HYSTERETIC MODELS FOR REINFORCED CONCRETE COLUMNS CONSIDERING AXIAL-SHEAR-FLEXURAL INTERACTION

Shi-Yu Xu[1] **and Jian Zhang**[2]

[1] *Graduate Student Researcher, Dept. of Civil & Environmental Engineering, UCLA, Los Angeles, USA*
[2] *Assistant Professor, Dept. of Civil & Environmental Engineering, UCLA, Los Angeles, USA*
Email: xushiyu@ucla.edu , zhangj@ucla.edu

ABSTRACT:

The seismic evaluation of RC bridges requires a column model capturing the nonlinear behavior and the induced damage due to the combined effects of axial-shear-flexural interaction under earthquakes. In this paper, two hysteretic models are developed to model the flexural and shear responses of columns respectively. The models consist of two distinctive primary curves and a set of loading and unloading rules to describe the cyclic behavior of columns including pinching, stiffness softening, and strength deterioration etc. The shear and flexural responses are coupled together at the element level through equilibrium. The primary curves are generated based on MCFT for constant axial load. The coupled axial-shear-flexural interaction model is implemented as a user element in the FE program, ABAQUS. The model is validated against the experimental results from static cyclic pushover tests and dynamic shaking table tests. The analytical results showed excellent agreement with the experimental results. The model can be used to evaluate the realistic seismic response of a bridge system.

KEYWORDS: hysteretic model, axial-shear-flexural interaction, nonlinear analysis, columns

1. INTRODUCTION

Columns are generally the most critical components in RC bridges whose failure will endanger the public safety and result in expensive repair cost. For the engineers to design the RC columns with sufficient capacity and ductility, a precise estimation of the seismic demands on columns must be made, requiring a deliberate element model considering the strength deterioration and stiffness degrading due to loading cycles, pinching behavior resulted from the crack opening and closing during loading reversals, as well as the interactions among the axial load, shear forces, and bending moments. Therefore, the adopted element model must take into consideration both the material damage and the axial-shear-flexural interaction (ASFI) in order to accurately predict the response of columns.

Either concentrated hinge model or fiber section formulation can be used to model the columns. The former uses a combination of translational and rotational springs to simulate the shear and flexural responses of the columns at the element level. The latter controls the element responses directly at the material level. The concentrated hinge model accumulates the total or the nonlinear inelastic deformation of a RC element at its critical sections (e.g., the plastic hinge area at two ends) and use equivalent springs representing different sub-elements accounting for flexural, shear, and bond slip deformation to model the force-displacement relationship of the entire member (D’Ambrisi and Filippou, 1999). The model usually consists of a primary curve (or envelope of the hysteretic loops) and a set of unloading and reloading rules to control the response of each sub-element. They are empirical and approximate. The model accuracy relies on how the primary curves are defined, the elaborateness of the unloading and reloading rules applied, and on the reasonableness of the assumptions the models are based. However, the coupling of the axial, shear, and flexural responses is difficult to be included in these models due to the fact that primary curves and unloading/reloading rules of sub-elements are predefined as the element properties, which can not be easily adjusted during an analysis according to the responses in the other sub-elements. Another difficulty for most existing models lies in the discontinuities at some critical force levels or displacement levels owing to the model defects, resulting in convergence issues due to numerical instability. Nevertheless, the concentrated hinge models are popular among structural engineers because they are relatively easy to be implemented and yield acceptable results.

The fiber section formulation discretizes a RC section into a large number of fibers, whose material behavior and loading history is individually defined. This approach relates the deformation of the material to the applied external loads from the constitutive law directly, hence can model the axial-flexural interaction of the RC column at the material level. Classical flexural fiber section models (Spacone et al., 1996) do not include shear deformation and shear capacity degrading so that they failed to capture very well the responses of the shear dominate RC members under cyclic loadings. To overcome this deficiency, Petrangeli et al. (1999) introduces the shear strain field and the lateral strain field, in addition to the traditional axial strain field and section curvature field, to make the new fiber element capable of accounting for the complex interactions among the axial force, shear forces, and bending moments. The fiber model is computationally demanding and it is typically implemented as a force-based element for better accuracy making it difficult to be implemented in prevailing finite element (FE) programs, which are often consistent with the displacement-based theory. Although it is possible to obtain the stiffness matrix of the force-based fiber element by inversing its flexibility matrix, this stiffness matrix can induce some error or cause convergence problems in nonlinear analyses if it is inverted from the flexibility matrix of the previous time increment due to the fact that the forces and moments of the current time increment are unknown variables in the displacement-based FE programs.

