# RISK BASED MINIMUM LIFE-CYCLE COST DESIGN OF ASEISMIC STRUCTURES

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**ABSTRACT**

The current reliability-based probabilistic limit state design is based on notional failure probability of structural components, which cannot explicitly consider the consequences of earthquake events in terms of seismic loss, life-cycle cost, or even fatalities rate. Therefore, it is necessary to move from *reliability-based design* toward *risk-based design* by using the quantitative risk analysis tools. This paper reviews some existing models of risk-benefit-cost criterion for seismic design of structures, and then put forward a new model of life-cycle cost, which includes the minimum initial cost and the expected seismic loss under the future earthquakes during the design reference period. A two-stage minimum life-cycle cost design procedure is presented, in which the decision of the optimal fortification intensity (OFI) is made during the first stage, while the minimum-cost design under the optimal fortification intensity is undertaken in the second stage. The functional relationship between the minimum initial cost and the fortification intensity is derived through a series of minimum-cost seismic design subjected to the codified provisions by successively adjusting the fortification intensity. The expected seismic loss is the sum of the products of damage state probabilities with the corresponding economic losses. The probabilistic seismic risk analysis is deconstructed into four constituents: probabilistic seismic hazard analysis (PSHA), probabilistic seismic fragility analysis (PSFA), probabilistic seismic safety analysis (PSSA) and probabilistic seismic damage analysis (PSDA). The proposed methodology is applied in seismic design optimization of steel frame buildings subjected to Chinese seismic design codes. A numerical example demonstrates the feasibility and prospect of this new paradigm.

**KEYWORDS:** Risk-Based Design, Life-Cycle Cost, Seismic Risk, Seismic Fragility, Seismic Loss, Optimal Fortification Intensity

## 1. INTODUCTION

The current seismic design practice based on level II probability-based limit states design has been to include the earthquake effect in load combinations, as with other loads (Ellingwood 2000, 2001a,b). However, the management of risk due to earthquakes by means of this load and resistance factor design (LRFD) formulation has been highly problematic. One of the major arguments is that the traditional codified LRFD methodology cannot explicitly consider the consequences of earthquake events in terms of seismic loss, life-cycle cost, or even fatalities rate, since it is based on notional failure probability of structural components by calibrating level I structural design codes and standards using first order reliability method (FORM). As we all know, in the case of low-probability and high-consequence natural disasters such as earthquakes, risk, rather than reliability, is most meaningful and useful when expressed in terms of potential economic losses and/or human sufferings. Therefore, it is necessary to move from the traditional *reliability-based design* procedure toward the new *risk-based design* paradigm by using the quantitative risk analysis tools (Ellingwood & Wen, 2005; Ang, 2007).
In this paper, some existing models of risk-benefit-cost criterion for seismic design of structures are reviewed, and then a new model of life-cycle cost is put forward, which includes the minimum initial cost and the expected seismic loss under the future earthquakes during the design reference period. A two-stage optimization methodology for seismic design of structures considering minimum life-cycle cost is proposed to cope with the new model. The probabilistic seismic risk analysis (PSRA) is deconstructed into four constituents: probabilistic

seismic hazard analysis (PSHA), probabilistic seismic fragility analysis (PSFA), probabilistic seismic safety analysis (PSSA) and probabilistic seismic damage analysis (PSDA). The proposed methodology is applied in seismic design optimization of steel frame buildings subjected to Chinese seismic design codes (GB50011-2001).

## 2. RISK-BENEFIT-COST CRITERIA FOR SEISMIC DESIGN OF STRUCTURES

### *2.1. Existing Models of Risk-Benefit-Cost Criteria for Seismic Design of Structures*

Significant progress has been made in the preceding decades in the area of minimum life-cycle cost design optimization for aseismic structures. The life-cycle cost model in the existing literature is the total summation of initial material/construction cost and the lifetime seismic damage cost. Liu & Neghabat (1972) are among the first researchers who introduced lifetime seismic damage cost into the initial design stage

$$E[C_T(\mathbf{d})] = C_I(\mathbf{d}) + E[C_D(\mathbf{d})] \tag{2.1}$$

where E[ ] = expectation operator; $\mathbf{d}$ = the design variable vector of the structure; $E[C_T(\mathbf{d})]$ = the expected total life-cycle cost; $C_I(\mathbf{d})$ = initial cost; and $E[C_D(\mathbf{d})]$ = the expected loss from seismic damage.

