SENSITIVITY ANALYSIS OF SDOF STRUCTURE PARAMETERS ON DAMAGE RATIO COEFFICIENT

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ABSTRACT

The structure response parameters (natural period, elastic base shear capacity, post-elastic stiffness and damping) of an SDOF model are obtained by analysing a large number of non-linear numeric structure responses using earthquakes of different intensities as load input.
An MLP neural network is used to model the relationship between the structure response parameters of the SDOF model and the damage ratio coefficient. A sensitivity analysis procedure is perfomed using the trained neural network to determine the influence of the individual structure response parameters on the damage level of a structure based upon an earthquake.

KEYWORDS: SDOF system, earthquake response, damage ratio, neural network, sensitivity analysis

1. INTRODUCTION

The structure response parameters of an SDOF model, obtained by analysing a large number of non-linear numeric structure responses using earthquakes of different intensities as load input, are input into a previously valorised new original deterministic declaration of damage ratio (*DR*), thereby interpreting the level of structure damage at the end of the earthquake.
With the aid of neural networks, a sensitivity analysis procedure can be applied to identify, qualitatively, the input parameters that have a greater influence on the damage level.
The various structure parameters (natural period, elastic base shear capacity, post-elastic stiffness and damping) which represent the input values of the neural network and the damage level, which represents the output of the neural network, are obtained from a databank of damage ratios, grouped by ground motion.
This would provide insight into the influence of the individual structure response parameters on the damage level of a structure based upon an earthquake load.

2. LEVEL OF STRUCTURAL DAMAGE

The seismic response analysis of regular structures (structure with symmetric plans and constant vertical stiffness) is acceptable if it is done as a simplified non-linear dynamic analysis with the time history function of ground motion as input load, and a Single Degree of Freedom (SDOF) model with known weight, elastic stiffness, damping, elastic base shear capacity and post-elastic stiffness representing the structure.
Usually, in literature, the problem of structural damage is solved by calculating the Damage Ratio (*DR*) coefficient, calculated for the structural element (partial) or for the whole structure (global). Damage ratios in various models ([1], [2]) are based on maximum values of structural response parameters or cumulative values and summing of non-linear deformation cycles. Based on some known damage models, an original formula for damage ratio is given [3].

The level of structural damage (damage ratio, *DR*) can be described as a function of the following calculated structure response parameters:

- Displacement ductility (*D*) defines the measure of post-elastic region in which a structure was during an earthquake.
- Maximum base shear force, $(BS)_{\max}$, and maximum top displacement ($u_{\max}$) defines the residual stiffness (K') of the structure at the end of the earthquake.
- Number of yield excursions (N_Y) and hysteresis energy (E_H) define the post-elastic cyclic nature of damage ratio developing.

The first two parameters define damage mechanism under monotonic load while the third parameter takes into account the cyclic failure. The damage ratio coefficient (*DR*) is defined as the linear combination of plastic deformations, stiffness degradation and energy dissipation of a structure during an earthquake [3],

$$DR = \frac{1}{30}\left[D + \Delta K + \sqrt[3]{\left(N_Y E_H / W\right)}\right], \tag{2.1}$$

where:

$D = \dfrac{u^{\max}}{u^{y}}$	-	the displacement ductility demand;
$\Delta K = \dfrac{K_e}{K'}$	-	the relative degradation of stiffness at the end of the earthquake;
$K_e = \dfrac{(BS)_y}{u^{y}}$	-	the initial structure stiffness;
$K' = \dfrac{(BS)_{\max}}{u^{\max}}$	-	the residual secant stiffness of a structure after an earthquake;
N_Y	-	the number of yield excursions reached during the earthquake;
$\dfrac{E_H}{W}$	-	the hysteresis energy per unit of structure mass, dissipated during an earthquake.

Its main function in the damage model is to describe the condition of a structure after an earthquake. In such an approach, the damage ratio value can be used to declare decreased residual seismic resistance and increased residual damping coefficient of a structure. Also, the damage ratio values (*DR*) can be implemented in pre or post earthquake damage analysis by relating the damage ratio values (*DR*) with the values of damage level identification (S), defined in the Croatian codes for post disasters damage assessment.
The valorisation of these assumptions and proposed formula for damage ratio was done by comparing the results with those of the CAMUS3 experiment, done by the Camus working group, in TRM-ECOEST 2 Research programme in EMSI Sacley, France [3], [4].

