STOCHASTIC INCREMENTAL DYNAMIC ANALYSIS CONSIDERING RANDOM SYSTEM PROPERTIES

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ABSTRACT

As an extension of static pushover analysis into nonlinear dynamic analysis to estimate more thoroughly structural performance under seismic loads, incremental dynamic analysis (IDA) has been widely applied in the field of Performance-Based Earthquake Engineering (PBEE). A single-record IDA cannot fully capture the behavior that a building may display in a future earthquake event. Therefore, the multi-record IDA is developed to consider the record-to-record variations in earthquake ground motions. However, neither single-record IDA nor multi-record IDA can take into account the random system properties of structures. In this paper, a stochastic IDA method is proposed, which is a coupling of point estimation method (PEM) based on Nataf transformation for approximating the statistical moments of random functions, and single-record IDA approach. The multi-variable random IDA curve is developed from single-variable random IDA ones according to the sampling strategy in PEM, and the fractile IDA curves are also advanced. The proposed methodology is applied in R.C. frame structures. A three-bay and five-storey plane R.C. frame is taken as an example in case study. It is demonstrated by this example that the approach proposed in this paper is an efficient and accurate tool for probabilistic seismic demand and capacity analysis of structures considering the inherent random system properties.

KEYWORDS: Incremental Dynamic Analysis, Nonlinear Dynamic Analysis, Random System Property, Point Estimation Method, Performance-Based Earthquake Engineering

1. INTODUCTION

Incremental dynamic analysis (IDA) is the dynamic equivalent to a static pushover analysis. Given a structure and a ground motion, single-record IDA is done by conducting a series of nonlinear time-history analyses. The intensity of the ground motion, measured using an IM, is incrementally increased in each analysis. An engineering demand parameter (EDP), such as global drift ratio, is monitored during each analysis. The extreme values of an EDP are plotted against the corresponding value of the ground motion IM for each intensity level to produce a dynamic pushover curve, namely single-record IDA curve, for the structure and the chosen earthquake record. A single-record IDA cannot fully capture the behavior that a building may display in a future earthquake event. Therefore, the multi-record IDA has been developed to consider the record-to-record variations in earthquake ground motions. Nowadays, IDA has already been an important tool for seismic demand and capacity assessment in the field of Performance-Based Earthquake Engineering (PBEE) (Fragiadakis et al., 2006; Mandel et al., 2007). Federal Emergence Management Agency guidelines (FEMA350, FEMA351, 2000) accepted the IDA method and took it as the state-of-the art method to determine the global collapse capacity of structures.
The idea of IDA was originally proposed by Bertero (1977). From then on, Cornell and his co-workers (Bazzurro & Cornell, 1994; Luco & Cornell, 1998, 2000) have been developing and improving the IDA method. Vamvatasikos and Cornell (2002, 2004) systematically summarized the basic theory and methodology of IDA, developed an applied IDA, and proposed a simple SPO2IDA analysis method (2005, 2006).

It is well known that there exist many factors influencing structural properties, such as structural configuration, structural dynamic properties (stiffness and damping), geometric sizes of structural elements, constitutive relationships of structural materials, modeling uncertainty of structures, and so on. Therefore, the system properties of structures are random in nature. However, the prevalent IDA method, whether it's a single-record IDA or a multi-record IDA, cannot considered the random system properties of structures. To consider this kind of randomness, in this paper, a stochastic IDA method is developed, which combines point estimation method (PEM) based on Nataf transformation (Liu & Der Kiureghian, 1986) with single-record IDA method. The proposed methodology is applied in R.C. frame structures. A three-bay and five-storey plane R.C. frame is taken as an example in case study.

2. POINT ESTIMATION METHOD BASED ON NATAF TRANSFORMATION

2.1. Nataf Transformation and its Application to Estimating Statistical Moments of Random Functions

Point estimation method (PEM) was proposed by Rosenblueth (1975) to approximate the lower-order moments of functions of random variables. It is a special case of numerical quadrature based on orthogonal polynomials. For normal variables, it corresponds to Gauss-Hermite quadrature. While the point estimate method is popular in practice, it has many detractors. Numerous modifications or improvements have been made for the original PEM. However, the early developments of PEM are all undertaken in the original space of random variables, requiring the higher order moments of random variables without considering the distribution information. To overcome these shortcomings, Zhao and Ono (2000) introduced a new point estimation method based on Rosenblatt transformation (Hohenbichler & Rackwitz, 1981), so that the Gauss-Hermite quadrature can be completed in standard normal space. Unfortunately, Rosenblatt transformation needs the joint probability distribution information of random variables; actually, it is difficult to get the joint PDF of random variables in practical engineering applications. Another transformation method, namely Nataf transformation (Liu & Der Kiureghian, 1986), is popular in the fields of structural reliability and probability analysis, since it only needs the marginal probability distribution information of each random variables and the covariance between them, which is easy to get in practice. Therefore, in this paper, the Nataf transformation is introduced into Zhao-Ono point estimation method.