The optimization models adopted in most of the subsequent research are the extensions of Eq. (2.1). For example, the International Standard "General Principles on Reliability for Structures (ISO 2394; 1998)" proposed a minimum lifetime cost objective function

$$C_{tot} = C_b + C_m + \sum P_f C_f \tag{2.2}$$

where $C_b$ = the building cost, $C_m$ = the cost of maintenance and demolition, $C_f$ = the cost of failure, $P_f$ = the lifetime probability of failure.

Eqs. (2.1) and (2.2) consider neither the random occurrence and the intensity variation in time of the hazards nor the discounted factor of over time $t$. Under the assumption that hazard occurrences can be modeled by a simple Poisson process with occurrence rate of $\nu$/year and that resistance is time-invariant, also considering discounting of cost over time, Wen and Kang (2001a,b) derived a closed analytical formulation of lifetime total expected cost model

$$E[C(t,\mathbf{x})] = C_0 + \frac{\nu}{\lambda}(1-e^{-\lambda t})\sum_i^k C_i P_i + \frac{(1-e^{-\lambda t})}{\lambda} C_m \tag{2.3}$$

where $C_0$ = initial cost for new building; $C_i$ = $i$th limit-state failure cost; $P_i$ = $i$th limit-state probability; $C_m$ = operation and maintenance costs per year; $e^{-\lambda t}$ = discounted factor of over time $t$; $t$ = design reference period of a new structure.

When used to decide the optimal target reliability for a new structure, Eqs. (2.1) to (2.3) can be transformed into the following formula

$$E[C_T(p_f)] = C_I(p_f) + C_m(p_f) + E[C_D(p_f)] \tag{2.4}$$

Many researchers have used Eq. (2.4) to decide the optimal reliability of structures, see Ang & De Leon (1997), Rackwitz (2000), Ang & Lee (2001), among others.

### *2.2. A New Model of Risk-Informed Decision-Making for Aseismic Structures*

There are some problems that must be carefully considered when Eqs. (2.1) to (2.4) are applied in practical engineering design. First, the closed analytical function between the initial cost $C_I$ and the design variable vector $\mathbf{d}$ or the target failure probability $p_f$ is difficult to obtain. Second, the closed analytical function between the expected damage cost $E[C_D]$ and $\mathbf{d}$ or $p_f$ is also difficult to reach. Third, the initial cost may be different with different design decision-makers. Fourth, the total lifetime expected cost should consider multiple limit states or performance levels of a structure. Although Eqs. (2.2) and (2.3) do consider this situation, they cannot be used to decide all acceptable risk levels for different limit states. In contrast, Eqs. (2.1) and (2.4) can be applied in obtaining the optimal target risk for one limit state. However, they cannot take into account the damage costs for different limit states.

To overcome the above difficulties, we take the *fortification intensity* $I_d$, instead of the target reliability, as the acceptable risk level of a structure. In other words, we choose the fortification intensity $I_d$ as one key decision-making variable when we make the minimum total expected life cycle cost optimization for seismic design of structures. From our experience, the fortification intensity is more convenient in use than the target reliability.

During the variable design stage of a structure, the design scheme, that is, the design variable vector **d**, can be denoted as the function $\mathbf{d}(I_d)$ of the fortification intensity $I_d$. Furthermore, the cost of maintenance and demolition can also be considered a kind of failure cost in some sense. Therefore, Eq. (2.3) can be transformed into the following form

$$E[C(t,I_d)] = C_I[\mathbf{d}(I_d)] + \frac{\nu}{\lambda}(1-e^{-\lambda t})L[\mathbf{d}(I_d)] \tag{2.5}$$

where $L[\mathbf{d}(I_d)]$ = expected total damage cost considering all seismic performance levels.

With respect to the same fortification intensity $I_d$, many kinds of feasible design schemes $\mathbf{d}_i(I_d)$ $(i = 1,2,\cdots)$ can be obtained, and accordingly, there are many cost functions. Therefore, structural initial cost $C_I[\mathbf{d}(I_d)]$ is a multi-value function of the intensity $I_d$, so it should not directly be used in Eq. (2.5) in a strict sense. On the other hand, given specific fortification intensity $I_d$, there should exit only one optimal design scheme in theory. Hence, from the viewpoint of more rational logical background, the function $C_I[\mathbf{d}(I_d)]$ should be replaced by its minimum counterpart $C_{\min}[\mathbf{d}(I_d)]$, which is a single-value function.