3. CLASSIFICATION OF STRUCTURES

Under the assumption that regular structures can be appropriately modelled using an SDOF system, a large number of non-linear numeric structure responses analysis are obtained using earthquakes of different intensities and dominant frequencies as load input on 2250 previously classified structures. All the structures are modelled as an SDOF system using the defined damping (ξ), weight (*G*) and elastic stiffness (K_e), yield base shear (S_Y) and post-elastic stiffness (K_2).

- A constant weight $G = 1000$kN is assumed for all SDOF systems.
- Damping is defined as 2%, 5% and 10% of critical (3 structure conceptions).
- Variation of the elastic stiffness is a function of the basic period of the system representing real regular structure. Elastic stiffness is modified in such a manner as to realise a change in basic period in steps of 0.1s, from 0.05s to 10s (15 structure conceptions).
- Yield base shear, which defines the yield point and the end of elastic stiffness, is modified in ten levels, from 0.1*G* to 1.0*G* (10 structure conceptions).

- Post-elastic stiffness, which represents residual stiffness after yield point is reached, is modelled as a percentage of initial elastic stiffness. Variations of this parameter is done, also, in five steps: $K_2 = 0.00$, $K_2 = 0.2K_e$, $K_2 = 0.4K_e$, $K_2 = 0.6K_e$, and $K_2 = 0.8K_e$ (5 structure conceptions).

As a result, 2250 various structures are obtained. The input seismic loads are ground acceleration time-histories of 20 real earthquakes.
All calculations were run using the program NONLIN [5], which implements step by step time-history numerical integration, thereby providing the analysis results: time-history of top displacements with FFT analysis in frequency domain, base shear–displacements hysteresis response, yield excursions and cumulative energy transformation. These structure response parameters were input into a previously valorised new original deterministic declaration of damage ratio (*DR*) resulting in a data bank of 45000 calculated values of damage ratios.

4. NEURAL NETWORKS

The field of neural networks has a history of some five decades but has found solid application only in the past fifteen years, and the field is still developing rapidly. Thus, it is distinctly different from the fields of control systems or optimization where the terminology, basic mathematics, and design procedures have been firmly established and applied for many years.
Neural networks are being successfully applied across an extraordinary range of problem domains, in areas as diverse as finance, medicine, engineering, geology and physics. Currently, anywhere where there are problems of prediction, classification or control, neural networks are being introduced.
A neural network consists of simple elements operating in parallel. These 'processing' elements are inspired by biological nervous systems or neurons. As in nature, the network function is determined largely by the connections between these elements. A neural network can be trained to perform a particular function by adjusting the values of the connections (weights) between elements. Commonly, neural networks are adjusted, or trained, so that a particular input leads to a specific target output.
The neural network structure used is a three-layered feed-forward neural network with full connectivity and bias, or simply, a three-layered Multilayer Perceptron (MLP) network (Figure 1).

