

# Seismic Design Under Time-Varying Epistemic Uncertainty and Safety Constraints

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## ABSTRACT:

Decisions involving complex systems with not well-known properties are complicated by the fact that epistemic uncertainties and risk regulations vary over time in an uncertain way. In practice, one often makes approximations by fixing the epistemic variables to most likely or worst-case values and ignores future changes in safety constraints. Such simplifications produce sub-optimal decisions. We propose a Bayesian framework for decision-making that explicitly accounts for the above random fluctuations and obtain numerical results for the optimal seismic design of low-rise and high-rise buildings. As expected, when future changes in epistemic uncertainty and regulatory constraints are considered, the optimal level of seismic protection exceeds the normative level at the time of construction.

**KEYWORDS:** seismic design, epistemic uncertainty, acceptable safety, life-cycle cost, Markov processes.

## 1. INTRODUCTION

Many complex technological and natural systems on which we depend are poorly understood. In some cases what is uncertain is the functioning of the system, while in others it is the environment in which the system operates. In either case, the risks posed by or to these systems are uncertain, due to what is commonly known as epistemic uncertainty or uncertainty due to ignorance [1, 2]. Most risk analysts deal with epistemic uncertainty using Bayesian methods [3]. While theoretically well founded, such methods are sometimes inappropriately used. This happens mainly when the utility is a nonlinear function of the level of risk and one erroneously evaluates the expected utility as the utility at the expected risk [4, 5]. A source of nonlinearity is the limit imposed by society on the acceptable risk [6]. The development of decision strategies in the presence of such limits is our main objective. For illustration, we consider the optimum seismic design of buildings. Section 2 presents the principles of optimal Bayesian decision for seismic design using the theory of Markov models with reward. An application example is given in Section 3, followed by conclusions.

## 2. A FRAMEWORK FOR THE OPTIMIZATION OF SEISMIC DESIGN

The seismic safety of a design  $S$  relative to some failure event (taken here to be partial or total collapse) is assessed by combining hazard and fragility information. The hazard function  $H(y)$  gives the rate at which some ground motion intensity at the site exceeds various levels  $y$  and the fragility function  $F(y)$  gives the probability of building failure for different  $y$ . When  $F$  and  $H$  are uncertain, the failure rate  $\lambda_f(F, H) = -\int_0^{\infty} F(y)dH(y)$  is also uncertain. For decision applications, what often suffices is the failure rate  $\lambda_f$ , which is defined as the expected value of  $\lambda_f(F, H)$  with respect to the epistemic uncertainties on  $F$  and  $H$  [7]. If  $F$  and  $H$  are independent,

$$\lambda_f = -\int_0^{\infty} E[F(y)]E[dH(y)] \quad (2.1)$$

The total failure rate  $\lambda_f$  varies randomly in time due to random variations in the epistemic uncertainty (new models and theories, newly collected data, etc.) and in the system properties (for example, due to earthquake-induced damages and retrofit actions). Also the regulatory limit,  $\lambda_{f,\max}$ , may vary in time in an uncertain way. We assume that, for the system to be allowed to operate at any time  $t$ , the safety factor  $SF(t) = \lambda_{f,\max}(t) / \lambda_f(t)$  must exceed 1.

At the time of construction  $t_0$ , the future rates  $\lambda_f(t)$  and  $\lambda_{f,\max}(t)$  and the future safety factor  $SF(t)$  are treated as random processes  $\lambda_f(t|t_0)$ ,  $\lambda_{f,\max}(t|t_0)$ , and  $SF(t|t_0)$  where conditionality on  $t_0$  indicates uncertainty at time  $t_0$ . It is complicated to represent these processes in an accurate way; so some simplification is warranted. We approximate the total process of future earthquake events as Poisson with rate equal to the expected rate calculated at time  $t_0$ . Similar simplifying approximations are made for the process of failure events and the evolution of the state of the system. These simplifications allow one to use the powerful theory of Markov processes with reward [8, 9], as described next.