The forward Nataf transformation T_N can be denoted by

$$T_N : \mathbf{u} = \mathbf{L}_0^{-1}\mathbf{\Phi}^{-1}[\mathbf{F_X}(\mathbf{x})] \tag{2.1}$$

where, $\mathbf{x}$ and $\mathbf{u}$ are the realizations of n dependent non-normal random variables $\mathbf{X}$ and independent standard normal random variables $\mathbf{U}$, respectively; $\mathbf{\Phi}^{-1}()$ represents the column vector composed of all inverse functions of standard normal random variables; $\mathbf{F_X}(\mathbf{x})$ is the column vector comprised of CDFs of random variables $X_i\ (i=1,\cdots,n)$; $\mathbf{L}_0$ is the lower triangle matrix of Choleski decomposition of correlation coefficients matrix $\mathbf{R}_0$ of dependent normal random vector $\mathbf{Y}=\mathbf{\Phi}^{-1}[\mathbf{F_X}(\mathbf{x})]$, i.e. $\mathbf{R}_0=\mathbf{L}_0\mathbf{L}_0^T$; the relationships between the elements $\rho_{0,ij}$ of $\mathbf{R}_0$ and the elements ρ_{ij} of $\mathbf{R}$, the correlation coefficients matrix of $\mathbf{X}$, are

$$\rho_{0,ij} = F_{ij}\rho_{ij} \tag{2.2}$$

where, the coefficient F_{ij} is function of correlation coefficient ρ_{ij} and marginal distributions $F_{X_i}(x_i)$ and $F_{X_j}(x_j)$ of random variables X_i and X_j. In general, $F_{ij} \geq 1$. Liu and Der Kiureghian (1986) gave the practical formula for computing coefficient F_{ij} corresponding to different probability distributions.

The inverse Nataf transformation T_N^{-1} can be denoted by

$$T_N^{-1} : \mathbf{x} = \mathbf{F_X}^{-1}[\mathbf{\Phi}(\mathbf{L}_0\mathbf{u})] \tag{2.3}$$

where, $\mathbf{F_X}^{-1}(\cdot)$ represents the column vector composed of all inverse CDFs of random variables $X_i\ (i=1,\cdots,n)$; $\mathbf{\Phi}(\cdot)$ denotes the column vector comprised of all CDFs of standard normal random variables.

For a random function $Z = g(\mathbf{X})$, we can make use of the inverse Nataf transformation to estimate its statistical moments:

$$\mu_Z = \int g(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int g\left[T_N^{-1}(\mathbf{u})\right] \varphi_n(\mathbf{u}) d\mathbf{u} \tag{2.4a}$$

$$\sigma_Z^2 = \int \left[g(\mathbf{x}) - \mu_Z\right]^2 f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int \left\{g\left[T_N^{-1}(\mathbf{u})\right] - \mu_Z\right\}^2 \varphi_n(\mathbf{u}) d\mathbf{u} \tag{2.4b}$$

$$\alpha_{kZ}\sigma_Z^k = \int \left[g(\mathbf{x}) - \mu_Z\right]^k f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int \left\{g\left[T_N^{-1}(\mathbf{u})\right] - \mu_Z\right\}^k \varphi_n(\mathbf{u}) d\mathbf{u} \quad k > 2 \tag{2.4c}$$

where, μ_Z and σ_Z are mean value and standard deviation of Z, respectively; α_{kZ} is the kth dimensionless central moment of Z; $f_{\mathbf{X}}(\mathbf{x})$ is the joint PDF of $\mathbf{X}$; $\varphi_n(\mathbf{u})$ is the joint PDF of standard normal vector $\mathbf{U}$.

2.2 Point Estimation for Single-Variable Function

For a single-variable function $Z = g(X)$, Nataf transformation reduces to iso-probability marginal transformation, so Eq. (2.4) becomes

$$\mu_Z = \int g\left\{F_X^{-1}[\Phi(u)]\right\} \varphi(u) du \tag{2.5a}$$

$$\sigma_Z^2 = \int \left[g\left\{F_X^{-1}[\Phi(u)]\right\} - \mu_Z\right]^2 \varphi(u) du \tag{2.5b}$$

$$\alpha_{kZ}\sigma_Z^k = \int \left[g\left\{F_X^{-1}[\Phi(u)]\right\} - \mu_Z\right]^k \varphi(u) du \quad k > 2 \tag{2.5c}$$