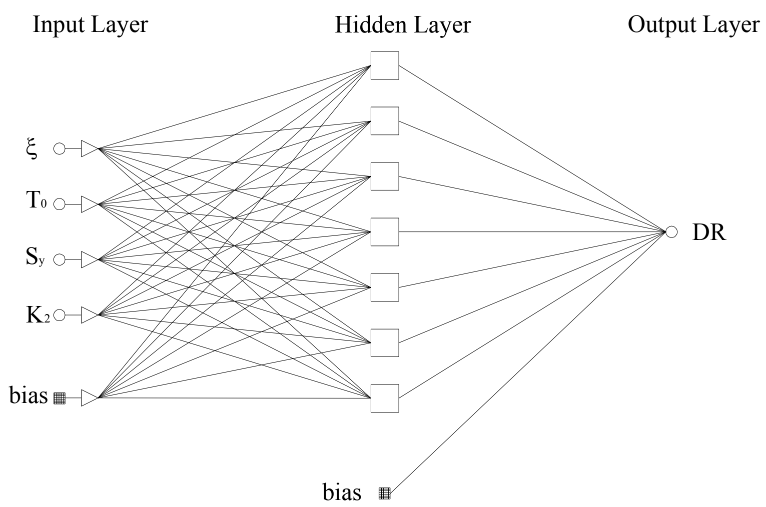

Figure 1 The structure of the MLP network implemented

As can be seen from Figure 1, this neural network structure has neurons arranged in a distinct layered topology. The input layer is not really neural at all: these units simply serve to introduce the values of the input variables. The hidden and output layer neurons are each connected to all of the units in the preceding layer.
Such networks can model functions of almost arbitrary complexity, with the number of layers, and the number of units in each layer, determining the function complexity. For a given earthquake accelerogram, the neural

network is used to model the function relating the natural period (T_0), yield base shear capacity (S_Y), post-elastic stiffness (K_2) and damping of a structure (ξ) to the Damage ratio (DR) obtained (i.e. a function of 4 inputs and 1 output). The neural network is then implemented in determining the relative importance (predictive importance) of the given inputs.

Terms such as “importance” and “sensitivity” do not have precise, widely-accepted meanings. A variety of methods have been proposed to measure the importance of inputs [6], but the list is by no means exhaustive. Different measures of importance are likely to be useful in different applications of neural nets.

The two most common notions of importance are predictive importance and causal importance.

- Predictive importance is concerned with the increase in generalization error when an input is omitted from a network.
  This is often performed by simply deleting an input unit from the network and retraining the network. The network error is compared with the error obtained when all inputs are used. Basically, the most important input is the one whose absence generates the greatest error.
- Causal importance is concerned with the change in output values when the input values are manipulated or changed (often referred to as sensitivity analysis).
  This is often performed by taking the partial derivative of the output value with respect to one of its input. However the partial derivative depends not only on the weights and biases but also on the current values of the input variables. It is thus difficult to generalize on the trend of the output value with respect to a change in a single input value. Hence, the distribution of the partial derivatives for the entire set can be used to describe qualitatively the sensitivity of the output value [7].

The number of combinations (the binomial coefficient) or the number of combinations of n parameters taken k at a time is given by:

$$\frac{n!}{(n-k)!\,k!}. \tag{4.1}$$

So for n input parameters there will be n combinations of $(n - 1)$ inputs. Hence, with only four inputs, there are only four combinations of 3-inputs.

5. RESULTS

The neural network used (Figure 1.) consisted of seven neurons in the hidden layer, with the tansig function used as the activation function for the neurons of this layer. The output layer consisted of only one neuron (since there is only one output) and the purelin function is used as the activation or transfer function for this neuron. The number of input neurons depended on the number of input parameters selected.

The data bank of 45000 calculated values of damage ratios was used (20 earthquakes and 2250 different structures). Thus for each earthquake, the neural network training dataset consisted of 2250 measurements (or data structures). ½ of the dataset was used for training, ¼ for validation and ¼ for testing.

Initially the neural network was trained using all four input parameters. The neural network was trained 15 times, each time with different weight initialization. The average network error of all training sessions was taken as the network error of the neural network structure when all inputs were used. This network error was used as the basis with which all future network errors were compared with.

For each combination of input parameters, the neural network was trained 15 times, each time with different weight initialization. The average network error of all training sessions was taken as the network error of the neural network structure. This procedure was repeated for all the 20 different earthquakes or earthquake accelerograms.

For the four inputs: structure damping (ξ), natural period (T_0), post-elastic stiffness (K_2) and yield base shear capacity (S_Y), the input combinations of per 3 parameters (and all four input parameters) used are shown in Table 1.

Table 5.1 Input combinations of three (and four) parameters

Combination	Input parameters
C11	ξ, T_0 , K_2
C12	ξ, T_0 , S_Y
C13	ξ, K_2 , S_Y
C14	T_0 , K_2 , S_Y
C15	ξ, T_0 , K_2 , S_Y

The results obtained are displayed on Figure 2 for combinations *C11* to *C15*.