Based on the above analysis, we herein propose a more scientific optimization objective function for seismic design of structures considering expected life cycle cost as follows

$$E[C(t,I_d)] = w_1 C_{\min}[\mathbf{d}(I_d)] + w_2\frac{\nu}{\lambda}(1-e^{-\lambda t})L[\mathbf{d}(I_d)] \tag{2.6}$$

where, $C_{\min}[\mathbf{d}(I_d)]$ is the initial minimum cost under the fortification intensity $I_d$; $w_1$ and $w_2$ are weights that consider the importance of the initial minimum cost $C_{\min}$ and the loss expectation $L$.

Because the minimum-cost design scheme $\mathbf{d}(I_d)$ of a structure is unique with respect to the given fortification intensity $I_d$, the objective function (2.6) can be expressed in the following simple formulation

$$E[C(t,I_d)] = w_1 C_{\min}(I_d) + w_2\frac{\nu}{\lambda}(1-e^{-\lambda t})L(I_d) \tag{2.7}$$

Eq. (2.6) or (2.7) is taken herein as the risk-informed decision-making model of minimum life-cycle cost design for aseismic structures.

## 3. MINIMUM LIFE-CYCLE COST DESIGN METHODOLOGY OF ASEISMIC STRUCTURES

### *3.1. Two-Stage Optimization Methodology for Minimum Life-Cycle Cost Design of Aseismic Structures*

The minimum expected life cycle cost optimization for seismic design of structures is divided into the following two design stages:

**Stage 1**: Decision-making for the optimal fortification intensity of aseismic structures considering expected life cycle cost. In this stage, the optimal fortification intensity is determined according to the following optimization model

$$E[C(T,I_d)] = C_{\min}(I_d) + \frac{\nu}{\lambda}(1-e^{-\lambda T})L(I_d) \to \min \tag{3.1}$$

where the minimum-cost function $C_{\min}(I_d)$ can be obtained from the regression analysis of a series of minimum initial cost seismic design by adjusting the fortification intensity $I_d$, which is an increasing function of $I_d$; the total loss expectation function $L(I_d)$ can be obtained from the regression analysis of a series of seismic damage probabilitity and loss assessment processes by adjusting $I_d$, which is a decreasing function of $I_d$.

The total expected life cycle cost curve $E[C(T, I_d)]$ by composing the above two curves generally have the lowest point. The fortification intensity $I_d$ corresponding to this point is called the *optimal fortification intensity* $I_d^*$, which represents the minimum acceptable seismic risk level of a structure.

**Stage 2**: Minimum initial cost seismic design under the optimal fortification intensity. Once the optimal fortification intensity $I_d^*$ has been obtained, the minimum initial cost seismic design can be made under this $I_d^*$. The optimization model in this stage then is

To find the design scheme $\mathbf{d}(I_d^*)$, so as to make the structural cost

$$C[\mathbf{d}(I_d^*)] \to \min \tag{3.2}$$

subjected to all constraints and requirements of design codes of structures.

The final solution is the optimal design scheme in consideration of the total loss expectation $L(\mathbf{d})$. Since the loss expectation has been taken into consideration when deciding $I_d^*$ during the first design stage, it is only necessary to counteract the optimal resistance $I_d^*$ by the minimum initial cost design scheme during the second design stage.

### *3.2. Minimum Initial Cost Design of Aseismic Structures*

Under the given fortification intensity $I_d$, the minimum initial cost design problem of a structure can be conceptually stated as

To find the design scheme $\mathbf{d}(I_d)$, make the initial cost of the structure

$$C[\mathbf{d}(I_d)] \to \min \tag{3.3}$$

subjected to the codified provisions.