The integration in Eq. (2.5) can be approximated by making use of Gauss-Hermite numerical quadrature in standard normal space:

$$\mu_Z \approx \sum_{j=1}^{m} \frac{w_j}{\sqrt{\pi}} g\left\{F_X^{-1}[\Phi(\sqrt{2}x_j)]\right\} \tag{2.6a}$$

$$\sigma_Z^2 \approx \sum_{j=1}^{m} \frac{w_j}{\sqrt{\pi}} \left[g\left\{F_X^{-1}[\Phi(\sqrt{2}x_j)]\right\} - \mu_Z\right]^2 \tag{2.6b}$$

$$\alpha_{kZ}\sigma_Z^k \approx \sum_{j=1}^{m} \frac{w_j}{\sqrt{\pi}} \left[g\left\{F_X^{-1}[\Phi(\sqrt{2}x_j)]\right\} - \mu_Z\right]^k \tag{2.6c}$$

where, $x_j (j = 1, \cdots, m)$ is the integration points of Gauss-Hermite quadrature, w_j is the weights of Gauss-Hermit quadrature, m is the number of integration points.

2.3 Point Estimation for Multi-Variable Function

For a multi-variable function $Z = g(\mathbf{X})$, two function approximation approaches can be used:

$$Z \approx g'(X) = Z_{\mu} \prod_{i=1}^{n} \left(\frac{Z_i}{Z_{\mu}}\right) \tag{2.7}$$

$$Z \approx g''(X) = \sum_{i=1}^{n} (Z_i - Z_{\mu}) + Z_{\mu} \tag{2.8}$$

in which,

$$Z_{\mu} = g(\boldsymbol{\mu}) = g(\mu_1, \cdots, \mu_i, \cdots, \mu_n) \tag{2.9}$$

$$Z_i = g\left[T_N^{-1}(\boldsymbol{u}_i)\right] = G(\boldsymbol{u}_i) = G(u_{\mu 1}, u_{\mu 2}, \cdots, u_{\mu i-1}, u_i, u_{\mu i+1}, \cdots, u_{\mu n}) \tag{2.10}$$

where, $T_N^{-1}()$ denotes inverse Nataf transformation; $\boldsymbol{\mu}$ represents the vector in which all the random variables take their mean values; $\mathbf{u}_i$ represents the vector in which only u_i is a random variable, while other variables take the corresponding transformed values of their mean values in standard normal space; $u_{\mu j} (j \neq i)$

is the *j*th element of the transformed vector $\mathbf{u}_\mu$ who corresponds the vector $\boldsymbol{\mu}$ in standard normal space $\mathbf{u}$; $G(\mathbf{u}) = g[T_N^{-1}(\mathbf{u})]$ is the formulation of random function $g(\mathbf{x})$ in standard normal space based on Nataf transformation.

Based on the product-rule as shown in Eq. (2.7), the mean value and the *k*th central moment of the function can be estimated:

$$\mu_Z \approx Z_\mu \prod_{i=1}^{n} \left(\frac{\mu_{Z_i}}{Z_\mu} \right) \tag{2.11a}$$

$$E\left[g^K(X) \right] \approx Z_\mu^K \prod_{i=1}^{n} \frac{E\left[Z_i^K \right]}{Z_\mu^K} \tag{2.11b}$$

Based on the non-product rule as shown in Eq. (2.8), the mean value and variance of the function can be estimated:

$$\mu_Z \approx \sum_{i=1}^{n} (\mu_{Z_i} - Z_\mu) + Z_\mu \tag{2.12a}$$

$$\sigma_Z^2 \approx \sum_{i=1}^{n} \sigma_{Z_i}^2 \tag{2.12b}$$

In Eqs. (2.11) and (2.12), μ_i and σ_i are mean value and standard deviation of G_i by using point-estimation of single-variable function.

3 IDA WITHOUT CONSIDERING RANDOM SYSTEM PROPERTIES

IDA is a parametric analysis method to estimate more thoroughly structural performance under seismic loads. For single-record IDA, one ground motion record is successively scaled to multiple spectral acceleration levels and the resulting maximum inter-storey drift angles are calculated in each case. The IDA curve connects the resulting inter-storey drift angles corresponding to the ground motion record. The procedure of single-record IDA without considering random system properties is summarized as follows:

Step 1. Choose one ground motion record, and select the Intensity Measure (IM) for the ground motion. There are many measures which can characterize the ground motion intensity, herein we choose the first mode spectra acceleration $Sa(T_1)$ as an IM.