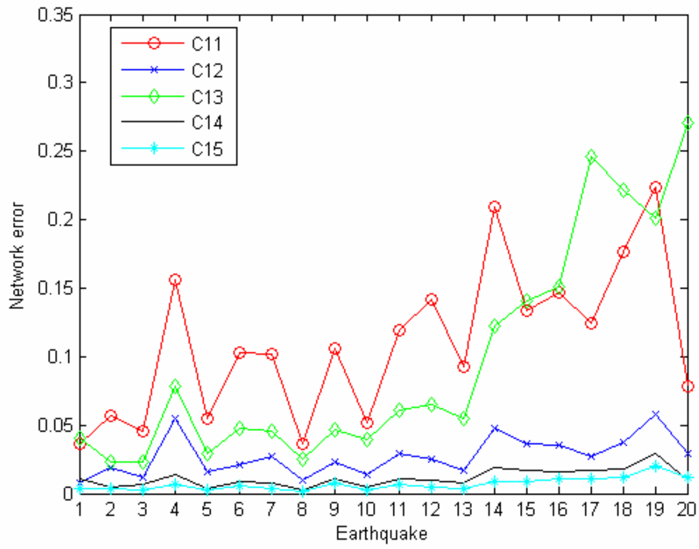

Figure 2 Network error on test data for combinations *C11 – C15*

Looking at Figure 2, combination *C15* (combination with all four inputs) has the lowest network error. However, when taking into account only the other input combinations of three parameters (*C11 – C14*), it is evident that combination *C14* has the least network error, indicating that the missing parameter of that combination (damping, ξ) is the least important parameter. The next combination with least network error after *C14* is *C12*, indicating that the post-elastic stiffness, K_2, is more important than ξ but less important than the other two parameters T_0 and S_Y. Hence by looking at the results displayed in the graph and comparing with the combinations given in Table 1 we can initially conclude that the parameters in order of importance (predictive importance) from the most important to the least important is:

$$S_Y - T_0 - K_2 - \xi.$$

In order to confirm this, it was decided to test further using input combinations of two parameters and one parameter and to train neural networks with these inputs and compare the network error to the reference error obtained with four inputs.
Table 2 shows all the other combinations used (including the original combinations of 3 inputs and 4 inputs).

Table 2. All input combinations of the parameters

Combination	Input parameters
C1	ξ
C2	T_0
C3	K_2
C4	S_Y
C5	ξ, T_0
C6	ξ, K_2
C7	ξ, S_Y
C8	T_0 , K_2

C9	T_0 , S_Y
C10	K_2 , S_Y
C11	ξ, T_0 , K_2
C12	ξ, T_0 , S_Y
C13	ξ, K_2 , S_Y
C14	T_0 , K_2 , S_Y
C15	ξ, T_0 , K_2 , S_Y

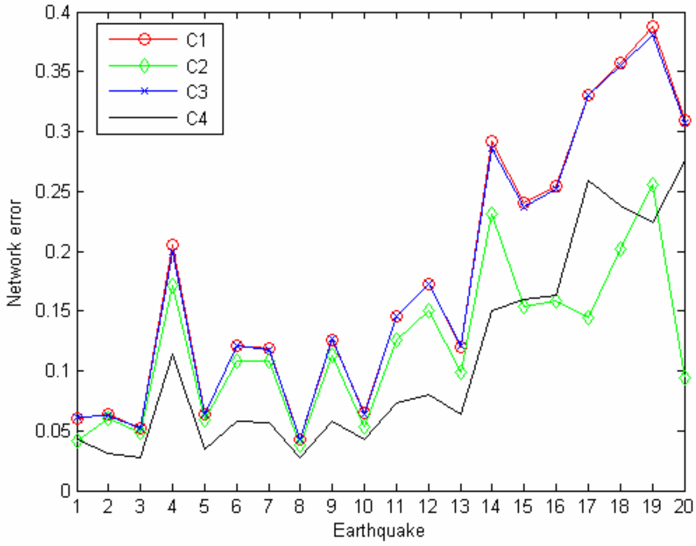

Figure 3 Network error on test data for combinations *C1* – *C4* for all earthquakes

For the combinations *C1* – *C4*, that is, when there is only one input, the most important input is obviously the input that gives the least network error. Looking at Figure 3, the parameters ordered in order of importance (predictive importance) from the most important to the least important are:

$$S_Y - T_0 - K_2 - \xi.$$

This order of importance can further be confirmed by looking at Figure 4, where the input combinations of two parameters are displayed.