## 2.1. The State Vector

The safety factor  $SF(t)$  is a fundamental quantity in our analysis. Its random variation in time is due to weakening/strengthening of the system, changes in epistemic uncertainty, and changes in the regulatory limit  $\lambda_{f,\max}$ . To separate these causes of variation, we express  $SF(t)$  as the combination of three frequency ratios:

$$SF(t) = \frac{\lambda_{f,\max}(t)}{\lambda_f(t)} = \frac{F_S(t) \cdot F_R(t)}{F_E(t)} \quad (2.2)$$

where  $F_S(t) = \lambda_{f,\max}(t_0) / \lambda_f(t|t_0)$ ,  $F_E(t) = \lambda_f(t) / \lambda_f(t_0)$  and  $F_R(t) = \lambda_{f,\max}(t) / \lambda_{f,\max}(t_0)$ . The ratio  $F_S(t)$  measures the safety of the system at time  $t$  using the state of uncertainty at time  $t_0$ . At time  $t = t_0$ ,  $F_S(t_0)$  equals  $SF(t_0)$ , the safety factor at the time of construction and may be taken as a basic variable to be optimized in design. For  $t > t_0$ ,  $F_S(t)$  tracks the changes in system strength due to damages, repairs, and retrofitting interventions. The factor  $F_E(t)$  is the ratio of the failure rates of the system at time  $t$  based on information available at times  $t$  and  $t_0$ . Changes in this factor are caused mainly by variations in epistemic uncertainty. Finally,  $F_R(t)$  tracks changes in the regulatory constraint on risk.

We view  $F_S$ ,  $F_E$  and  $F_R$  as components of a state vector that evolves randomly in time. To facilitate calculations, we replace these continuous state variables with discrete variables  $S$  (for structural strength, with values  $1, \dots, n_s$ ),  $E$  (for epistemic uncertainty, with values  $1, \dots, n_E$ ) and  $R$  (for the regulatory limit on risk, with values  $1, \dots, n_R$ ), and denote by  $X = [S, E, R]$  the resulting state vector. Each discrete value of  $X$  is associated with a specific value of the frequency ratios  $F_S$ ,  $F_E$  and  $F_R$  in Eqn. 2.2; see Section 3 for details. Low-case letters  $x = [s, e, r]$  indicate specific integer values of  $X$  and its components. In dealing with state transitions, we generally use primed symbols (e.g.  $x'$ ) for initial values and double-primed symbols (e.g.  $x''$ ) for terminal values.

## 2.2. Structural and Nonstructural Damages

As a result of earthquakes, the system may sustain structural damages. Depending on the repair strategy, the system is returned to pre-earthquake conditions or strengthened to a higher level [9]. To save on storage and computation, we condense out the structural damage levels depending on what repair is made. This is done by creating a duplicate state variable  $S^*$  of  $S$  and considering  $S = s'$  to transition to  $S^* = s''$  whenever, as a result of structural damages and repairs, the system is brought from state  $s'$  to state  $s''$ . In this way no approximation is made and the number of possible state values is only doubled. Once in  $S^* = s''$ , the system has the same properties as if it were in  $S = s''$  and the roles of  $S$  and  $S^*$  reverse. This is the familiar technique to account for repairs in Markov models with reward [8], which we extend here to include multiple possible damage levels. In what follows, the  $S^*$  states are included in the  $S$  state variable, which thus has  $2n_s$  possible values.

In addition to structural weakening and strengthening, one must consider non-structural losses (including damage to non-structural building components, economic losses from downtime, social losses from injuries and fatalities etc.). One could use additional state variables to track such nonstructural damage conditions, but if damages are instantly repaired and losses are instantly incurred one can account for these damages and losses without further augmentation of the state vector  $X$ ; see below.

### 2.3. State Transitions, Losses and Rewards

Changes in  $X$  originate from events of three types: (1) large earthquakes in the region, which may induce damages and subsequent repairs and may additionally trigger new studies of regional seismicity and tightening of the safety regulations, (2) studies of regional seismicity conducted independently of earthquake occurrences in the region, which lead to changes in the epistemic uncertainty, and (3) public safety reviews made independently of the above events, which may lead to changes in the acceptable risk level. Events of Type 1 can possibly modify all three state variables, whereas events of Type 2 and 3 affect directly only  $E$  and  $R$ , respectively (but if changes in  $E$  and  $R$  are such that the regulatory constraints are violated, then also  $S$  changes due to retrofitting).

