Shear Failure Model for Flexure-Shear Critical Reinforced Concrete Columns

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ABSTRACT :

In areas of high seismicity around the world, the collapse vulnerability of older RC structures built prior to the advent of more effective seismic design codes is under careful investigation. The sheer number of these older structures, coupled with the overwhelming proportion of them that would require retrofit if assessed according to today's conservative codes of practice, are hindering region-wide mitigation efforts to the point of rendering them ineffectual. Fortunately, in direct contradiction with the predictions of current codes of practice, post earthquake reconnaissance studies show that only a low portion of these buildings collapse. This observation highlights failings in our current codes in assessing structural collapse and suggests that more refined engineering tools might be useful to identify the small portion of buildings that are most collapse prone. In this way resources could be focused on seismic mitigation of those buildings and by consequence render region-wide mitigation efforts more tractable. Presented here is a newly developed shear failure initiation model for reinforced concrete columns with light transverse reinforcements. This model takes into account key variables that contribute to the loss of shear strength and is specifically aimed at columns with details and dimensions allowing them to yield in flexure prior to reaching their shear and axial capacities. This failure model breaks away from current global deformation or drift-based approaches to relate shear and failure to local deformations within the column critical sections.

KEYWORDS: Concrete, column, collapse, axial, shear, failure

1. INTRODUCTION

In areas of high seismicity around the world, the collapse vulnerability of older RC structures built prior to the advent of more effective seismic design codes in the 1980's is under careful investigation. The sheer number of these older structures (estimated at 40,000 in California alone), coupled with the overwhelming proportion of them that would require retrofit if assessed according to today's conservative codes of practice, are hindering region-wide mitigation efforts to the point of rendering them ineffectual.

Fortunately, in direct contradiction with the predictions of current codes of practice, post earthquake reconnaissance studies (Otani (1999)) show that only a low portion of these buildings collapse. This observation highlights failings in our current codes in assessing structural collapse and suggests that more refined engineering tools might be useful to identify the small portion of buildings that are most collapse prone. In this way resources could be focused on seismic mitigation of those buildings and by consequence render region-wide mitigation efforts more tractable.

A newly developed shear failure initiation model for reinforced concrete columns with light transverse reinforcements is presented. This model takes into account key variables that contribute to loss of shear strength of reinforced concrete columns with light transverse reinforcements. This model is specifically aimed at columns with details and dimensions allowing them to yield in flexure prior to reaching their shear and axial capacities (referred to as flexure-shear critical columns or FSC columns). The interest in this class of column stems not only from their ability to withstand moderate to large deformations prior to axial collapse (Elwood and Moehle (2005a); Sezen and Moehle (2006)) but from the fact that current codes of practice and design guidelines under-estimate their deformation capabilities, often leading to overly conservative predictions of structural collapse.

The proposed failure model breaks away from current global deformation or drift-based approaches to relate shear and axial failure to local deformations within column critical sections.

2. SHEAR FAILURE INITATION MODEL

Flexure-shear critical columns, which are typically more slender columns, have higher shear strength than flexural strength, which allows them to yield in flexure prior to shear failure. Shear failure in these columns will only occur after the plastic hinge region deteriorates sufficiently, resulting in degradation in shear strength. Several models for shear strength have been proposed [Watanabe and Ichinose (1992); Aschheim and Moehle (1992); Priestley et al. (1994); Sezen (2002)] to model this shear degradation with respect to increasing deformation demands. While these adequately model shear strength as function of drift demand, they do not produce a reliable estimate of displacement ductility at shear failure (Elwood and Moehle (2005b)). Given the aforementioned limitation, several displacement-based models have been developed to estimate deformation capacity of FSC columns given a shear force demand (Pujol et al. (1999); Kato and Ohnishi (2002); Elwood and Moehle (2005b)). These models relate horizontal drift ratio at shear failure to various configuration and demand parameters. These models were derived empirically from column tests with essentially fixed-fixed end conditions.