Since gradient information can greatly improve the optimization efficiency, the Polak-Ribiere conjugate gradient direction algorithm (Nocedal and Wright, 1999) is herein made use of, which performs the optimization loop according to the search direction

$$\mathbf{r}^{(j)} = -\nabla Q\left(\mathbf{d}^{(j)}\right) + \theta_{j-1}\mathbf{r}^{(j-1)} \tag{3.4}$$

where $\mathbf{r}^{(j)}$ = search direction vector in the $j$th iteration; $\mathbf{d}^{(j)}$ = design variable vector in the $j$th iteration; $Q(\ )$ = the dimensionless, unconstrained objective function via penalty function method; $\nabla Q(\ )$ = the gradient vector of the function $Q$ with respect to design variable vector; $\theta_{j-1}$ = the conjugate direction coefficient in the ($j$-1)th iteration, whose formula is

$$\theta_{j-1} = \frac{\left[\nabla Q(\mathbf{d}^{(j)}) - \nabla Q(\mathbf{d}^{(j-1)})\right]^T \nabla Q(\mathbf{d}^{(j)})}{\left\|\nabla Q(\mathbf{d}^{(j-1)})\right\|^2} \tag{3.5}$$

where $\|\bullet\|$ represents $l_2$ norm.

### *3.3. Earthquake Loss Assessment of Structures*

In general, five seismic damage states of engineering structures are specified: (1) nonstructural damage, (2) slight damage, (3) moderate damage, (4) severe damage, and (5) collapse.

Let $B_j$ represent the $j$th damage state. Then the earthquake loss of a structure in damage state $B_j$ can be evaluated as follows:

$$D_j = D_j^{(1)} + D_j^{(2)} + D_j^{(3)} \qquad (j = 1, \cdots, 5) \tag{3.6}$$

where $D_j^{(1)}$ = the direct loss from both structural and non-structural damage as well as the cost of maintenance and demolition; $D_j^{(2)}$ = the indoor loss induced by the structural damage; $D_j^{(3)}$ = the indirect loss induced by the structural damage.

To simplify the earthquake loss evaluation approach, the three kinds of economic losses for five seismic damage states can be assessed according to the *loss coefficients method* which depends on the earthquake filed investigations and experts' judgment (Wang, et al., 2003).

For direct economic loss, the cost is evaluated by

$$D_j^{(1)} = \xi(B_j)C_I(I_d) \tag{3.7}$$

where $\xi(B_j)$ = the direct loss coefficient for damage state $B_j$ ; $C_I(I_d)$ = the initial cost designed according to the fortification intensity $I_d$.

For indoor economic loss, the cost is evaluated by

$$D_j^{(2)} = \eta(B_j)C_{eq} \tag{3.8}$$

where $\eta(B_j)$ = the indoor loss coefficient for damage state $B_j$; $C_{eq}$ = the equivalent merit of the indoor asset.

For indirect economic loss, the cost is evaluated by

$$D_j^{(3)} = \gamma(B_j)D_j^{(1)} \tag{3.9}$$

where $\gamma(B_j)$ = the indirect loss coefficient for damage state $B_j$.

### *3.4. Expected Failure Cost Analysis of Aseismic Structures*

The values of loss coefficients in Eqs. (3.7) to (3.9) depend on the types and importance of the buildings. The loss $D_j$ should be evaluated according to the specific situation of a structure and the seismic damage states. The total loss expectation value with five seismic damage levels can be obtained by

$$L[\mathbf{d}(I_d)] = \sum_{j=1}^{5} P_f[B_j, \mathbf{d}(I_d)] \cdot D_j \tag{3.10}$$

## 4. PROBABILISTIC SEISMIC RISK ANALYSIS OF STRUCTURES

### *4.1. Probabilistic Seismic Hazard Analysis of Sites*

*4.1.1 PSHA for general sites in mainland of China*

The seismic hazard at a building site is displayed through a cumulative distribution function (CDF) or its complimentary one (CCDF) of earthquake ground motion parameters, e.g., seismic intensity, peak ground acceleration, spectral acceleration, etc. For general building sites in mainland of China, Gao & Bao (1986) analyzed 45 cities in the northern, northwestern and southwestern China by probabilistic seismic hazard analysis (PSHA) method, derived a conclusion that the cumulative distribution function of the seismic intensity during the design reference period in mainland of China is type III extreme value distribution, which takes the form of

$$F_I(i) = \exp\left[-\left(\frac{\omega - i}{\omega - \varepsilon}\right)^k\right] \tag{4.1}$$

where exp( ) = exponent distribution function; $\omega$ = the upper limit value of the random variable $I$, it takes 12 for seismic intensity; $\varepsilon$ = characteristic value of $I$, which equals to the basic intensity $I_0$ minus 1.55, i.e., $\varepsilon = I_0$ - 1.55; $k$ = shape parameter of the distribution function depending on the basic intensity $I_0$ of the building site, see table 4.1.