To simplify the analysis, we assume that events of different types occur according to independent Poisson processes and that the state transitions caused by different events are independent. Then the state vector  $X$  evolves in time according to a Markov process, discrete in state and continuous in time. For each event type  $i$  ( $i = 1, 2, 3$ ) one must specify the rate  $\lambda_i$  and the transition probabilities  $P_{x'x''}^i = \Pr[x' \Rightarrow x'' | \text{event of Type } i]$ . How these rates and transition probabilities are assigned will be explained in Section 3 in the context of a specific application example.

Markov processes with reward allow one to further account for the benefits and costs accrued during the lifetime of the system. Such earnings and losses are discounted at a specified rate  $\gamma$  meaning that 1 dollar earned at a future time  $t$  is worth  $e^{-\gamma(t-t_0)}$  dollars at time  $t_0$ . While operating in state  $x$ , the system earns at a rate  $\epsilon_x$  (dollars/year). Whenever an event of Type 1 (an earthquake) occurs and causes a transition from state  $x'$  to state  $x''$ , a lump-sum cost  $C_{x'x''}^1$ , expressed as negative earned dollars, is incurred. This lump-sum cost includes structural and non-structural repairs. If the transition  $x' \Rightarrow x''$  can occur under different (damage, repair) scenarios, the total transition probability  $P_{x'x''}^1$  is the sum of the probabilities of all such scenarios and  $C_{x'x''}^1$  is the expected cost over the same scenarios. Similarly, when events of Type 2 or 3 occur, a state transition may result due to changes in the calculated risk or in the regulatory limit on risk. If the changes violate the condition  $SF(t) \geq 1$ , the structure must be strengthened according to a specified retrofit policy, at lump-sum costs  $C_{x'x''}^2$  and  $C_{x'x''}^3$  (the dollar amounts differ from  $C_{x'x''}^1$  because events of Types 2 and 3 cause no physical damage). We assume that strengthening is done instantaneously.

What matters for the present worth of the system is the expected earning rate  $e_x$  when the system is in state  $x$ , which is found from the rate  $\epsilon_x$  and the lump-sum negative earnings  $C_{x'x''}^i$  as  $e_x = \epsilon_x + \sum_{i=1}^3 \lambda_i \sum_{x''} P_{x'x''}^i C_{x'x''}^i$ . The theory of Markov processes with reward [8] says that  $Q_x$ , the expected actualized reward earned in the infinite future by a system that is initially in state  $x$ , satisfies the following set of linear algebraic equations:

$$(\gamma + \lambda)Q_x = e_x + \sum_{i=1}^3 \lambda_i \sum_{x''} P_{xx''}^i Q_{x''} \quad (2.3)$$

where  $\gamma$  is the discount rate and  $\lambda = \lambda_1 + \lambda_2 + \lambda_3$  is the total rate of events that can possibly induce state changes. We use these asymptotic actualized rewards, incremented by the (negative) cost of construction, to rank alternative designs and repair/retrofit strategies.

### 3. APPLICATION EXAMPLE

To illustrate, we consider eight 9-story steel designs for Los Angeles, California. These are a subset of the 12 designs of Wen and Kang [10]. We denote the selected designs as  $S_1, \dots, S_8$ , in order of increasing strength. We also consider nine designs  $S_1, \dots, S_9$  of a 2-story reinforced-concrete building. Table 3.1 lists some basic characteristics of both the “high rise” and “low-rise” designs. In both cases, design,  $S_1$ , just satisfies the regulatory requirements at the time of construction.

Table 3.1 Characteristics of high-rise and low-rise designs.