Step 2. Determine the scaling rule, and adjust the levels of λ_i:

$$\ddot{u}_g^{(i)}(t) = \lambda_i a(t) \tag{3.1}$$

where, $a(t)$ is the original ground motion record, λ_i is a scaling parameter, $\ddot{u}_g^{(i)}(t)$ is the ground motion after adjusting. The scaling rule can be divided into two kinds: equal-step rule and unequal-step rule. Vamvatasikos and Cornell (2002) proposed a Hunt & Fill rule based on unequal-step rule.

Step 3. Choose an Engineering Demand Parameter (EDP). EDPs can be divided into global EDPs and local EDPs. On the other hand, EDPs can also be divided into load-based EDP, displacement-based EDP and damage-based EDP.

Step 4. Perform parametric analysis. Subject the structural model to the group of ground motions $\ddot{u}_g^{(i)}(t)$, and an IDA curve of EDP versus IM can be produced. IDA curves can be divided into global IDA curves and local IDA curves. In order to save computation resource, the technique of spline interpolation is often made use of.

For multi-record IDA, we only need to choose a group of ground motion records and repeat the steps above.

4 IDA CONSIDERING RANDOM SYSTEM PROPERTIES

4.1 Basic Methodology

In this paper, the EDP is seen as an implicit and complex function of random vector $\mathbf{X}$:

$$EDP = g(\mathbf{X}) = g(X_1, X_2, \cdots, X_n) \tag{4.1}$$

where, $g(\cdot)$ is the function which characterizes the relationship between EDP and X, and it is generally implicit; $\mathbf{X}=[X_1, X_2,...,X_n]^T$ is the basic random variables which influence structural properties, such as material strength, geometry size, structural modeling uncertainty, and so on.
Since IDA can determine the relationship between EDP and IM, if random system properties are taken into account, then the relationship between of EDP and IM will be a complex and implicit random function.
The mean value μ_{EDP} and the *n*th central moment $M_{n\text{EDP}}$ of EDP can be calculated by

$$\mu_{\text{EDP}} = \int g(\mathbf{X}) f(\mathbf{X}) d\mathbf{X} \tag{4.2}$$

$$M_{n_{\text{EDP}}} = \int [g(\mathbf{X}) - \mu_g]^n f(\mathbf{X}) d\mathbf{X} \qquad n \geq 2 \tag{4.3}$$

where, $f(\mathbf{X})$ is the joint PDF of $\mathbf{X}$.
In general, we can not directly obtain μ_{EDP} and $M_{n\text{EDP}}$ by Eq. (4.2) and Eq. (4.3) for the highly implicit function $g(\mathbf{X})$. In this paper, a kind of stochastic IDA method considering random system properties is proposed, which combines the above-mentioned point estimation method based on Nataf transformation, with the singe-record IDA method. We call it as a stochastic IDA method, which has two key issues, one of which is how to consider random system properties, the other of which is how to estimate the statistical moments of EDP.
If random system properties are considered, we will firstly determine structural random variables and their distribution information. Then, Gauss-Hermite integration points and their corresponding weights are selected. The structural sample matrix $\mathbf{u}_{ij}$ $(i=1,2,...,n;\ j=1,2,...,m)$ in standard normal space is thus obtained, where, i is the index of random variable, j is the index of Gauss-Hermite integration points, n is the number of random variables, and m is the number of integration points. And then, the inverse Nataf transformation $T_N^{-1}(\mathbf{u}_{ij})$ is made use of, and structural sample matrix $\mathbf{x}_{ij}$ $(i=1,2,...,n)\ (j=1,2,...,m)$ in general random space is obtained. If the matrix is combined with structural finite element models, then n groups and m×n structural random samples will be obtained.
The statistical moments of an EDP can be divided into two parts: single-variable statistical moments and multi-variable statistical moments. Carry on IDA for n groups of structural random samples, and then *n* groups of EDP results will be obtained. The single-variable mean value and standard variance of EDP by Eq. (2.6) is estimated for each variable. Based on the single-variable statistical moments of EDP for each variable, the multi-variable mean value and standard variance of EDP can be approximated by Eq. (2.10) and Eq. (2.11). During the analysis procedure above, a series of dispersed IDA curves are depicted, which are named as stochastic IDA curves. The flowchart of the IDA method considering random system properties is summarized in Figure 1.