Table 4.1 Values of shape parameter $k$

| Basic intensity $I_0$ | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| $k$ | 9.7932 | 8.3339 | 6.8713 | 5.4028 |

*4.1.2 PSHA for specific sites*

For specific building sites, it has been shown by many researchers that at moderate to large values of ground acceleration, there is a logarithmic linear relation between annual maximum earthquake ground acceleration $A$ and the exceedance probability $H_A(a)$ (Ellingwood, 2001a; Cornell et al., 2002). This relationship implies that $A$ is described by a Type II distribution of largest values

$$H_A(a) = 1 - \exp[-(x/k_0)]^{-k} \approx k_0 x^{-k} \tag{4.2}$$

where $k_0$ = characteristic extreme, $k$ = shape parameter.

The second formula in Eq. (4.2) implies that the hazard curve is approximately linear on a log-log plot in the region of interest.

### *4.2. Probabilistic Seismic Fragility Analysis of Structures*

The seismic fragility of a structural system is defined as the conditional failure probability of the system for a given intensity of the ground motion. In a performance-based seismic design approach, the failure event is said to have occurred when the structure fails to satisfy the requirements of a prescribed performance level. If the intensity of the ground motion is expressed as a single variable (e.g., seismic intensity or the peak ground acceleration), the conditional failure probability expressed as a function of *intensity measure* (IM) is described by a seismic fragility curve.

*4.2.1 PSFA for general structures*

For general structures in common, there is usually lack of information on laboratory or field observations. We put forward a simplified seismic fragility analysis method using the information provided by the current seismic design code of building, in which three seismic design levels are prescribed (Wang & Lu 2001; Lu et al., 2007). The relationships of the three-level design intensity and the basic seismic intensity are summarized in Table 4.2.

Table 4.2 Three earthquake levels in Chinese seismic design code of buildings

| Earthquake levels | Minor earthquake ($I_S$) | Moderate earthquake ($I_M$) | Major earthquake ($I_L$) |
|---|---|---|---|
| Exceedance probability in 50 years | 0.632 | 0.10 | 0.02 to 0.03 |
| Relationships with the basic intensity $I_0$ | $I_s = I_0 - 1.55$ | $I_m = I_0$ | $I_L \approx I_0 + 1$ |
| Performance objectives | Do not be damaged | Can be repaired | Do not collapse |

Let $B_j^*$ represent the state that equals or is larger than the seismic damage state $B_j$, $P_f[B_j^*,\mathbf{d}(I_d)|i]$ be the conditional exceedance probability, i.e., the seismic fragility $F_{Rj}(i)$ for the above damage state $B_j^*$ ($j$ = 1,2,3,4) of scheme $\mathbf{d}(I_d)$ designed according to the fortification intensity $I_d$ when subjected to the seismic intensity $I = i$. Considering the regulations of the three-level performance objectives shown in Table 4.2, four simplified seismic fragility curves are proposed as shown in Figure 1. Figure 1(a) to (d) represent the fragility curves for four limit states corresponding to the slight, moderate, severe, collapse damage states, respectively. From Figure 1, it is easy to give the seismic fragility functions for four limit states.

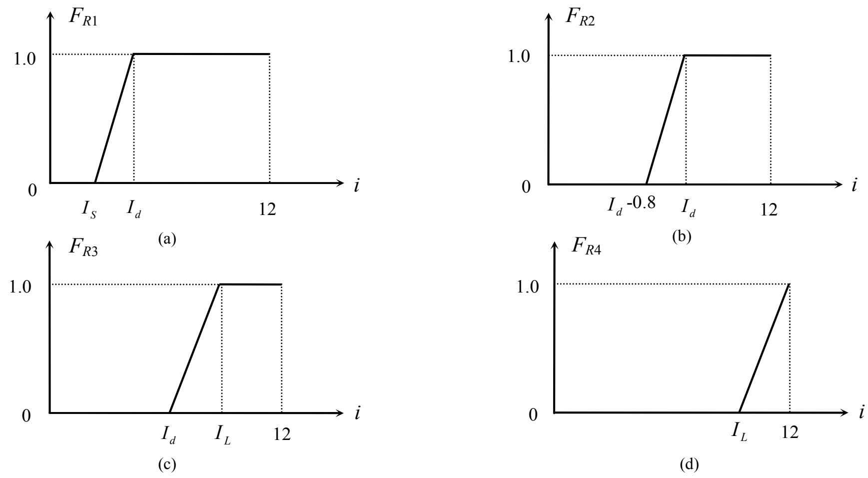


Figure 1 Fragility curves for limit states

*4.2.2 PSFA for specific structures*

For specific structures, it is necessary to use analytical fragility procedures. The conditional exceedance probability $P_f\,[B_j^*,\,\mathbf{d}(I_d)|I]$ is generally obtained using the global displacement limit states in terms of the maximum inter-storey drift ratio for building structures