Design Level <i>S</i>	Period (sec)		Mass (tons)		Total construction cost <i>TC</i> (\$1000)	
	High Rise	Low Rise	High Rise	Low Rise	High Rise	Low Rise
1	2.32	0.4	5183	219.6	11,056	246
2	2.06	0.38	5223.8	225.7	11,145	253
3	1.88	0.35	5267.4	231.8	11,238	259
4	1.77	0.33	5311.8	237.9	11,333	266
5	1.66	0.3	5356.1	244	11,426	273
6	1.57	0.28	5398.7	250.1	11,536	280
7	1.50	0.25	5440.3	256.2	11,643	287
8	1.20	0.23	5730.4	262.3	12,300	293
9	-	0.2	-	268.4	-	300

As in [10], the structural and non-structural damage  $d$  caused by an earthquake is described using a seven-point scale:  $d = 1$  (no damage), 2 (slight damage), 3 (light), 4 (moderate damage), 5 (heavy damage), 6 (major damage), and 7 (collapse). Each damage level corresponds to a range of the maximum interstory drift ratio  $\Delta$ ; see Table 1 of [10, Part II]. Here we use the same drift-ratio intervals, except for the collapse state, for which we assume that the minimum drift ratio increases linearly with  $S$ , from 5% for  $S_1$  to 8% for  $S_8$  (high-rises) or  $S_9$  (low-rises). The drift ratio of  $\Delta = 0.7\%$  (damage level  $d = 4$ ) is also the threshold for structural damage. Lower drift ratios (damage levels  $d \leq 3$ ) involve only non-structural losses. For the drift ratios used and the central damage factors associated with different damage levels, see Table 4.2 of Agarwal [11].

The rate at which each design suffers damage at level  $d$  is evaluated using Eqn. 2.1, where  $y$  is spectral acceleration at the elastic period of the structure. The hazard functions  $H(y)$  for high-rise and low-rise buildings are estimated based on the 2002 USGS national seismic hazard maps and curves. The fragility function  $F(y)$  for each building and for each damage level is derived using time history analysis (THA) of equivalent single-degree-of-freedom (SDOF) models; for details see Chapter 4 of [11]. The functions  $H(y)$  and  $F(y)$  are used in Eqn. 2.1 to calculate the rate at which each design suffers different damage levels  $d$ . Table 3.2 presents the damage rates for high-rise designs. For the damage rates of the low-rise designs see Table 4.4 of [11].

Table 3.2 Damage rates (events/year) for high-rise designs in Los Angeles, California.

Design Level <i>S</i>	Damage Level $d$							$F_S$ Ratio
	1	2	3	4	5	6	7	
1	0.0172	0.0597	1.00E-02	1.03E-02	2.16E-03	5.64E-04	9.82E-06	1
2	0.0281	0.0522	8.52E-03	9.08E-03	1.69E-03	3.87E-04	5.52E-06	$10^{1/4}$
3	0.0362	0.0470	7.29E-03	7.95E-03	1.26E-03	2.82E-04	3.11E-06	$10^{2/4}$
4	0.0408	0.0438	6.87E-03	7.29E-03	1.04E-03	2.26E-04	1.75E-06	$10^{3/4}$
5	0.0441	0.0412	6.69E-03	6.90E-03	9.18E-04	1.94E-04	9.82E-07	10
6	0.0504	0.0369	6.18E-03	5.75E-03	7.02E-04	1.41E-04	5.52E-07	$10^{5/4}$
7	0.0539	0.0346	5.85E-03	4.95E-03	5.76E-04	1.14E-04	3.11E-07	$10^{6/4}$
8	0.0564	0.0329	5.55E-03	4.51E-03	5.30E-04	1.10E-04	9.82E-08	100