Ghannoum (2007) however observed that shear failure appears to be related to inelastic rotational demands of critical FSC column end regions. In column tests with fixed-fixed boundary conditions, column end rotations are equal at both ends and closely related to drifts. However, in RC frames this is rarely the case (Ghannoum (2007)). In frame structures, end rotational demands are usually different at opposite ends of a column because of differing frame-imposed boundary and loading conditions. These observations imply that local end rotational demands rather than global drift demands may be better suited for shear failure initiation estimates in FSC columns.

A new shear failure initiation model that relates shear strength degradation to column end rotation in flexure-shear critical columns is proposed. This model is deformation-based and intended to be used in estimating column end rotations at which shear failure initiation occurs. Shear failure initiation is defined where shear strength loss commences and is associated with the development of a large shear crack. A database of 56 column tests is used in a parametric regression analysis to determine factors that most significantly affect the rotation capacity of this type of column.

2.1 Column Database

A database of 56 tests conducted on flexure-shear critical columns with light transverse reinforcement (ρ''<0.007) is used to develop the proposed shear failure initiation model. All tests were conducted under reversed cyclic deformations, and four were conducted dynamically. All tests were subjected to horizontal deformations in a single plane. The first fifty column properties and test results were compiled by Sezen and Moehle (2004). The database includes columns with the following range of properties:

- shear span to depth ratio: $2.0<a/d<4.0$ (mean=3.0)
- transverse reinforcement spacing to depth ratio: $0.2<s/d<1.2$ (mean=0.62)
- concrete compressive strength: $1900<f'_c<6500$ psi (mean=3700 psi)
- longitudinal-reinforcement yield stress: $47<f_{yl}<80$ ksi (mean=60 ksi)
- longitudinal reinforcement ratio: $0.01<\rho_l<0.04$ (mean=0.023)
- transverse-reinforcement yield stress: $46<f_{yt}<100$ ksi (mean=65 ksi)
- transverse reinforcement ratio: $0.0010<\rho''<0.0065$ (mean=0.0028)
- maximum nominal shear stress: $2.8< v/f'^{0.5}_c$,psi<8.6 (mean=5.5)
- axial load ratio: $0.0<P/A_g f'_c<0.6$ (mean=0.2)

where a is the shear span, d is the column depth from extreme compression fiber to centerline of outermost tension reinforcement, s is the spacing of transverse reinforcement, $\rho_l=A_{sl}/bh$; A_{sl} is the area of the longitudinal reinforcement; b is the column section width, h is the column section depth; $\rho''=A_{st}/bs$, A_{st} is the area of transverse reinforcement in the direction of lateral load at spacing s, v is the maximum nominal shear stress in psi, f'_c is the concrete cylinder compressive strength, P is the axial load, and A_g is the gross cross-sectional area of the column.

2.2 Analytical Model Description

For most column tests in the database, the available test data were limited to the global relation between shear force and lateral displacement. To better understand local distributions of deformations, the fiber-section column model developed by Ghannoum (2007) was used to model all database columns. This analytical model was highly accurate in modeling frame and column behavior of a three-bay, three-story RC frame structure. High levels of accuracy were achieved with this model at the frame level, column level, and more importantly at the column end rotation level. The main components of this model are a fiber-section implementation of column elements and a novel zero-length fiber-section implementation of longitudinal-bar anchorage slip. These anchorage slips produce rigid body rotations in columns and can account for 50% of column lateral deformations. Bond stress at the interface between longitudinal bars and anchorage regions was assumed to be uniform in both the elastic and plastic deformation ranges. More detail on the analytical model can be found in Ghannoum (2007).

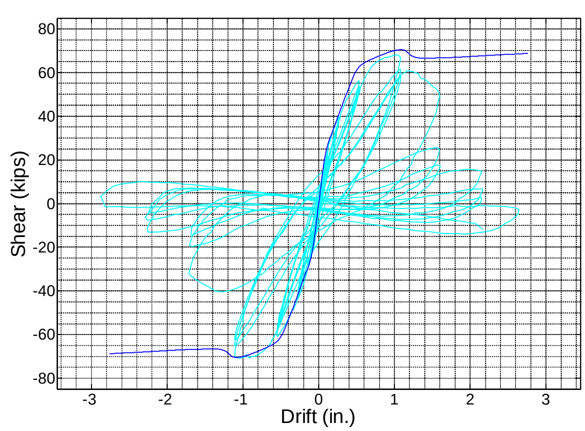

Figure 1 Sample pushover curve vs. experimental shear-drift relation (test by Sezen and Moehle (2006))

