibration characteristics in order to improve the seismic capacity of structure by correct theory and method. Method based on frequency topology optimization includes two meanings: frequency could map structural vibration characteristics and topology optimization is the base of correct design. Therefore, study on the method of frequency topology optimization, being a way to seek the best structure form, could provide evidence for structural seismic design powerfully.

View structural dynamics as a whole, a good structural dynamic mathematical model consists of three parts such as response computation, load pre-estimation and stability analysis. However, in the whole actual process of structural design, engineer provides structural design proposal in accordance with principle of structural static limit loads, and then makes dynamic strength check and modifies the proposal by dynamic test. So it usually takes a lot of time, human and material resources on completing the process of from preliminary design to the final construction. Differs from traditional design method, the method of structural dynamic optimum design meet special requirement of quality, natural frequency, dynamic stress of some points and dynamic response. To realize this function, constraint condition which includes frequency, response and parameter size will be set beforehand as an important task. A problem worthy to be pointed out is that the method could be used to both size optimization and shape topology.

Liquid container, the common structural forms of which consist of liquid storage tank, oil tank, pool, high pressure water-tower, and aqueduct and so on, is important structure in lifeline engineering. It seems that this kind of structural form is simple, however, its dynamic characteristics and response under seismic load is quite complicated. The main reason has nothing to do with but state nonlinearity that the material of liquid and container are different phases. that is to say fluid structure interaction (FSI) is considered during seismic response analysis. FSI has great impact on natural frequency of the whole liquid container structure system to a certain degree; furthermore, it may increase seismic response and bring much difficult to structural seismic design. Thus, if the information of seismic load could be pre-estimated, using frequency topology optimization method to make seismic design can yet become an effective way.

In this paper, classical ESO (evolutionary structural optimization) [1] method of structural dynamic optimum design is used for design space optimization for frequency topology of liquid container. To reduce overall computational time, the author proposed an improved ESO method and unit connectivity parameterization. In this method, a small design domain with a small number of design variables evolves to a larger one to make target natural frequency of the container far way from of the seismic loads. Also, optimization result can decrease seismic response of the whole FSI system significantly.

2. CONSTRAINT FUNCTION IN TOPOLOGY OPTIMIZATION

Three main factors of Structural dynamic optimum design are structured by optimization variable, optimization objective and constraint function, they are complementing each other. When it comes to specific execution process, there have also three parts as follows: choosing appropriate initialization structural model to analyze accord with the demands of optimization, using approximate method such as sensitivity analysis to create correct objective and constraint function which have linear relation with design variables, obtaining the minimum optimum design solution of objective function from an effective mathematical programming.

Because constraint function has great influence on optimum property and computational efficiency, it become the key point regardless of whether in optimization method or the specific process. General speaking, from the view of constraint variables, constraint function can be classified into state of material strength, static stiffness, dynamic stiffness and stability.

In the field of optimization research, how to choose proper constraint function has become core main factor of structural dynamic optimum design calculation. In the literature up to now, many papers have been published on this topic which mainly concentrated in optimal control method rather than specific numerical method and application technology. Unmerited constraint function can cause to large computational cost to limit the scale of engineering system will be treated. Bendsoe [2] and FOX [3] suggested method to simplify the form of external transient load to solve the function dependency on time. In this circumstance, minimizing the quality of initial structure can be finished with the condition of displacement or stress constraint. To solve this problem effectively, Mill-curran and Schmit [4] tried to make analysis of structural dynamic response in incoherent space with condition of harmonic load. Haug and Hang [5] performed integrations on the constraint function with total effective time and transformed it to equivalent function. However, it still made realization process of numerical optimization face a serious challenge. Hsieh [6] proposed sensitivity analysis method with state variable of inverse point if effective constraint function was implicited in certain time. In theoretically, this method is called worst-case design which uses the maximum constraint value of local point to replace those of every inverse point of the whole numerical model. This has been proven very popular and efficient to obtain the final solution that satisfies optimization condition. Furthermore, Hsieh and Arora presented an adjoint variables mixing formula to study the problem of constraint handling of state variable. This formula, which is suitable to sensitivity analysis, combined constraint function in a certain time and equivalent function into a complete model successfully. In addition to this, it doesn’t have to define the local maximum constraint value precisely. Adelman and Haftka [7] summarized the application situation of sensitivity analysis which is used during the dynamic optimization work well under arbitrary loads. As an efficient tool, three basic approximate methods are classified to this application. Although results from the three methods are consistent, the respective computational cost, which depends on the number of design variables, freedom and constraint, differs widely. Here, in order to realize frequency topology optimization in seismic designon of liquid container, on one hand, we use this method to increase the number as much as possible, on the other hand, we propose element connectivity parameterization method to reduce the time that single-cycle takes as little as possible.