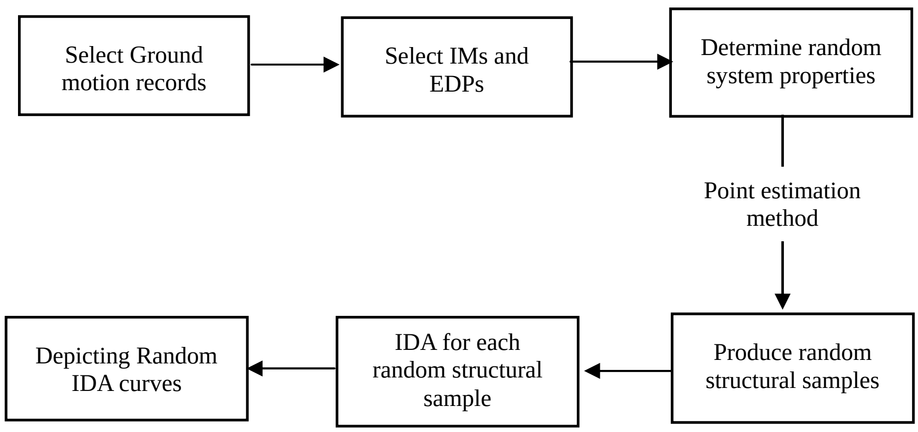

Figure 1 Flow chart of stochastic IDA method considering random system properties

4.2 Random IDA Curves

The random IDA curves by stochastic IDA method considering random system properties can be divided into three categories: single-variable IDA curves, multi-variable IDA curves, and fractile IDA curves.

4.2.1 Single-variable Random IDA Curves

Make IDA for m×n structural random samples, then *n* groups and m×n IDA curves will be obtained. Based on the single-variable statistical moments of EDP, *n* single-variable IDA curves are attained. As a result, *n*+1 IDA curves can be gained for each random variable, which are named as single-variable random IDA curves. The dispersion of single-variable random IDA curves reflects the individual effects of the basic random variable for structural dynamic responses.

4.2.2 Multi-variable Random IDA Curves

Based on the multi-variable statistical moments of EDP, multi-variable mean IDA curve will be gained. Combine it with *n* single-variable mean IDA curves obtained above, then multi-variable random IDA curves can be acquired. The dispersion of multi-variable random IDA curves reflects the total effects of random system properties on structural dynamic responses.

4.2.3 Fractile IDA Curves

Through IDA considering random system properties, the mean value μ_{EDP} and standard variation σ_{EDP} of EDP have been known. Then the logarithmic mean value λ_{EDP} and logarithmic standard variance ζ_{EDP} of EDP can be computed by

$$\lambda_{\mathrm{EDP}} = \ln\left(\frac{\mu_{\mathrm{EDP}}}{\sqrt{1+\delta_{\mathrm{EDP}}^2}}\right) \tag{4.4}$$

$$\zeta_{\mathrm{EDP}} = \sqrt{\ln\left(1+\delta_{\mathrm{EDP}}^2\right)} \tag{4.5}$$

where, $\delta_{\mathrm{EDP}} = \sigma_{\mathrm{EDP}} / \mu_{\mathrm{EDP}}$ is the COV of the EDP. EDP is often assumed to submit to log-normal distribution (Cornell, et al, 2002), as a result, it's easy to draw 84% fractile IDA curve, 16% fractile IDA curve and 50% fractile IDA curve, respectively, such that

$$\Phi\left(\frac{\ln \mathrm{EDP}_{50\%} - \lambda_{\mathrm{EDP}}}{\zeta_{\mathrm{EDP}}}\right) = 50\% \tag{4.6a}$$

$$\Phi\left(\frac{\ln \mathrm{EDP}_{84\%} - \lambda_{\mathrm{EDP}}}{\zeta_{\mathrm{EDP}}}\right) = 84\% \tag{4.6b}$$

$$\Phi\left(\frac{\ln \mathrm{EDP}_{16\%} - \lambda_{\mathrm{EDP}}}{\zeta_{\mathrm{EDP}}}\right) = 16\% \tag{4.6c}$$

where, $\mathrm{EDP}_{50\%}$, $\mathrm{EDP}_{84\%}$ and $\mathrm{EDP}_{16\%}$ are the values of EDP on 50%, 84% and 16% fractile IDA curves.

Because of Eq. (4.6), we can get $\mathrm{EDP}_{50\%}$, $\mathrm{EDP}_{84\%}$ and $\mathrm{EDP}_{16\%}$ by

$$\mathrm{EDP}_{50\%} = \exp(\lambda_{\mathrm{EDP}}) \tag{4.7a}$$

$$\mathrm{EDP}_{84\%} = \exp(\lambda_{\mathrm{EDP}} + \zeta_{\mathrm{EDP}}) \tag{4.7b}$$

$$\mathrm{EDP}_{16\%} = \exp(\lambda_{\mathrm{EDP}} - \zeta_{\mathrm{EDP}}) \tag{4.7c}$$

Based on Eq. (4.7b) and Eq. (4.7c), we can compute ζ_{EDP} by

$$\zeta_{\mathrm{EDP}} = \frac{1}{2}\left[\ln \mathrm{EDP}_{84\%}(x) - \ln \mathrm{EDP}_{16\%}(x)\right] \tag{4.8}$$

ζ_{EDP} is the logarithmic deviation of EDP, it can show how much the random system properties affect the structural seismic performance. As a result, the dispersions of three fractile IDA curves are helpful to evaluate the influence of the random system properties on structural dynamic responses.