The integer levels of the state variables  $S$ ,  $E$  and  $R$  for high-rise and low-rise designs correspond to different frequency ratios  $F_S$ ,  $F_E$  and  $F_R$  in Eqn. 2.2. After calculating the exact frequency ratio  $F_S$  for different designs  $S$ , we approximate these ratios as simple powers of 10. We then discretize  $F_E$  and  $F_R$  for different  $E$  and  $R$  using similar powers of 10. Specifically, we set  $F_E = 10^{(E-(n_E+1)/2)/4}$  and  $F_R = 10^{(R-n_R)/4}$  where  $E = 1, \dots, n_E$  and  $R = 1, \dots, n_R$ . The upper limit of  $S$  is  $n_S = 8$  for the high-rises and  $n_S = 9$  for the low-rises. In choosing  $n_E$  and  $n_R$ , we have imposed that for any value of the state variables  $E$  and  $R$  there is at least one admissible design  $S$ . These considerations have led us to set  $(n_E, n_R)$  to (11,4) for the high-rises and to (17,5) for the low-rises; for further details see [11]. Next we describe how we model state transitions due to events of Type 1 (earthquakes), Type 2 (changes in epistemic uncertainty) and Type 3 (changes in the regulatory constraints).

### 3.1. Transition Rates and Transition Probabilities

For simplicity, we assume that changes in  $S$ ,  $E$  and  $R$  due to earthquakes (to events of Type 1) are independent. Hence, using a super-script to indicate the causative event type, the transition probabilities  $P_{x',x''}^1$  have the form

$$P_{x^i, x^n}^1 = P_{s^i, s^n}^1 P_{e^i, e^n}^1 P_{r^i, r^n}^1 \quad (3.1)$$

This should be an acceptable approximation if one excludes small events with little chance of producing changes in any of the state variables. Here we consider earthquakes whose site intensity is exceeded with rate  $\lambda_1 = 1/(5 \text{ yr})$  and set the probabilities in Eqn. 3.1 as follows. We assume that earthquakes initiate changes in the epistemic uncertainty  $E$  and the regulatory constraints  $R$  on average once every 20 and 40 years, respectively. The magnitude of the change is random, but for  $R$  we assume that the regulatory constraints can only become more stringent (this reflects the general trend towards tightening of the codes, although in some cases the safety standards have also been relaxed). The assumed transition probabilities  $P_{e^i, e^n}^1$  and  $P_{r^i, r^n}^1$  for high-rises and low-rises are listed in Tables 4.5 and 4.6 of [11].

Earthquake-induced changes in the seismic strength of the building structure  $S$  occur when the building suffers structural damages ( $d \geq 4$ ) and is repaired to the strength of one of the initial designs, or when  $E$  and  $R$  change and cause the building to violate the regulatory constraints, requiring retrofitting. If the system experiences only nonstructural damage (drift ratio less than 0.7% and  $d = 1, 2, 3$ ), the building remains in the pre-earthquake  $S = s$  state. This occurs with probability  $P_{ss}^1 = \sum_{d=1}^3 P(d|s)$ . If the drift ratio exceeds 0.7% (an event that happens with probability  $1 - P_{ss}^1$ ), the building is repaired to some structural state  $s''$  that depends on the repair and retrofit strategies.

The epistemic uncertainty state  $E$  may change also due to events of Type 2 (studies of seismic hazard not triggered by earthquakes). We assume that also these events occur on average once every 20 years; hence  $\lambda_2 = 1/(20\text{yr})$ . Table 4.7 of [11] gives the transition probabilities  $P_{e^i, e^n}^2$  that are assumed for high-rise and low-rise buildings. Similarly, the minimum safety standards may be revised due to events of Type 3 (reassessments not triggered by earthquakes). These events are assumed to occur with mean rate  $\lambda_3 = 1/(40\text{yr})$  and have transition probabilities  $P_{r^i, r^n}^3$  given in Table 4.8 of [11].

### 3.2. Costs and Earnings

There are three cost components to be specified: the cost of construction, the cost of structural and non-structural repairs, and the cost of retrofitting. In addition one must specify the earning rate of the system and the discount rate.

The total cost of construction for high-rise and low-rise designs are listed in Table 3.1. The maximum difference among alternative designs is about 11% for the high-rises and 20% for the low-rises. Table 3.3 presents the cost of repairing damage  $d$  for the high-rises,  $C_{d|S}$ . This cost, is obtained as the sum of several terms, which include structural and non-structural damage and repair, content losses, rental loss, as well as losses from injuries and fatalities. Damage costs for the low-rise designs are obtained by reducing the high-rise costs in proportion to the floor area, see Table 4.9 of [11]. We recognize that there is a fixed cost of upgrading to make the structural system accessible (this is set to 15% of the total cost of construction of the pre-earthquake building) and a cost that depends on the amount of upgrading. The latter is set to twice the cost of providing the same increased protection in the initial design. At any time, the building is assumed to earn at a fixed rate  $\epsilon = 0.06TC_1/\text{yr}$  irrespective of the state  $X$ . Results for cases when the earning rate depends on  $X$  are given in [11]. The discount rate for future costs and earnings is set to 3% per year.