3. ESO METHOD AND IMPROVED ESO METHOD

Researchers have been studying Evolutionary Structural Optimization (ESO) method for a long time. This is very simple in the idea and use. ESO in its original form optimizes a structure by slowly removing elements with low or none efficiency of stress, approaching towards a fully stressed design. In the early stage of ESO, it occurred that an element once removed could not be recovered when necessary. To solve this problem, Querin and Steven[8] extended the ESO method to add as well as remove elements, namely bidirectional ESO. In this paper, we need to make recovering the deleted elements timely in single step iterative the truth rather than to add more new elements. Thus, the method used in this paper is equivalent to partial bidirectional ESO. A classical full stress ESO under gravity is applied to structure in FG 1. And this example is also called apple full stress optimization.

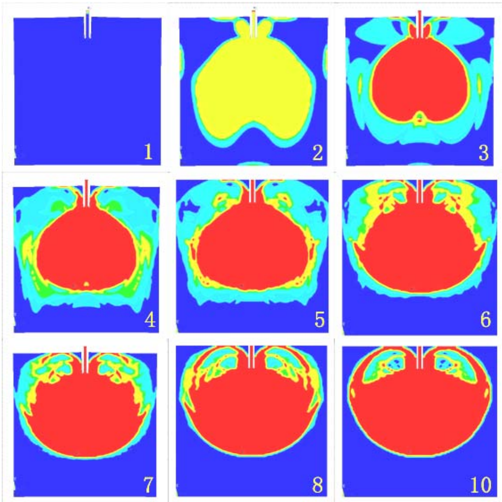


Figure 1 Classical full stress ESO application

In the civil engineering field of optimization application, ESO includes basic object ESO and advanced ESO. Basic ESO is mainly used to solve those problems that oriented to static response such as stress level of those structures which have simple shapes. In specific, given a continuous FEM which has been enough meshed to meet the requirement of calculation, optimization will be executed to the end point with the ratio of the minimum value of element stress to the maximum is less than a certain value under static load. That is to say, the objective optimization result will satisfy condition of full stress that is just the definition of Basic ESO. Based on this, advanced ESO is extended under the range of its optimization constraint conditions and objectives such as integral rigidity and distribution range of structural frequency. To show superiority of the performance over the traditional seismic design on liquid container, advanced ESO abut frequency topology optimization is used to this study.

After given sample space of design variables which is shown in Eq. (3.1)

$$A=\{\beta_1,\beta_2,\beta_3\cdots\cdots\beta_N\} \qquad \beta_i\in\{0,1\},\, i=1,2,3\cdots,N \tag{3.1}$$

The optimization objective in basic ESO can be defined as in Eq. (3.2)

$$W=\sum_{i=1}^{N} w_i\beta_i \tag{3.2}$$

The origin iterated condition in single step to express full stress requirement can be depicted as follows inequality.

$$\frac{\sigma_i^{von}}{\sigma_{max}^{von}}<R_i\,,\;\; \sigma_{max}^{von}<[\sigma] \tag{3.3}$$

If it is necessary to modify the FEM to save calculation resource, the corresponding delete rate should be set in different step.

$$R_{i+i} = R_i + \Delta R \tag{3.4}$$

Firstly, in actual application, sometimes delete rate only based on full stress may cause attenuation gradient of iteration convergence to reach a very small value. This means it will take so much time on computation. Secondly, if the degree of sensitivity deformation is greater than stress for a structure, full stress condition is not the best way to make topology optimization. However, based on character of implicit computation, flexibility matrix is much sensitive than stiffness matrix can be easily proved. So we put forward an improved method that replace strain with stress to solve the above two problems.

$$\frac{\varepsilon_i^{von}}{\varepsilon_{max}^{von}} < R_i \,,\ \ \varepsilon_{max}^{von} < [\varepsilon] \tag{3.5}$$