$$P_f[B_j^*,\mathbf{d}(I_d)\,|\,I_k] = P[\Delta(I_d,I_k) > \Delta_j] \tag{4.3}$$

where $\Delta(I_d,\ I_k)$ is the maximum inter-storey drift ratio of structures with fortification intensity $I_d$ given occurrence of earthquake $I_k$; $\Delta_j$ is the corresponding drift ratio limit value, according to the Chinese Seismic Code of Buildings (GB50011-2001), $\Delta_j$ is listed in Table 4.3.

Table 4.3 Performance and damage levels in terms of inter-storey drift ratio

| Performance level | Damage State | Drift ratio (%) |
|---|---|---|
| I | $B_1$: None | $\Delta$<0.2 |
| II | $B_2$: Slight | 0.2<$\Delta$<0.4 |
| III | $B_3$: Moderate | 0.4<$\Delta$<0.8 |
| IV | $B_4$: Severe | 0.8<$\Delta$<2.0 |
| V | $B_5$: Collapse | $\Delta$>2.0 |

Eq. (4.3) can be computed by finite element reliability method based on FORM or Monte Carlo simulations. After obtaining the conditional exceedance probabilities $P_f\,[B_j^*,\,\mathbf{d}(I_d)|I]$, the fragility curves can then be fitted to the available data. It has been customary to model seismic fragility by a lognormal cumulative distribution function (CDF):

$$F_R(x) = \Phi[\ln(x / m_R) / \beta_R] \tag{4.4}$$

where Φ[] is the standard normal probability integral, $m_R$ is the median (50th percentile) fragility, and $\beta_R$ is the logarithmic standard deviation.

### *4.3. Probabilistic Seismic Safety Analysis of Structures*

The central content of probabilistic seismic safety analysis (PSSA) of structures is to compute the limit state probabilities corresponding to damage states or performance objectives, which are the convolution integral of hazard function and fragility ones. According to the formulations of seismic hazard and fragilities, different integral schemes should be adopted.

*4.3.1 Numerical integration method of PSSA*

For general structures and building sites, the hazard function of seismic intensity Eq. (4.1) and the simplified fragility curves shown in Figure 1 are adopted. We use the numerical integration method to compute the convolution integral. The failure probability for limit state $B_j^*$ is obtained with the summation of product of the discrete seismic hazard and fragilities:

$$P_f[B_j^*, \mathbf{d}(I_d)] = \sum_{I_k} P_f[B_j^*, \mathbf{d}(I_d) \mid I_k] \cdot P(I_k) \tag{4.5}$$

where $P(I_k)$ is the occurrence probability of the earthquake when earthquake intensity $I$ takes discrete value $I_k$. $I_k$ usually takes from 6 degree to 9 degree with 0.5 spacing in Chinese seismic design practice (GB50011-2001). To compute $P(I_k)$, the following formula are taken:

$$\begin{aligned} P(I_k = 6.0) &= F_I(6.25), \\ P(I_k = 6.5) &= F_I(6.75) - F_I(6.25), \\ P(I_k = 7.0) &= F_I(7.25) - F_I(6.75), \\ P(I_k = 7.5) &= F_I(7.75) - F_I(7.25), \\ P(I_k = 8.0) &= F_I(8.25) - F_I(7.75), \\ P(I_k = 8.5) &= F_I(8.75) - F_I(8.25), \\ P(I_k = 9.0) &= 1 - F_I(8.75) \end{aligned} \tag{4.6}$$

*4.3.2 Analytical Approximate method of PSSA*

For specific structures and building sites, the hazard function Eq. (4.2) of earthquake ground acceleration $A$ and the analytical fragility function Eq. (4.4) are adopted. Then the limit state probability can be obtained by a closed analytical formulation (Cornell et al., 2002):

$$P_f[B_j^*, \mathbf{d}(I_d)] = \int_0^\infty F_{Rj}(x) d \mid H_A(x) \mid = H_A(m_R) \exp[(k\beta_R)^2 / 2] \tag{4.7}$$

The above equation says that the limit state probability is equal to the seismic hazard, evaluated at the median fragility, multiplied by a correction factor that considers the inherent randomness in structural capacity.