### 3.3. Numerical Results

First we discuss base-case results and then make sensitivity analyses with respect to future changes in epistemic uncertainty and the regulatory limits on safety, and different retrofit strategies.

#### 3.3.1 Base-Case

One can use Eqn. 2.3 to obtain the actualized net rewards  $Q_x$  for the base-case parameters and repair/retrofit strategies. In the base case, we consider repairing to the larger of the pre-earthquake state and the lowest admissible strength consistent with the post-earthquake values of  $E$  and  $R$ . Alternative designs  $s$  are compared using the return per dollar invested,  $RPDI_s = (Q_{s, e_o, r_o} - TC_s) / TC_s$ , where  $TC_s$  is the total cost of construction and  $e_o$  and  $r_o$  are the values of  $E$  and  $R$  at the

Table 3.3 Damage cost  $C_{dIS}$  (\$1000) for different high-rise designs and different damage levels.

Design Level $S$	Damage Level $d$						
	1	2	3	4	5	6	7
1	0	259	1,428	5,548	14,212	30,089	132,635
2	0	260	1,432	5,566	14,252	30,161	132,724
3	0	260	1,437	5,585	14,294	30,235	132,817
4	0	261	1,442	5,604	14,337	30,311	132,912
5	0	261	1,446	5,622	14,379	30,385	133,005
6	0	262	1,452	5,644	14,428	30,473	133,115
7	0	262	1,457	5,666	14,476	30,559	133,222
8	0	265	1,490	5,797	14,772	31,085	133,879

time of construction. The solid lines in Figure 1 show  $RPDI$  for high-rise and low-rise buildings. Levels 7 and 6 are the best high-rise and low-rise designs, respectively, with a return on investment of about 70%.

Considering that design  $S_1$  already meets current safety standards, these optimal designs may appear unduly conservative. Conservatism is dictated by the high likelihood of future adverse changes in the assessed hazard ( $E$ ) and the regulatory limits on risk ( $R$ ) assumed in the analysis. The risk of violating the regulations in the near future (and to a lesser extent the risk of significant damages) decrease for stronger designs and this decrease more than offsets the additional cost of construction.

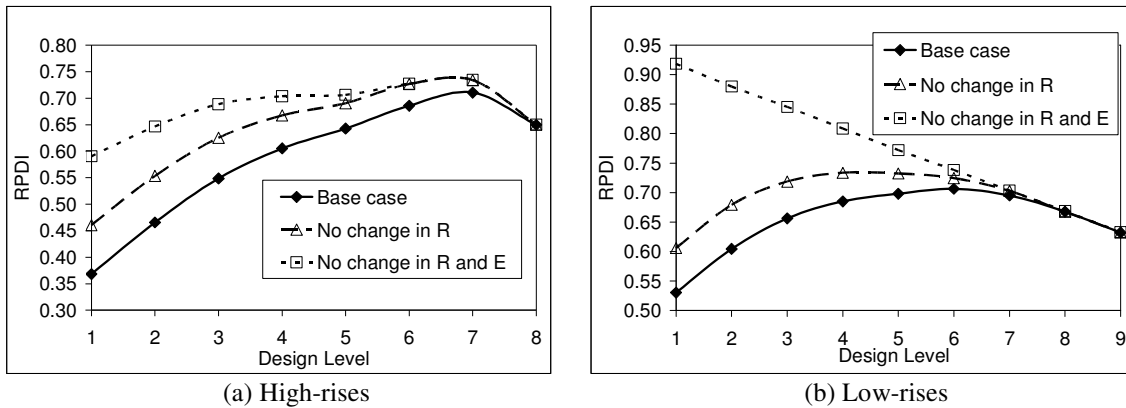


Figure 1 – Return per dollar invested ( $RPDI$ ) for different initial designs: base case and sensitivity to the future volatility of the epistemic uncertainty and the regulatory limit on risk.