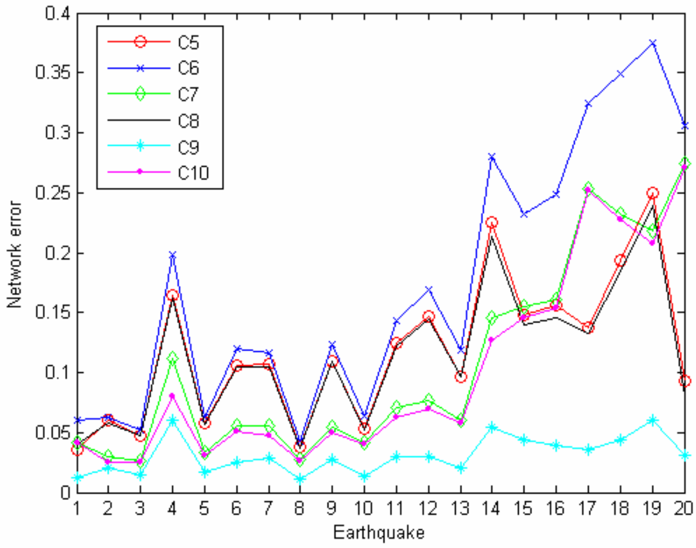

Figure 4 Network error on test data for combinations *C5* – *C10* for all earthquakes

Figure 5 displays the surface plot of the network error for all input combinations for all earthquakes.

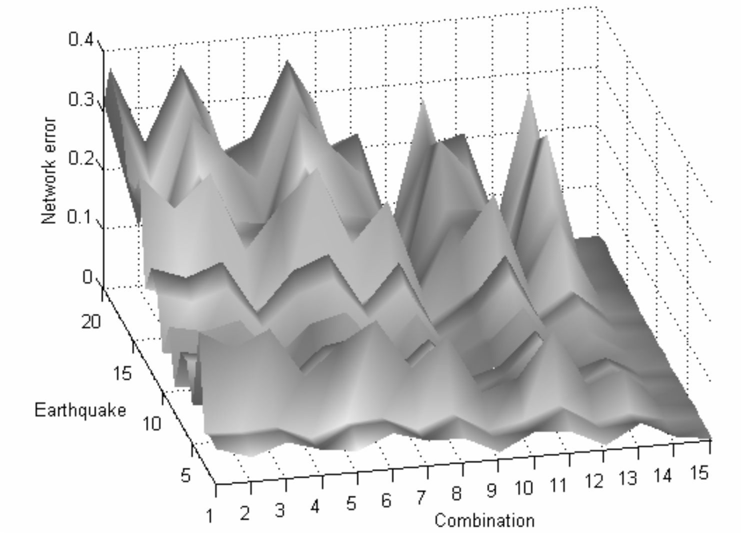

Figure 5 Network errors for all input combinations for all earthquakes

6. CONCLUSION

A three-layered MLP neural network is used to model the function relating the structure response parameters of an SDOF model (natural period, elastic base shear capacity, post-elastic stiffness and damping) obtained by analysing a large number of non-linear numeric structure responses using earthquakes of different intensities as load input and the corresponding damage level obtained by using a previously valorised new original deterministic declaration of damage ratio.
With the aid of the neural network, a sensitivity analysis procedure is applied to identify, qualitatively, the input parameters that have a greater influence on the damage level.
This is performed by using different input combinations of these input parameters from the network and retraining the network. The network error is compared with the error obtained when all inputs were used. Initially, input combinations of three parameters are used. Then only one input parameter is used at a time and finally input combinations of two parameters are used.
In all situations, using predictive importance, it was noticed that the most important parameter was yield base shear capacity (S_Y) followed by natural period (T_0). The post-elastic stiffness (K_2) and damping of a structure (ξ) were the least important parameters.

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