Pushover analyses were performed on all column analytical models with increasing lateral deformations applied at cantilever tips. The bond stress that resulted in the best fit over all columns between analytical and experimental column stiffnesses was $u_e=11\ f'_c{}^{0.5}$ (psi). The resulting error between column analytical and experimental elastic stiffnesses is defined as: $e_s = (K_{y\text{-}analysis} - K_{y\text{-}experiment})/\ K_{y\text{-}experiment}$ ($K_{y\text{-}analysis}$ = column analytical elastic stiffness; $K_{y\text{-}experiment}$ = column experimental elastic stiffness). The choice of $u_e=11\ f'_c{}^{0.5}$ (psi) produced a mean error in elastic stiffness across all database columns of 0.01 with a standard deviation of 0.39. Figure 1 presents a sample pushover curve overlaid on the experimental shear-drift relation for a column tested by Sezen and Moehle (2006). This figure shows very close agreement between model and experiment.

2.3 Analytical Model Results

Analytical models of database columns were used to extract column end rotations at the drift (δ_{max}) that caused shear failure initiation experimentally. Pushover analyses yielded several column rotation measures that were evaluated over a plastic hinge length L_P=h=column section height. This length was used for its simplicity and to insure that all plastic deformations would be contained within this length. The extracted rotations were: θ_{Totmax}=column total rotations at shear failure initiation including bar-slip induced rotations; $\theta_{fTotmax}$=column flexural rotations at shear failure initiation excluding bar-slip induced rotations; $\theta_{BSTotmax}$=bar-slip component of θ_{Totmax} and equals θ_{Totmax}-$\theta_{fTotmax}$; $\theta_{TotPlmax}$=plastic rotation component of θ_{Totmax}; θ_{fPlmax}=plastic rotation component of $\theta_{fTotmax}$.

To assess the sensitivity of these rotations to assumed longitudinal-bar bond-stress values within anchorage zones (i.e., footings), column pushover analyses were undertaken for $u_e= \lambda\ (f'_c)^{0.5}$ (psi), with the bond-stress parameter taken as λ=6, 11, and 18. From these analyses, the total rotation at δ_{max} (θ_{Totmax}), which includes flexural rotations over L_p and bar-slip rotations, was found to be insensitive to bond-stress assumption. Figure 2 plots the relative differences in θ_{Totmax} for the various bond-stress parameters. These relative differences are defined for each column as: ($\theta_{Totmax}(\lambda$=6 or 18)- $\theta_{Totmax}(\lambda$=11))/ $\theta_{Totmax}(\lambda$=11). Figure 2 shows variation of less than 2% in θ_{Totmax} between the various bond-stress scenarios. This indicates that θ_{Totmax} values are insensitive to the bar-slip model assumptions and can be used to define shear failure initiation without bar-slip model constraints.