### 3.3.2 Sensitivity to the Future Volatility of the Epistemic Uncertainty and the Regulatory Limits

If one excludes changes in the regulatory environment and thus keeps  $R$  constant over time, the expected returns on investment become as shown by the dashed lines in Figure 1. The  $RPDI$  values increase for all designs, except the strongest ones, which also under base-case assumptions never require retrofitting. The gains are especially large for the weaker structures, causing some shift in the optimal designs. For the high-rises one is essentially indifferent between designs 6 and 7, whereas for the low-rises the optimum design becomes 4, with a region of insensitivity between 3 and 7.

If also the epistemic uncertainty (state variable  $E$ ) is kept fixed over time – a common assumption in seismic design decisions [10, 12-14] – then losses come exclusively from damage repair. In this case one obtains the dotted line in Figure 1. For the high-rises the region of optimality extends now from design 3 to 7, whereas for the low-rises minimum coverage of the regulatory requirements (design 1) becomes optimal, with an expected return on investment above 90%.

The reason why design 1 is not optimal for the high-rises is somewhat complex: high-rises have a lower cost increment from one design level to the next (except for level 8) and this should favor the use of stronger structures. However, high-rises have long natural periods, which makes damage less sensitive to the strength of the structure and favors weaker designs. A third (and

dominant) factor is that high-rise structures are more susceptible to damage than low-rise buildings and for them additional seismic protection is more advantageous. These issues are discussed in further detail in [11].

In their approach to optimal seismic design, Wen and Kang [10] neglect regulatory constraints and rank the designs according to the expected actualized cost  $C$ . In their formulation, future changes in the assessed risk are inconsequential. To compare with the present methodology, we calculate  $RPDI$  in the present method for the case when  $E$  and  $R$  do not change in time and obtain the expected actualized cost in Wen and Kang’s approach for the initial costs, damage rates, damage costs and discount rate used to compute  $RPDI$ . Figure 2 shows the  $RPDI$  values divided by the  $RPDI$  of the optimum design ( $RPDI_{max}$ ) and Wen and Kang’s expected cost of the optimum ( $C_{min}$ ) divided by the expected cost  $C$  (in the figure, we call these normalized quantities “benefit indices”). As the Wen and Kang method does not account for future earnings, the comparison is only qualitative, but results are in generally good agreement. In particular, the optimum high-rise and low-rise designs are the same.

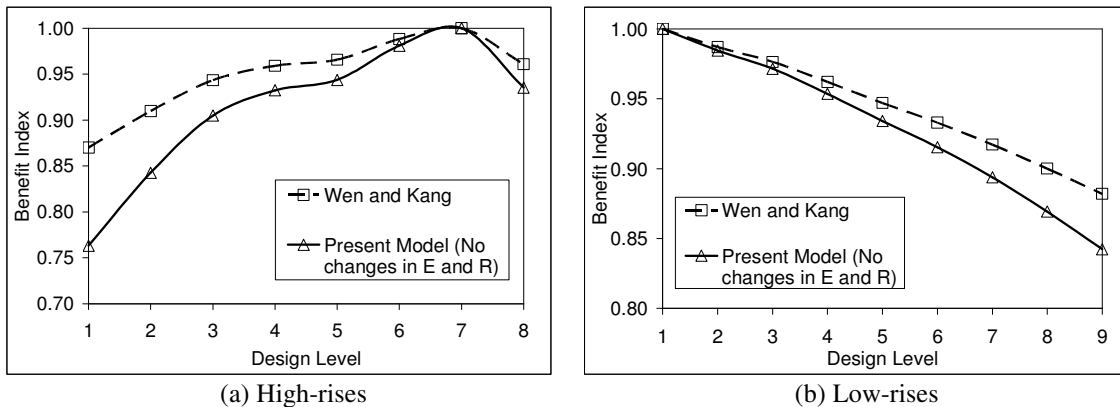


Figure 2 – Comparison of the relative economic value of different designs using the present model with no change in  $E$  and  $R$  and the model of Wen and Kang [10].