An axial-shear-flexural interaction (ASFI) scheme and two hysteretic models for flexural and shear reversals in RC columns, taking into consideration the axial load variation, the strength deterioration and stiffness degrading, and the pinching behavior, are proposed in this paper. The proposed scheme and hysteretic models have been implemented as a user element in a displacement-based FE program, ABAQUS. Comparisons between the analytical and the experimental results of a number of cyclic and shaking table tests of columns are made. The satisfactory matching of the data not only validates the developed user element but also demonstrates the feasibility of modeling the ASFI in the displacement-based FE programs.

2. AXIAL-SHEAR-FLEXURAL INTERACTION (ASFI) SCHEME

Ozcebe and Saatcioglu (1989) reported that shear displacement can be significant even if a RC member is not governed by shear failure. They also indicated that RC members with higher shear strength than flexural strength do not guarantee an elastic behavior in shear deformation. Based on their observation, RC members controlled by flexural behavior (as is the case in most of the current RC design codes) may still have significant shear displacement which goes into the inelastic stage and thus should not be left ignored. ElMandooh Galal and Ghobarah (2003) further pointed out that the dynamic variation of axial force in RC columns will cause significant change in the lateral hysteretic moment-curvature relationship and consequently the overall structural behavior. These observations dictate the importance of including nonlinear ASFI in the analysis of RC columns.

2.1 Deficiency of Current Prevailing Numerical Models

Table 1 summarizes the details of the tested columns used for validation in this paper. The first three are from static cyclic pushover experiments while the last one is from a shaking table test. Two common modeling techniques for nonlinear analysis of RC columns are examined. Fig. 1 (left) compares the experimental results (blue lines) of column TP-021 (Yoneda et al., 2001) and the simulated results (red lines) from a commercial FE program, ABAQUS, using the Timoshenko beam elements with nonlinear moment-curvature relationship and constant shear stiffness. Fig.1 (right) compares the experimental results (blue lines) of the same column and the simulated results (red lines) from an open source FE program, OpenSees, using the force-based fiber element formulation aggregated with linearly elastic shear modulus. It is shown that the result from the first model by capturing the nonlinear flexural behavior of columns is far away from the realistic column response while the second model by capturing the axial-flexural interaction, provides a much improved prediction yet still fails to capture the often observed pinching behavior and strength deterioration due to cyclic loading reversals.

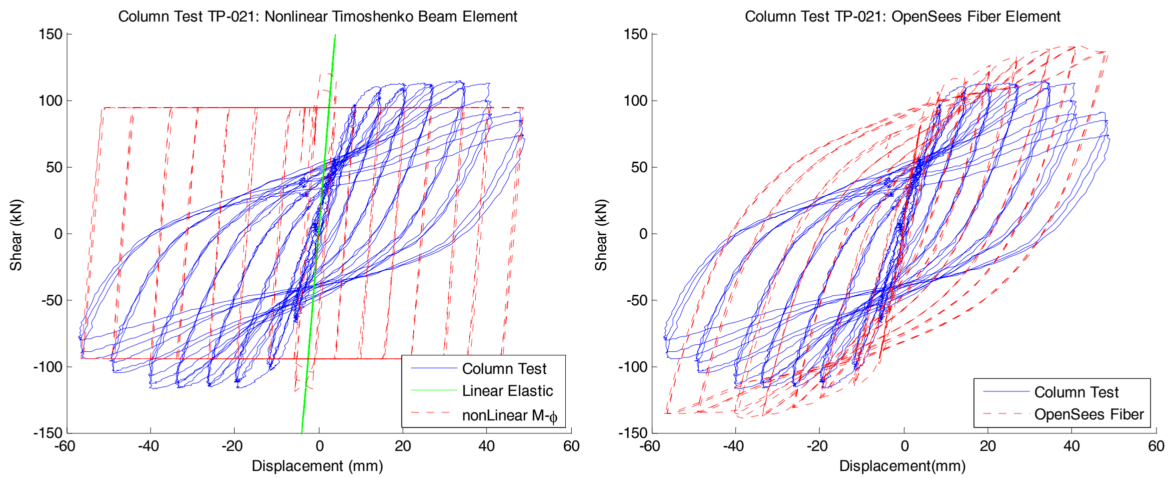


Figure 1 Analytical hysteretic responses using nonlinear Timoshenko beam and fiber element for TP-021

Table 1 Specimen geometry, reinforcement, material properties, and applied load of the examined column tests

Column Index	Column Size (mm)	Column Height (mm)	Number of Steel Rebars	Longitud. Steel Diameter	Transverse Steel Diameter	Longitud. Reinforce. Ratio	Transverse Reinforce. Ratio	fy (MPa)	fc' (MPa)	Axial Load (kN)	Axial Load Ratio
TP-021	400 circ.	1350	12	16	6	1.89 %	0.26 %	374	30.0	185	5.0 %
TP-031	400x400	1350	20	13	6	1.58 %	0.79 %	374	22.9	470	12.8 %
TP-032	400x400	1350	20	13	6	1.58 %	0.79 %	374	23.0	-170	-4.6 %
UNR-9F1	406.4 circ.	1828.8	20	12.7	6.35	1.95 %	1.00 %	448	37.4	355.86	10.0 %