5 APPLICATION OF THE METHODOLY TO A R.C. FRAME STRUCTURE

5.1 Basic Data

A five-storey and three-bay R. C frame is taken as an example. It is designed according to Chinese seismic design code of Buildings (GB50011-2001). The load distributions and the elevation of the structure are shown in Figure 2.

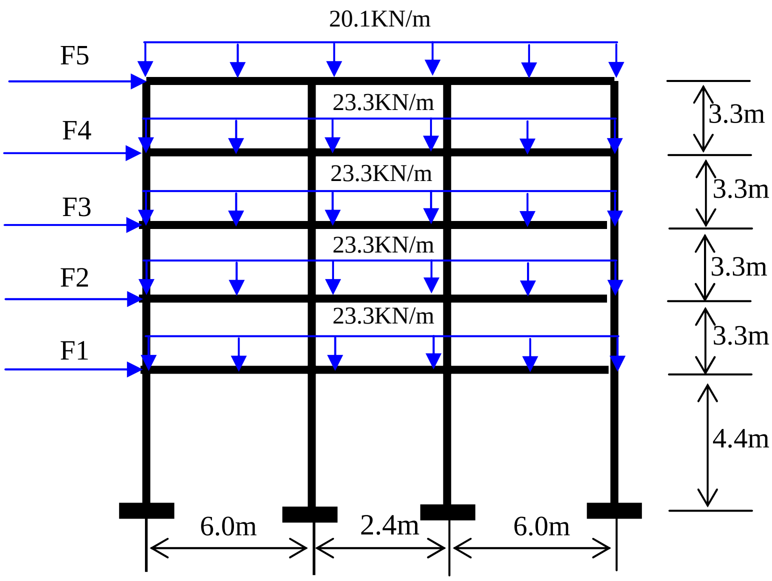

Figure 2 Five-storey and three-bay R.C. frame

Table 5.1 Statistics of random variables

Variable	Distribution parameters		Variable	Distribution parameters	
	Mean value	Variance value		Mean value	Variance value
f_y(N/mm^2)	384.80	28.59	f_{cr}(N/mm^2)	27.32	4.44
E(N/mm^2)	204000	2040	ε_c	0.0022	0.000308
f_c(N/mm^2)	26.10	4.44	ε_{cu}	0.021	0.00274

Table 5.2 Random samples of the structure

Random variables	Structural samples						
	1	2	3	4	5	6	7
f_y(N/mm^2)	384.80	417.80	351.80	452.46	317.14	492.02	277.58
E(N/mm^2)	204000	206355	201645	208828	199172	211651	196349
f_c(N/mm^2)	26.10	31.23	20.97	36.61	15.59	42.75	9.45
f_{cr}(N/mm^2)	27.32	32.44	22.19	37.83	16.81	43.97	10.67
ε_c	0.0022	0.0026	0.0018	0..0029	0.0015	0.0034	0.0010
ε_{cu}	0.021	0.024	0.018	0.027	0.015	0.031	0.011

El Centro ground motion record is selected as the ground motion record, its original intensity measures are: PGA= 0.29g； Sa(T_1, 5%) = 0.67g. Sa(T_1, 5%) is chosen as the IM herein.

For convenience, only random material properties are considered, which include six basic random variables: yield strength f_y and initial elastic modulus E of steel; compressive strength f_c, crushing strength f_{cr}, strain at compressive strength ε_c, strain at crushing strength ε_{cu} of concrete. The statistics of the variables are listed in Table 5.1. All variables are assumed to follow normal distribution, and to be independent on each others. Seven estimation points are chosen. The random samples of the structure in general random space are generated as shown in Table 5.2. Combine the matrix with the structural finite element model, $7\times6=42$ structural random samples are obtained.

5.2 Single-variable Random IDA Curves

Six groups of single-variable random IDA curves are shown in Figures 3 to 8.

5.3. Multi-variable Random IDA Curves

The multi-variable random IDA curves are shown in Figure 9. From Figure 9 we can see that the mean IDA curves by product-rule and by non-product rule are approaching to each other. However, the multi-variable mean IDA curves are in different shapes, compared with the single-variable mean IDA curves. Therefore, it's necessary to investigate the mean performance by IDA method considering random system properties.