Before explaining detailed optimization work of this paper, we need to define design variables sensitivity. Design variable sensitivity (DVS) refers to the effect of the change of design viable on the objective function or constraints with a fixed design space and is obtained as follows:

$$\left.\begin{array}{c} ([K]-\omega_j^2[M])\{\varphi_j\}=\{0\} \\ \omega_j^2=\dfrac{k_j}{m_j} \\ k_j=\{\varphi_j\}^T[K]\{\varphi_j\} \\ m_j=\{\varphi_j\}^T[M]\{\varphi_j\} \end{array}\right\} \Rightarrow \left\{\begin{array}{c} \Delta\omega_j^2=\dfrac{\Delta k_j}{m_j}-\dfrac{k_j\Delta m_j}{m_j^2}=\dfrac{1}{m_j}(\Delta k_j-\omega_j^2\Delta m_j) \\ \Delta k_j \approx \{\varphi_j\}^T[\Delta K]\{\varphi_j\}=-\{\varphi_j^i\}^T[K^i]\{\varphi_j^i\} \\ \Delta m_j \approx \{\varphi_j\}^T[\Delta M]\{\varphi_j\}=-\{\varphi_j^i\}^T[M^i]\{\varphi_j^i\} \end{array}\right\}$$

$$\Rightarrow \Delta\omega_j^2=\frac{1}{m_j}\{\varphi_j^i\}^T(\omega_j^2[M^i]-[K^i])\{\varphi_j^i\} \tag{3.6}$$

There are two directions for design variable sensitivity. To delete elements with low efficiency is positive and to add is negative. The value of α_j^i can be obtained from Eq. (3.7).

$$\alpha_j^i=\Delta\omega_j^2, \Delta\omega_j^2>0 \quad , \quad \alpha_j^i=-\Delta\omega_j^2, \Delta\omega_j^2<0 \tag{3.7}$$

When it comes to iterative attenuation during the programming control, we suggest two decay methods such as absolute value to slowly and relative ratio to fast which are expressed in Eq. (8).

$$\alpha_j^i=\Delta\omega_j^2=\omega_j^2-(\omega_j^i)^2 \quad , \quad \alpha_j^i=\Delta\omega_j^2=\frac{\omega_j^2-(\omega_j^i)^2}{\omega_j^2} \tag{3.8}$$

4. CALCULATION FLOW AND ELEMENT CONNECTIVITY

According to the above theory and technical requirement, we use ANSYS APDL language to compile program which includes basic ESO and advanced ESO, and the calculation flow chart of FEM numerical optimization is expressed in FG. 2.

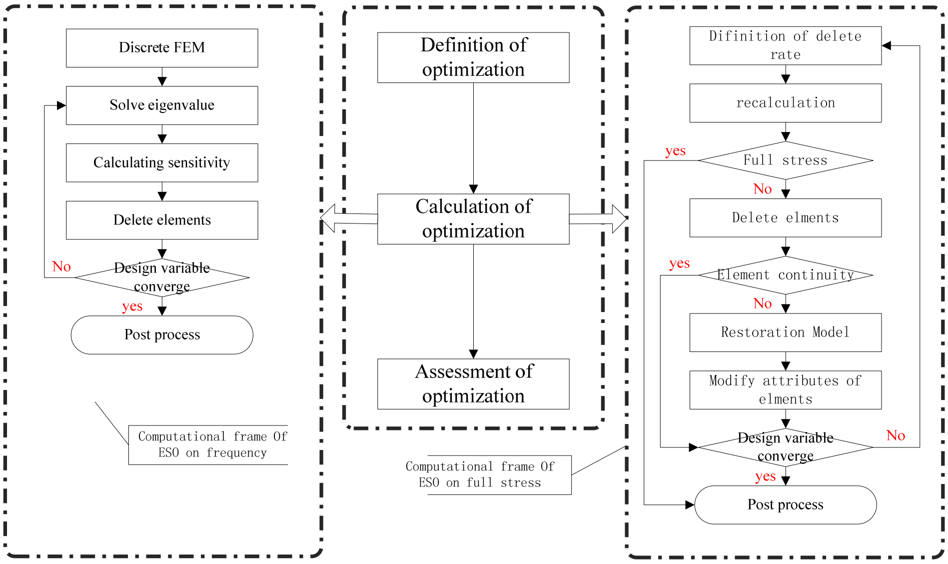
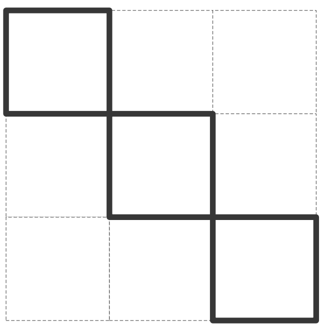
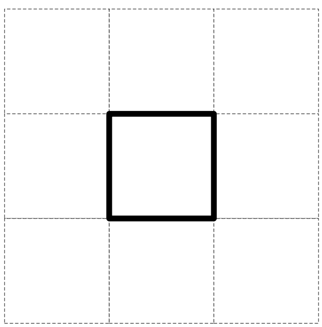