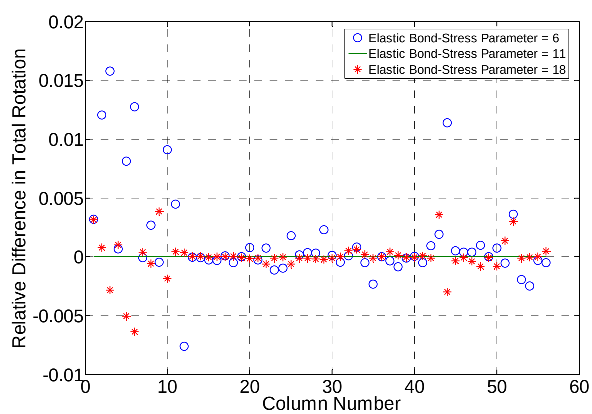

Figure 2 Variations in θ_{Totmax} with changes in bond-stress assumption

The proportions of column flexural rotations over L_p ($\theta_{fTotmax}$) and bar-slip rotations ($\theta_{BSTotmax}$) that comprise θ_{Totmax} were found to vary significantly with bond-stress parameter. This indicates that analytically derived $\theta_{fTotmax}$ or $\theta_{BSTotmax}$ values are specific to the bar-slip model and bond-slip parameters, and should not be used to define shear failure initiation in conjunction with other bar-slip models. The ratio of bar-slip to total rotations $r_{bst}=\theta_{BSTotmax}/\theta_{Totmax}$ or the converse ratio of flexural to total rotations $r_{ft}=\theta_{fTotmax}/\theta_{Totmax}$ were found to be fairly constant across all columns in the database for each bond stress value. The mean values of these ratios for the bond stress value of interest (i.e., λ=11) are mr_{bst}=0.54 and mr_{ft}=0.46, with most values within +/– 0.1 from mean. This close clustering of rotation ratios indicates that direct scaling from θ_{Totmax} to either $\theta_{BSTotmax}$ or $\theta_{fTotmax}$ by the ratios r_{bst} or r_{ft} may be done without much loss in accuracy.

2.4 Shear Failure Initiation Model

A model to estimate column deformation at initiation of shear failure in flexure-shear-critical columns with light transverse reinforcement is proposed. This model is intended to estimate mean response values. In this model it is postulated that shear strength (V_r) of flexure-shear critical columns prior to shear strength degradation is a sum of shear strength of column sections under tensile strains (V_{ct}), column sections under compression (V_{cc}), and transverse ties (V_s). These terms are illustrated in Figure 3. V_r can be written out as:

$$V_r = V_{ct} + V_{cc} + V_s \qquad (2.1)$$

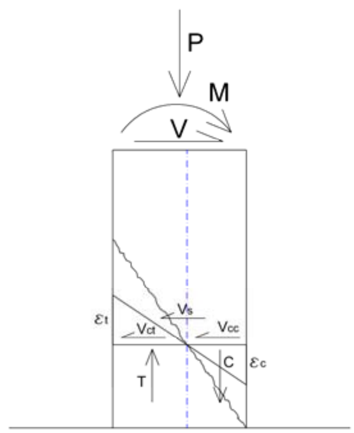

Figure 3 Illustration of column shear strength components

Research into the effects of tensile strains on concrete member shear strength (Vecchio and Collins (1986); Belarbi and Hsu (1995)) has shown strength decreases with increasing axial tension strains. Studies on effects of high compressive axial stresses on shear strength (Gupta and Collins (2001)) show a drop in shear strength of members under high compression loads. It is also well know that concrete compressive strength and transverse reinforcement play an important role in shear strength of reinforced concrete members under either tension or compression. Thus, V_{ct} can be written out as function of concrete compressive strength (f'_c), concrete tensile strains at critical shear-failure section (ε_t), and transverse reinforcement properties (s/d, ρ", f_{yt}). Likewise, V_{cc} can be written out as a function of concrete compressive strength (f'_c), concrete compressive stresses or strains at critical shear-failure section (σ_c, or ε_c), and transverse reinforcement properties (s/d, ρ", f_{yt}). This leads to a relation of the following form for V_r:

$$V_r = f\,(s/d,\ \rho'',\ f_{yt},\ f'c,\ \varepsilon_t,\ \sigma_c,\ \varepsilon_c) < V\ (\text{=imposed shear force}) \qquad (2.2)$$

Several proxies for the shear-strength predictor variables listed in Eqn. 2.2 are investigated to generate the proposed shear failure model. Namely, column flexural rotations evaluated over a critical column length are investigated as proxies for (ε_t). $v/f'^{0.5}_c$ is used a measure of V (v=column shear stress= V_{max}/bd at shear failure initiation). Transverse reinforcement ratio (ρ"), spacing of transverse ties (s) normalized by d or s/d, and $\rho'' f_{yt}$ are explored to represent the effects of transverse ties. $P/(A_g f'_c)$, the normalized average concrete compressive stress in base-column-section fibers covering a distance h/4 from extreme compression fiber ($\sigma_{c\text{-}h/4}/f'_c$), and the corresponding normalized strains ($\varepsilon_{c\text{-}h/4}/\varepsilon_c$; ε_c=concrete strain at f'_c) are used to represent σ_c and ε_c. The ratio a/d is investigated as a variable that accounts for shear to moment ratio in columns. The proposed model thus takes the following form at shear failure initiation:

$$\theta_{max} = f\,(s/d,\ \rho'',\ \rho'' f_{yt},\ a/d\ ,\ P/(A_g f'_c),\ v/f'^{0.5}_c,\ \sigma_{c\text{-}h/4}/f'_c,\ \varepsilon_{c\text{-}h/4}/\varepsilon_c) \qquad (2.3)$$

with θ_{max}=column rotation measure at initiation of shear failure evaluated over a plastic hinge length h. The proposed model is presented in four formulations that use four different column rotation measures to define shear failure initiation i.e., $\theta_{max} = \theta_{Totmax}$, $\theta_{fTotmax}$ $\theta_{TotPlmax}$, or θ_{fPlmax} (defined previously). These rotation measures were considered as they would allow use of the proposed model with most lumped-plasticity or fiber-section column analytical implementations. The model formulation with θ_{Totmax} has the advantage over the other three model formulations of being insensitive to bar-slip modeling parameters. The remaining three formulations must be used in conjunction with analytical bar-slip models that produce similar bar slip rotations as the ones obtained with the previously described analytical model.

Predictor variables listed in Eqn. 2.3 were plotted versus θ_{Totmax}, $\theta_{fTotmax}$, θ_{fPlmax}, and $\theta_{TotPlmax}$ for all columns in the database. These plots are not reproduced for brevity. Trends observed for all rotation measures were similar. Little relation between ρ", or ρ"f_{yt} and column rotations at shear failure initiation was observed. Rotations at shear failure initiation however were found to increase with decreasing s/d. This suggests that the spacing of column ties is more influential on shear failure initiation than the amount and strength of these ties. The shear-span to column-depth ratio (a/d) showed no clear relation to rotations. Rotations were noted to decrease as values of $v/f'_c{}^{0.5}$ increased. This illustrates the detrimental effect of shear stresses on column deformation capacity prior to shear failure. Rotation values decreased significantly with increasing $P/A_g f'_c$ values. The detrimental effect of axial load on deformation capacity of flexure-shear critical columns is also expressed in observed trends between $\sigma_{c\text{-}h/4}/f'_c$ and $\varepsilon_{c\text{-}h/4}/\varepsilon_c$ and rotations. These plots indicate that columns with compression blocks under high compressive stresses and strains have reduced rotational capacity prior to shear failure.

Based on the aforementioned observations, a forward stepwise linear regression of the form [$\bar{O}_{max} = b_0 + b_1 X_1 + b_2 X_2 + \ldots\ldots$] (with, $\bar{O}_{max}$ = least squares estimate of θ_{max}; b_0, b_1, … = linear regression parameters; X_1, X_2, … = predictor variables) is used with variables that exhibited trends with respect to column rotations. These variables are: ρ", s/d, ρ"f_{yt}, $P/(A_g f'_c)$, $v/f'_c{}^{0.5}$, $\sigma_{c\text{-}h/4}/ f'_c$, and $\varepsilon_{c\text{-}h/4}/\varepsilon_c$. Forward stepwise regression starts with no model terms ($b_i X_i$) and adds the most statistically significant term (the one with the highest F statistic or lowest p-value) at each step until no significant terms are left. This regression technique is applied to all four column rotation measures at shear failure initiation (i.e., θ_{Totmax}, $\theta_{fTotmax}$, $\theta_{TotPlmax}$, or θ_{fPlmax}). The most significant predictor variables for θ_{Totmax} and $\theta_{fTotmax}$ were found in this way to be s/d, $P/(A_g f'_c)$, $\sigma_{c\text{-}h/4}/ f'_c$, and $\varepsilon_{c\text{-}h/4}/\varepsilon_c$ while those for $\theta_{TotPlmax}$, and θ_{fPlmax} were s/d, $P/(A_g f'_c)$, and $v/f'_c{}^{0.5}$. Given the redundancy of the terms $P/(A_g f'_c)$, $\sigma_{c\text{-}h/4}/ f'_c$, and $\varepsilon_{c\text{-}h/4}/\varepsilon_c$, as they all relate to column compression-block state, and considering that $\sigma_{c\text{-}h/4}/f'c$, and $\varepsilon_{c\text{-}h/4}/\varepsilon_c$ are difficult to extract analytically, the stepwise regression for θ_{Totmax} and $\theta_{fTotmax}$ were re-evaluated without $\sigma_{c\text{-}h/4}/ f'_c$, and $\varepsilon_{c\text{-}h/4}/\varepsilon_c$. The end result for all rotations under consideration was that s/d, $P/(A_g f'_c)$, and $v/f'_c{}^{0.5}$ surfaced as the most significant terms to include in the model. Thus, the least squares estimate of θ_{max} takes the final form:

$$\bar{O}_{\max} = b_0 + b_1\left(\frac{s}{d}\right) + b_2\left(\frac{P}{A_g f'_c}\right) + b_3\left(\frac{v}{\sqrt{f'_c}}\right) \tag{2.4}$$

An iteratively re-weighted least squares algorithm was used to determine the regression parameters b_0, b_1, b_2, and b_3. This technique minimizes the effects of outliers and produces a better fit through the bulk of data points. Table 2.1 lists the linear regression parameters b_0, b_1, b_2, and b_3 obtained from the robust regression fits performed on all four rotations (θ_{Totmax}, $\theta_{fTotmax}$, $\theta_{TotPlmax}$, or θ_{fPlmax}). This table also presents the coefficients of multiple determination (R^2) for these fits.

Table 2.1 Column rotation weighted regression parameters

	θ_{Totmax}	$\theta_{fTotmax}$	$\theta_{TotPlmax}$	θ_{fPlmax}
b_0	0.0437	0.0210	0.0317	0.0149
b_1	-0.0171	-0.0077	-0.0138	-0.0067
b_2	-0.0211	-0.0088	-0.0165	-0.0065
b_3	-0.0020	-0.0011	-0.0016	-0.00082
R^2	0.53	0.49	0.44	0.43

From Table 2.1 one can note that $\theta_{fTotmax}$ regression parameters relate to those of θ_{Totmax} by an approximate factor of 0.45. θ_{fPlmax} regression parameters are also found to relate to those of $\theta_{TotPlmax}$ by a ratio of 0.45. Hence ofr simplification, Eqn. 2.4 was defined for θ_{Totmax} and $\theta_{TotPlmax}$ while $\theta_{fTotmax}$ and θ_{fPlmax} values were related to them by a factor of 0.45. This simplification resulted in little loss of accuracy. A simple relation between total rotations and plastic rotations could not be implemented as it resulted in significant accuracy losses in plastic rotation estimates. Thus, the least squares estimates of column rotations at shear failure initiation are given by:

$$\theta_{Total} = 0.044 - 0.017\left(\frac{s}{d}\right) - 0.021\left(\frac{P}{A_g f'c}\right) - 0.0020\left(\frac{v}{\sqrt{f'c}}\right) \geq 0.009$$

$$\theta_{Flexural} = 0.45\theta_{Tot\,\max} \geq 0.00405$$

$$\theta_{Total-Plastic} = 0.032 - 0.014\left(\frac{s}{d}\right) - 0.017\left(\frac{P}{A_g f'c}\right) - 0.0016\left(\frac{v}{\sqrt{f'c}}\right) \geq 0.0$$

$$\theta_{Flexural-Plastic} = 0.45\theta_{TotPl\,\max} \geq 0.0$$

(2.5)
psi units

with, θ_{Total}=least squares estimate of θ_{Totmax}=column total rotation at shear failure initiation measured over a plastic hinge length h including bar-slip induced rotations; $\theta_{Flexural}$=least squares estimate of $\theta_{fTotmax}$=column flexural rotation at shear failure initiation measured over a plastic hinge length h excluding bar-slip induced rotations; $\theta_{Total\text{-}Plastic}$=plastic rotation portion of θ_{Total}; $\theta_{Flexural\text{-}Plastic}$=plastic rotation component of $\theta_{Flexural}$; s=transverse reinforcement spacing, d=column depth from extreme compression fiber to centerline of outermost tension reinforcement; P=column axial load; A_g=column gross section area; f'_c=concrete cylinder compressive strength; v=column section shear stress=V/bd (V=column shear force). Eqn. 2.5 is based on shear failure initiation that is defined where shear strength loss commences and is associated with the development of a large shear crack.