### 3.3.3 Changing the Retrofitting Strategy

Figure 3 shows the return on investment of high-rise and low-rise designs for different retrofitting strategies. In the base case, retrofitting strengthens the structure to the minimum level required by the regulations that apply at the time. The alternative strategies retrofit to a level no less than  $S_{min}$ , where  $S_{min} = 5, 6, 7,$  or  $8$ . Providing this extra level of protection may be preferable to retrofitting by minimum allowed amounts, due to the high fixed costs of retrofitting and the possibility of needing additional retrofit interventions in the future. The rationale is similar to that for choosing an initial design that is conservative relative to the minimum seismic protection required by regulations.

Figure 3 shows that conservative retrofitting strategies are generally superior to minimal retrofitting. In particular, using  $S_{min} = 7$  typically outperforms other choices of  $S_{min}$ . For the high-rises, the global optimum is still attained for initial design 7 (for this design,  $S_{min}$  is ineffective). However, for the low-rises the optimal initial design is lowered to 4. This means that for the low-rises it is best to choose a relatively weak initial design ( $S_4$ ) and then upgrade to  $S_7$  if and when needed, rather than making a larger initial investment and directly designing for level 7.

Agarwal [11] presents additional sensitivity results with respect to the costs of construction and retrofitting, the level of seismicity in a region and the repair strategies.

## 4. CONCLUSIONS

The introduction of regulatory limits on the acceptable risks causes the utility of a decision to be a non-linear function of the risk. In this case it is necessary to make decisions accounting for the possible future evolution of the epistemic uncertainties. The net effect is that optimal designs tend to be more conservative than those based on the assumption that uncertainties remain the same during the lifetime of the project. Another consequence is that sequential (wait-and-see) decisions become superior to one-time (here-and-now) decisions.

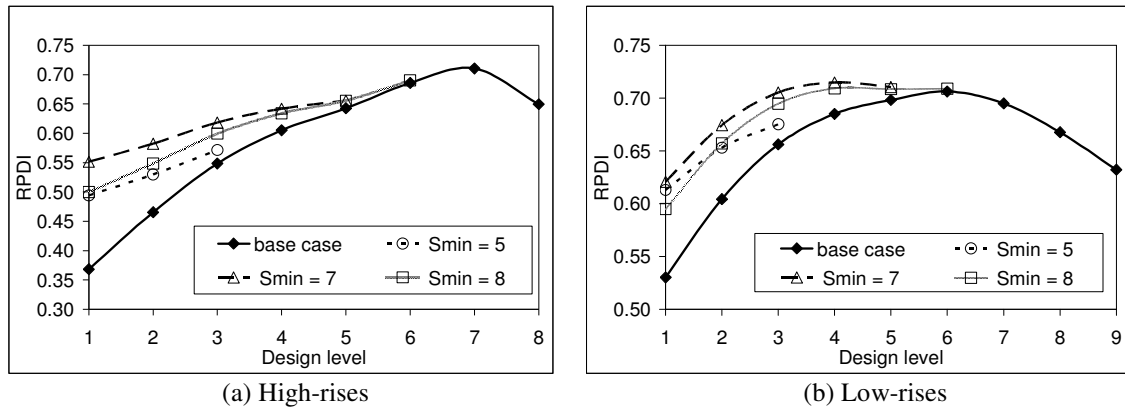


Figure 3 –Dependence of  $RPDI$  on the retrofitting strategy.

We have proposed a framework for decision-making that explicitly accounts for the random temporal evolution of the epistemic uncertainties and minimum safety standards and illustrated the effects of these factors on the optimal design of low and high-rise buildings against earthquake loads. Accounting for future changes in epistemic uncertainty and regulatory constraints makes the optimal initial design more conservative than when these future changes are ignored. Some of the parameters used in the numerical examples were judgmentally assigned. A more detailed and objective derivation using data or models would make the conclusions more useful in practice.

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