Figure 2 Flow chart for full stress ESO and advanced frequency ESO

It is the high efficiency that makes ESO is fully parameterized and popular. However, there are still many difficult problems urgent to be solved. For example, to keep integral continuity of FEM during each substep after low efficient elements have been deleted is a key and difficult problem. In general, the continuity is the precondition of the whole process for that each step must be carried on with a stable stable equilibrium structure. Then, it will cause the model has more than one closed boundary and make the whole system being a state of transient equilibrium. As the common problems during ESO process, it often induces procedure interrupt. Therefore, it is necessary to introduce a special filter module to check boundary attributes of the structure. Only to confirm the continuity of FEM, the optimization procedure can be executed successfully.

In this paper, element connectivity parameterization (ECP) is used to solve this problem. As it is known, there are three discontinuous types may appear in the procedure. Firstly, it is single point or single node connection which is shown in FG.3. Secondly, sub-element is isolated which is shown in FG.4. The last is that sub-elements are detached from parent elements which is show in FG.5. To overcome the three difficulties, we make the whole FEM parameterized and then use the ECP to scan the boundary to make sure whether the number of closed boundary is greater than 1 or not. If the number is equal to 1, the FEM is continuous and suitable to the next step optimization. Otherwise, it will come back to the upper step and attributes of some elements will be modified to accord with optimization principle. Meanwhile, during the process of scan and check, the number and coordinate all of the elements will be recorded and parameterized as the inquiry mark in the cycle solution. In this method, the continuity of the model could be guaranteed to the end.





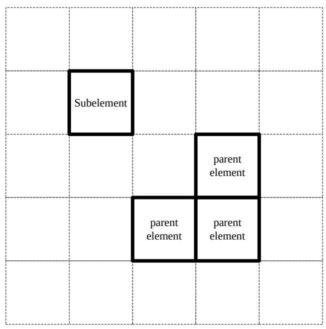
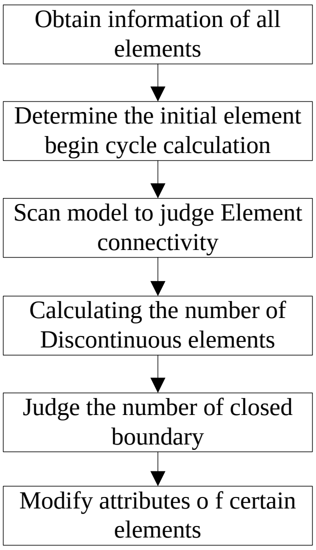
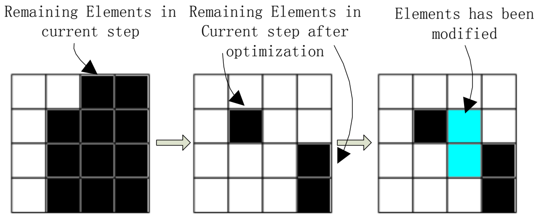


Figure 3 Single node connection   Figure 4 Isolatedsubelment Figure 5 Element detached from parent elements

The specific flow chart of ECP in sub-step is shown as FG. 5. (Note: part of sentence of the chart is illustrative which is not in accordance with source code strictly)