5.4. Fractile IDA Curves

The fractile IDA curves are shown in Figure 10. From Figure 10, we can derive two conclusions. First, before structural collapse, fractile IDA curves get close, so the random material properties don't influence the response of the structure very much. It's valid to make IDA without considering random material properties. Second, when structure is approaching the state of collapse, fractile IDA curves disperse widely, therefore, it's necessary to investigate structural collapse considering random material properties.

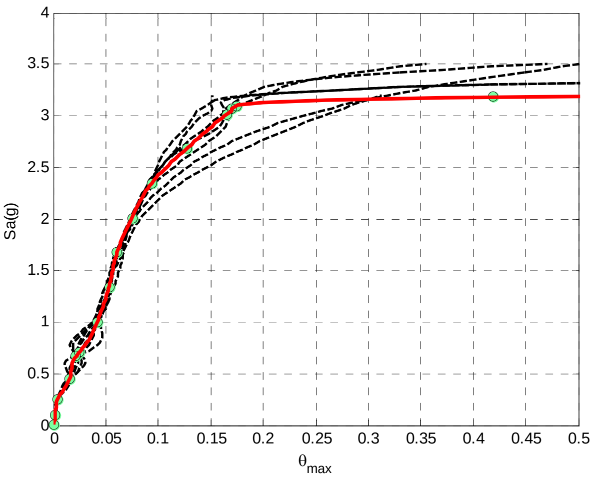

Figure 3 Random IDA curves for f_y

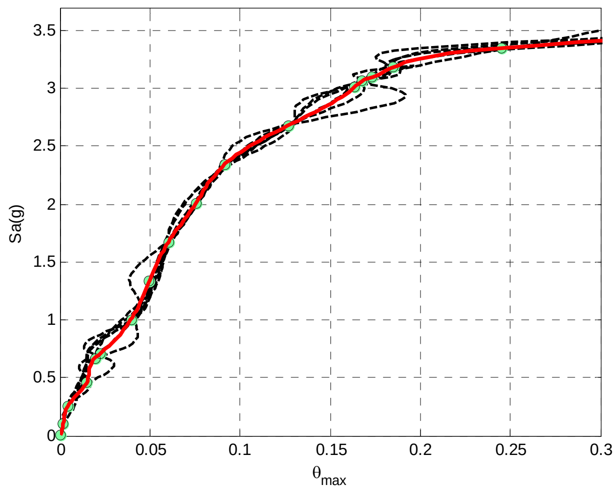

Figure 4 Random IDA curves for E

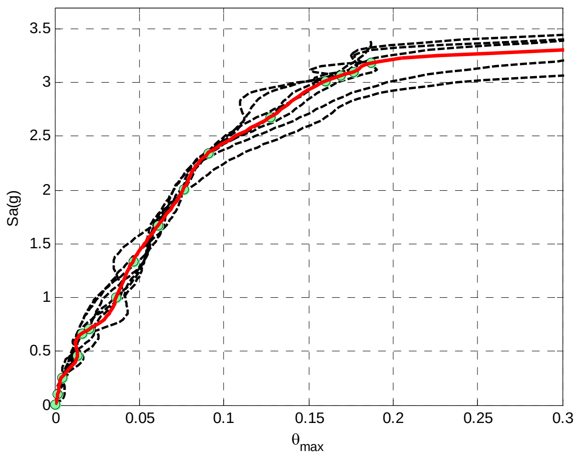

Figure 5 Random IDA curves for f_c

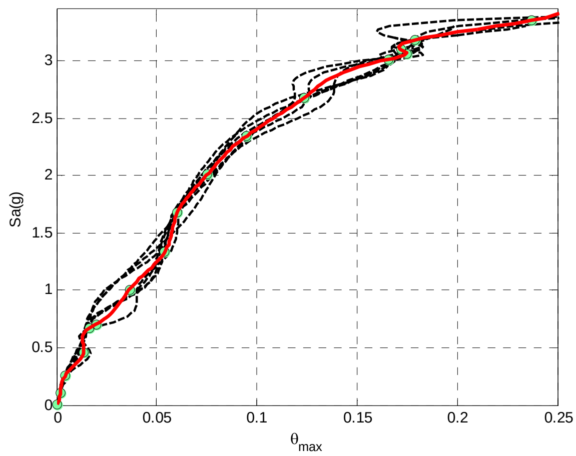

Figure 6 Random IDA curves for f_{cr}

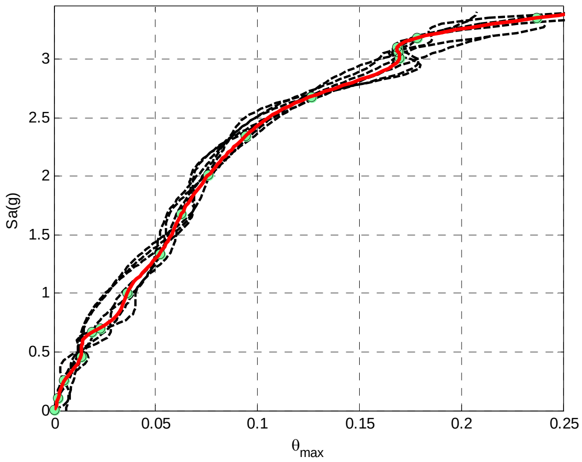

Figure 7 Random IDA curves for ε_c

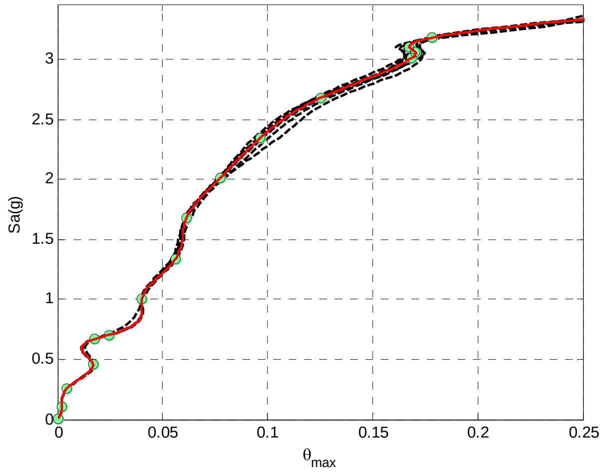

Figure 8 Random IDA curves for ε_{cu}

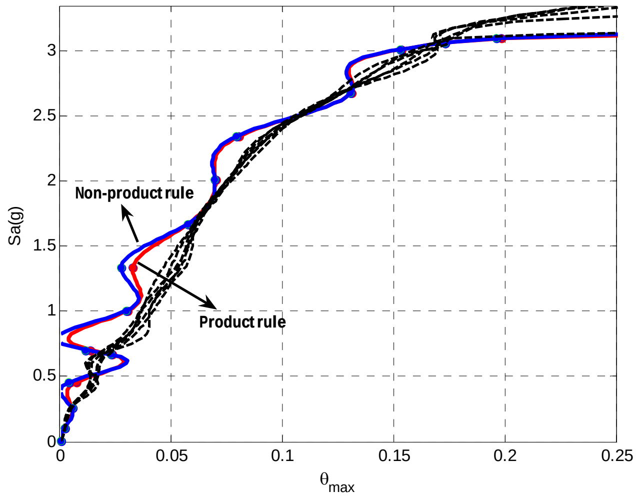

Figure 9 Multi-variable random IDA curves

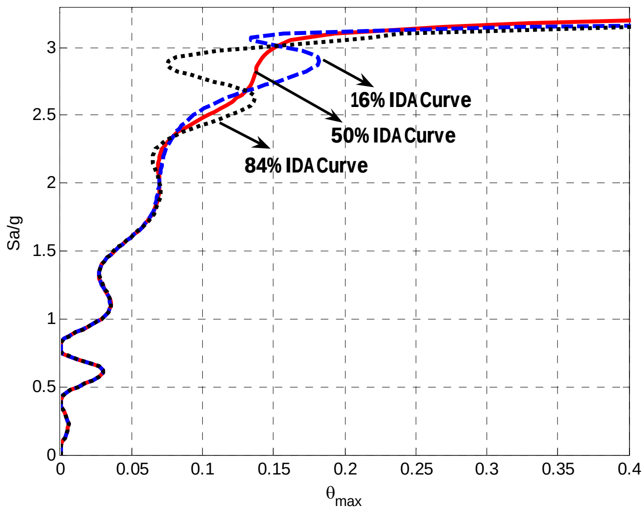

Figure 10 Fractile IDA curves

6. CONCLUSION

In this paper, a stochastic IDA method combining point estimation method based on Nataf transformation with single-record IDA approach is proposed. The methodology is applied to a five-storey and three-bay R.C frame structure considering random material properties. The conclusions are summarized as follows.

(1) The yield strength f_y of Steel and the compressive strength f_c of concrete has great influences on the dynamic responses of the structure.

(2) Before structural collapse, the random material properties don't influence structural responses very much. It's valid to make IDA without considering structural material random property in this case. When structure is approaching the state of collapse, fractile IDA curves disperse widely, so it's necessary to study structural collapse considering random material properties.

(3) The proposed method doesn't consider the randomness in ground motions. It is necessary to make further investigation on IDA considering both ground motion record-to-record variations and random system properties of structures.

7. ACKNOWLEDGEMENTS

The support of National Science Foundation of China through projects (Grant No. 90715021, 50678057, 50108005) is greatly appreciated. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the National Science Foundation of China.

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