### *4.4. Probabilistic Seismic Damage Analysis of Structures*

The damage state probabilities are the main contents of probabilistic seismic damage analysis (PSDA) of structures. They are final results of probabilistic seismic risk analysis (PSRA) as well. After obtaining the limit state probabilities, the damage state probabilities for seismic risk can be evaluated as:

$$P_f[B_1, \mathbf{d}(I_d)] = 1 - P_f[B_1^*, \mathbf{d}(I_d)] \tag{4.8a}$$

$$P_f[B_j, \mathbf{d}(I_d)] = P_f[B_{j-1}^*, \mathbf{d}(I_d)] - P_f[B_j^*, \mathbf{d}(I_d)] \; (j = 2,3,4) \tag{4.8b}$$

$$P_f[B_5, \mathbf{d}(I_d)] = P_f[B_4^*, \mathbf{d}(I_d)] \tag{4.8c}$$

## 5. APPLICATIONS OF THE METHODOLOGY IN STEEL FRAME BUILDINGS

We have applied the developed methodology in the seismic design optimization of steel frame buildings

considering the minimum life-cycle cost subjected to the provisions of Chinese design codes (Lu et al., 2007). A three-bay and four-story plane steel frame structure, as shown in Figure 2, is demonstrated in this paper. All beams of the frame are made of Q235B steel, while all columns are made of Q345B steel. The soil type of the building site is type III, and the basic seismic intensity is 7 degree. The equivalent static horizontal seismic forces are calculated using base shear method according to the Chinese seismic design code (GB50011-2001).
At the stage of initial optimum design, the finite element model for this structure is built in ANSYS. All beams and columns are modeled using Beam3 element during the elastic design stage under minor earthquake, while they are modeled using Beam24 element during the elastoplastic design stage under major earthquake. The minimum initial cost seismic design is performed using ANSYS design optimization tool. The global optimization strategy is used to treat with both elastic and elastoplastic inter-storey drift angle constraints simultaneously. The first-order optimization method is adopted, in which the gradients are calculated by the forward finite difference method.

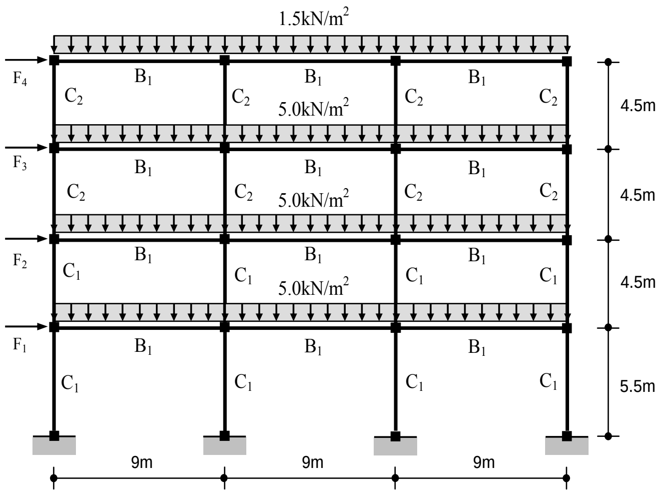


Figure 2 Three-bay and four-storey plane steel frame

From Table 4.1, the shape parameter $\zeta$ = 8.3339. The design reference period $T$ = 50 years. The mean occurrence rate of earthquake $\nu$ = 0.03 /year. Assume that $\lambda$ = 5%, $C_{eq}$ = 1.5$C_I$. The loss coefficients for five damage states are listed in Table2.