Procedure of Element connectivity



Before delete After delete After modification

```
     difine  E = E_i(x_i, y_i, z_i)
     i = 1
     get N, Elem, sum
100  N = N
     E_i = E_i(x_i, y_i, z_i)
     get E_j = E_j(x_i ± Δl, y_i ± Δl, z_i ± Δl)   j ∈ (1,2···26)
       do j = 1,26
       if E_i(x_i, y_i, z_i) & E_j(x_j, y_j, z_j) are continuous
       select all effective N elment , except E_j(x_j, y_j, z_j)
       get N, Elem, sum
       N = N - 1
          if M ≥ N ,then
          M = N
          else
          M = M
          endif
        enddo
     i = i + 1
     if i ≤ N & M ≠ N ,then
     go to 100
     else
     endif
     end
     select all effective elment
     get N, Elem, sum
     T = M - N
     if T ≠ 0 ,then
     modify the character of element been deleted in this step
     else
     endif
     end
```

Flow chart of program for element connectivity

Figure 6 Flow chart for element connectivity parameterization

By this means, we can get the final value of T, if the value is not equal to 0, there is discontinuity in the model and attributes of some elements should be modified. Of course, if the value of T is equal to 0, that is to say, the optimization during this sub-step is success. The most advantage of the method is that the number of closed boundary can be judged through only once cycle avoiding complex calculation. And this reduces computational cost greatly.

5. EXAMPLE APPLICATION OF OPTIMIZATION

We have shown that the core technology of ESO and advanced ESO. In this paper, as the object of study, the rectangle and cylinder pool are optimized in the field of frequency with the two methods. On the other hand, rectangle and cylinder pool are also the important structures of lifeline engineering. According to Structure Seismic Design Codes (GB500ll-2001), the hoop stress design is set to the constraint condition of basic ESO, and this will be the executed as the sub objective of optimization design. The main object is to make target natural frequency of the container far way from of the seismic loads.

For the determination of elementary parameter of FEM, the curve of natural frequency to depth-span ratio which is shown in FG.7 is used as the foundation. It is seen that the two conditions of pool with consideration of FSI and without differs greatly. Because the frequency has maximum value when depth-span ratio comes to 1, we choose the preliminary parameter value of depth-span ratio equal to 1. In this method, maximizing the value of natural frequency of the pool can put it far from that of seismic loads to avoid the resonance and check the correctness of the result.

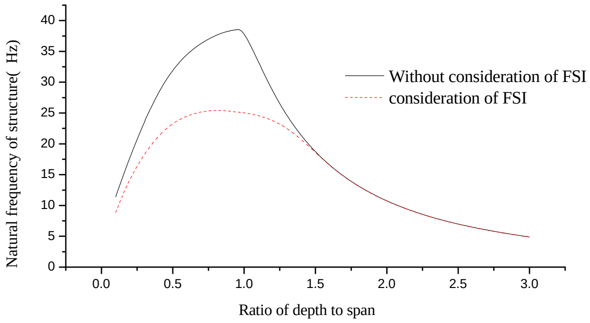


Figure 7 Natural frequency to depth-span ratio

To the calculating results, the comparison between the efficiency of ESO and improved ESO method has been made to prove that the latter is the better. As shown in Table I, the optimization ratio has 8 grades from 10% to 80%. And for the speed of attenuation in flexibility gradient is fast than in rigid, the maximum number of recycle will be reduced by 7%. FG.8 to FG.9 show the preliminary FEM and the final optimization result. And also, from the final result, we can see that the value of depth-span ratio is close to 1.