In this relation the lower value of θ_{Total} is limited to 0.009, which is a bound observed in the database. This bound safeguards against estimating shear failure much prior to flexural yielding. It is good to note that total rotations in Eqn. 2.5 are bounded by 0.009 and 0.044 (which correspond roughly to drift ratios of 0.9% and 4.4%). Total plastic rotations (plastic bar-slip + plastic flexural rotations over h) are bound by 0.0 and 0.032. The shear failure initiation model presented in Eqn. 2.5 should only be used with columns that have material and geometric parameters in the same ranges as database column. Figure 4 compares measured rotations with those estimated using Eqn. 2.5.

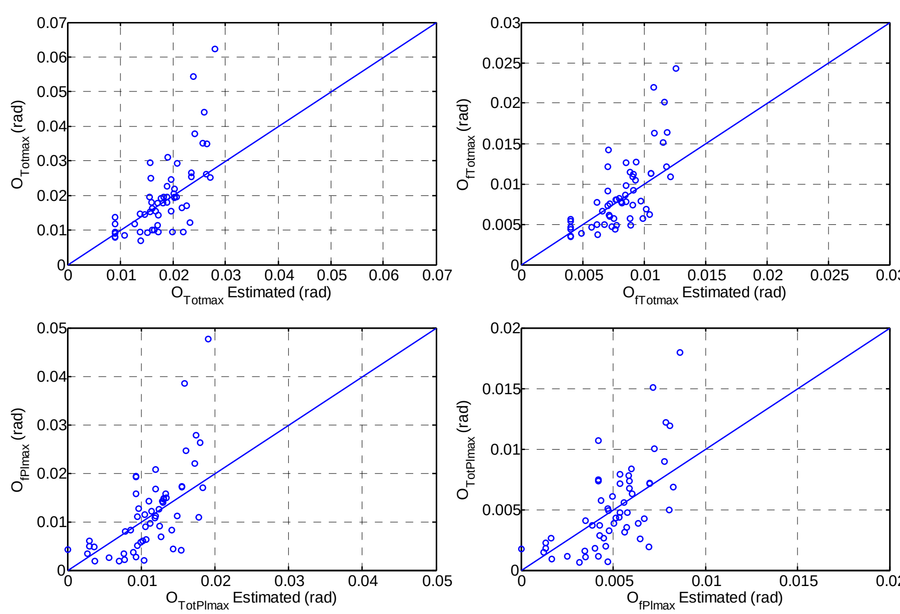

Figure 4 Shear failure initiation model estimates of rotations versus database column rotations

3. CONCLUSIONS

A new shear failure model is introduced that determines column rotations at which shear strength degradation (or shear failure) in flexure-shear-critical columns is initiated. This model is intended for use in performance-based design applications in which structural performance objectives are related to element critical-section rotations. The proposed model is based on a parametric regression analysis that was performed on a database of 56 column tests. This analysis demonstrated that column rotational capacity prior to shear failure initiation is negatively correlated with transverse reinforcement spacing, compressive axial loads, and shear stresses. The failure model is presented for four rotation measures (elastic, plastic, with and without bar-slip rotations) sat hat is can be used with most lumped-plasticity or fiber-section column analytical implementations. The model formulation with total end rotation including bar-slip rotations has the advantage over the other three model formulations of being insensitive to bar-slip modeling parameters. The 56-column database elastic stiffnesses were best matched analytically with an elastic constant anchorage bond stress $u_e=11\ f'^{0.5}_c$ (psi).

4. ACKNOWLEDGEMENTS

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