Table 5.1 Loss coefficients

| Damage State | $\xi$ | $\eta$ | $\gamma$ |
|---|---|---|---|
| $B_1$: None | 0.00 | 0.00 | 0.00 |
| $B_2$: Slight | 0.10 | 0.05 | 0.00 |
| $B_3$: Moderate | 0.30 | 0.15 | 0.50 |
| $B_4$: Severe | 0.90 | 0.50 | 2.00 |
| $B_5$: Collapse | 1.00 | 0.95 | 6.00 |

The conditional limit state probabilities are calculated by FORM-based finite element reliability method with nonlinear static procedure based on nonlinear fiber-section beam-column elements in OpenSees platform (Haukaas & Der Kiureghian, 2007). The computed seismic risk probabilities for five damage states are listed in Table 5.2.
The original initial cost, the minimum initial cost, the expected damage cost as well as the total expected life cycle cost are summarized in Table 5.3 and Figure 3. From Figure 3, it can be readily seen that the optimal fortification intensity corresponding to the minimum total expected life cycle cost point is $I_d^*$ =6.5. The optimum solution corresponding to this optimal earthquake intensity is the final seismic design scheme considering the expected life cycle cost. The result of this paper is consistent to that by using simplified fragility method (Lu et al., 2007).

Table 5.2 Seismic risk probabilities for five damage states

| Intensity $I_d$ | $P_f[B_1]$ | $P_f[B_2]$ | $P_f[B_3]$ | $P_f[B_4]$ | $P_f[B_5]$ |
|---|---|---|---|---|---|
| 6.0 | 0.1754 | 0.3776 | 0.0001 | 0.4443 | 0.0026 |
| 6.5 | 0.2381 | 0.3203 | 0.2798 | 0.1615 | 0.0003 |
| 7.0 | 0.1250 | 0.6624 | 0.1758 | 0.0364 | 0.0004 |
| 7.5 | 0.6000 | 0.3014 | 0.0710 | 0.0260 | 0.0016 |
| 8.0 | 0.7229 | 0.2417 | 0.0249 | 0.0105 | 0.0000 |
| 8.5 | 0.7254 | 0.2417 | 0.0287 | 0.0042 | 0.0000 |
| 9.0 | 0.7287 | 0.2680 | 0.0033 | 0.0000 | 0.0000 |

Table 5.3 Total expected life cycle cost ($\times 10^5$RMB)

| Intensity $I_d$ | $C_0$ | $C_{\min}$ | $L$ | $E[C_T]$ |
|---|---|---|---|---|
| 6.0 | 27.812 | 21.032 | 34.0905 | 39.8073 |
| 6.5 | 35.213 | 23.875 | 19.2103 | 34.4551 |
| 7.0 | 45.897 | 30.846 | 11.2136 | 37.0219 |
| 7.5 | 73.911 | 36.336 | 7.4071 | 40.4155 |
| 8.0 | 87.296 | 40.374 | 3.8489 | 42.4938 |
| 8.5 | 97.553 | 55.867 | 4.2548 | 58.2103 |
| 9.0 | 115.27 | 61.254 | 3.0009 | 62.9113 |

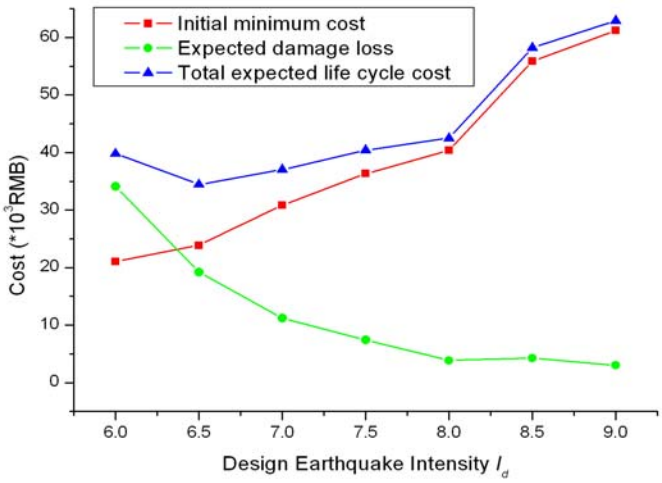


Figure 3 Decision-making of optimal fortification intensity

## 6. CONCLUSIONS

This paper presents a risk-based minimum life-cycle cost design methodology for seismic design of structures. The conclusions can be summarized as follows:

(1) Rational model of total expected life cycle cost should include the minimum initial cost plus the total expected damage cost.

(2) The optimal fortification intensity represents the acceptable risk level more flexible and convenient than the target reliability.

(3) The division of total optimum design process based on life cycle cost into the stage of the decision-making of the optimal fortification load and the stage of minimum initial cost design can greatly overcome some difficulties in the conventional design methods.

## 7. ACKNOWLEDGEMENTS

The support of National Science Foundation of China through projects (Grant No. 90715021, 50678057, 50108005) is greatly appreciated. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the National Science Foundation of China.

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