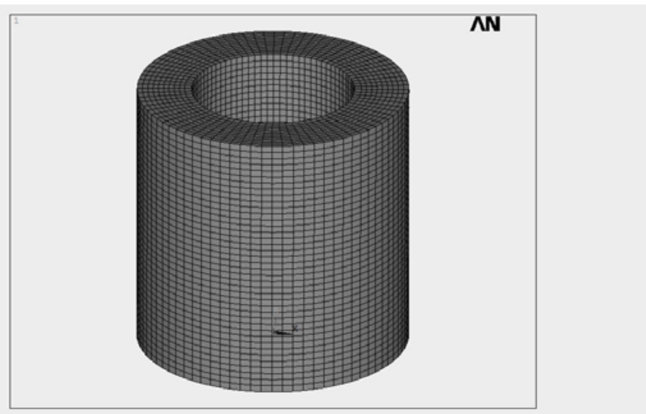


Figure 8 Initial FEM of the cylinder pool

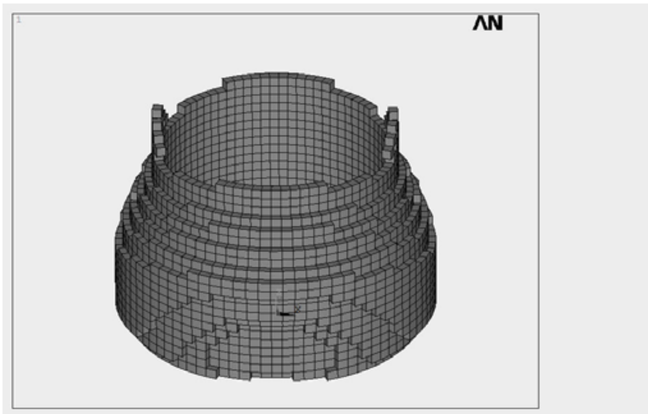


Figure 9 Final optimization FEM of the cylinder pool

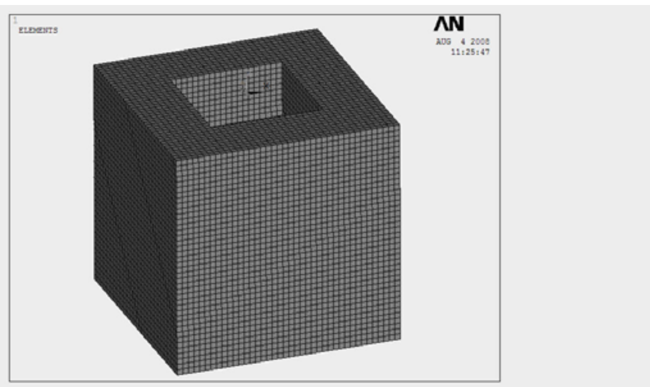


Figure 10 Initial FEM of the rectangle pool

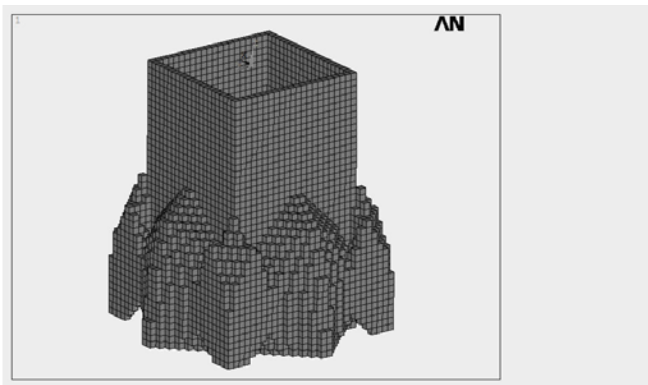


Figure 11 Final optimization FEM of the rectangle pool

Table 5.1 Comparison of solution efficiency (between basic ESO and improved ESO method

Delete rate	Cylinder pool				Rectangle pool			
	ESO		Improved ESO		ESO		Improved ESO	
	SST	TT（h）	SST	TT（h）	SST	TT（h）	SST	TT（h）
10%	4	6.2	4	6.7	7	13.3	7	13.3
20%	7	7.9	7	8.4	11	15.4	11	14.9
30%	12	9.6	11	8.9	14	16.5	15	16.1
40%	18	14.0	14	13.1	19	17.5	20	16.9
50%	25	17.1	21	15.4	26	22.0	27	21.2
60%	33	20.1	27	18.0	27	27.3	29	25.8
70%	40	26.9	34	24.8	49	33.1	44	33.0
80%	45	32.6	37	30.7	61	40.1	52	39.7
SST——the maximum cycle times in the single step TT —— total computational cost								

6. CONCLUSIONS

In this paper, we extend a structural topology optimization for designing vibrating structures. Based on the information of preliminary design and pre-estimated load, structural frequency ropology optimization is studied in details. Basic ESO and improved ESO method are both used to numerical calculation. The constraint condition of stress is changed to strain. Furthermore, element connectivity parameterization is applied into the optimization combined with finite element method. In order to judge integrality of the whole model, the only closed boundary technique is introduced. And we achieved the following:

(1) The improved ESO method has higher efficiency than the classical full stress ESO method. In this method, the maximum value of single recycle times will be reduced by 7%, and the total computational time by 5-10% when the delete rate is equal to 80%.
(2) Element connectivity parameterization has the advantage that judge the boundary integrality without integrating stiffness matrix.
(3) To rectangle and cylinder pool, structural frequency topology optimization in seismic design is so successfully to make the structure avoid resonance and meet with the seismic design code.

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