2.2. Proposed Axial-Shear-Flexural Interaction Scheme

To mend for the deficiencies in the current models, an analysis scheme is developed to include the nonlinear ASFI. This scheme couples the axial force, shear forces, and bending moments at the section level, similar to the fiber section formulation, and produces much improved results. The basic idea is to introduce the ASFI into the scheme through the primary curves, which can be perceived as the constitutive law of the RC element. The total primary curve of a column is equivalent to its monotonic pushover curve by considering the combined effects of axial, shear and moment loads. It is broken into a flexural and a shear primary curve that can be applied to the flexural and shear springs in the proposed scheme. This is achieved by adopting the modified compression field theory (MCFT) (Vecchio and Collins, 1988). Given the geometry of the target RC section, the reinforcement configuration, the material properties, and the applied external loads, MCFT can yield the moment-curvature ($M-\phi$) and the shear force-shear strain ($V-\gamma$) relationships of the section subject to the given loading conditions. In a cantilever column, although the axial and shear forces along the element might be the same, the induced bending moment at each section is different. Therefore, the derived $M-\phi$ and $V-\gamma$ curves are different due to the varying combinations of axial, shear and moment loads at each section. The flexural deformation (Δ_f) and shear deformation (Δ_s) can then be obtained by integrating the curvature and shear strain in each section along the column. Subsequently, one can obtain the bending moment-to-rotation angle (M-θ) and shear force-to-shear displacement (V-Δ_s) relationships. They can be regarded as the primary curves for the flexural and shear springs respectively. It should be noted that the MCFT is a force-based approach which will stop once the peak strength of the section is reached (i.e., starting to undergo some softening). To estimate the descending branch of the primary curves, empirical equations for flexural and shear displacement can be used as alternatives (Sezen, 2008). The detailed analytical procedure is illustrated in Fig.2. If the inflection point of a RC column is known (or simply assuming to be at the mid height of the column), the column can be broken at its inflection point into two cantilever columns and simulated by a rigid bar and a combination of flexural sub-elements (F-UELs) and shear sub-elements (S-UELs), as demonstrated in Fig.2. The primary curves for the upper springs and lower springs can be the same if the inflection point is at the middle of the column, or can be different if not.

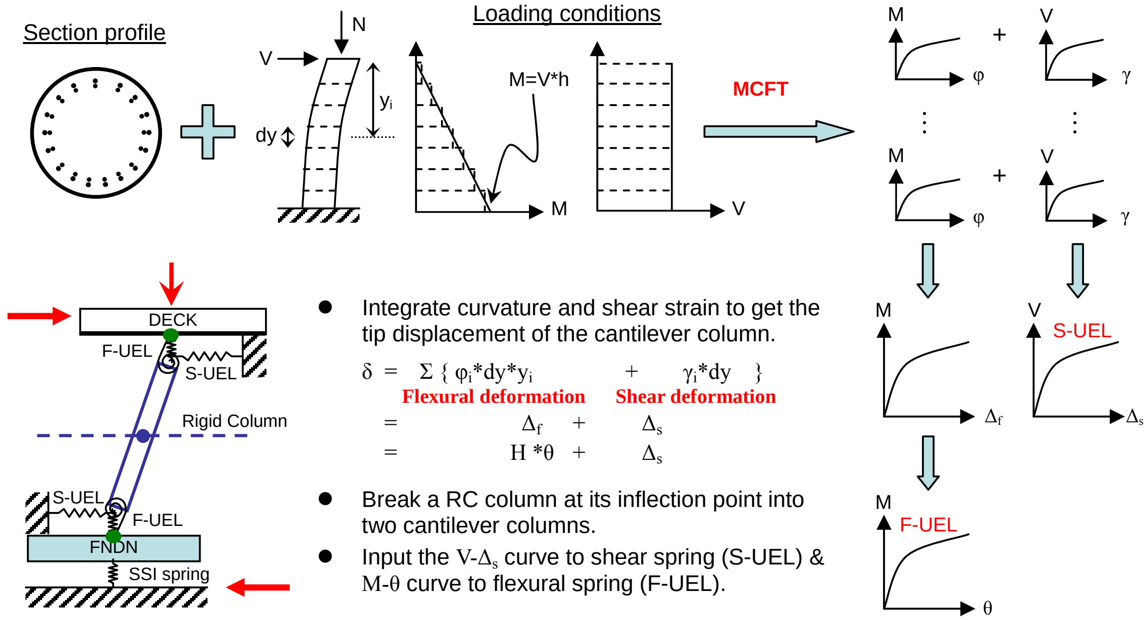


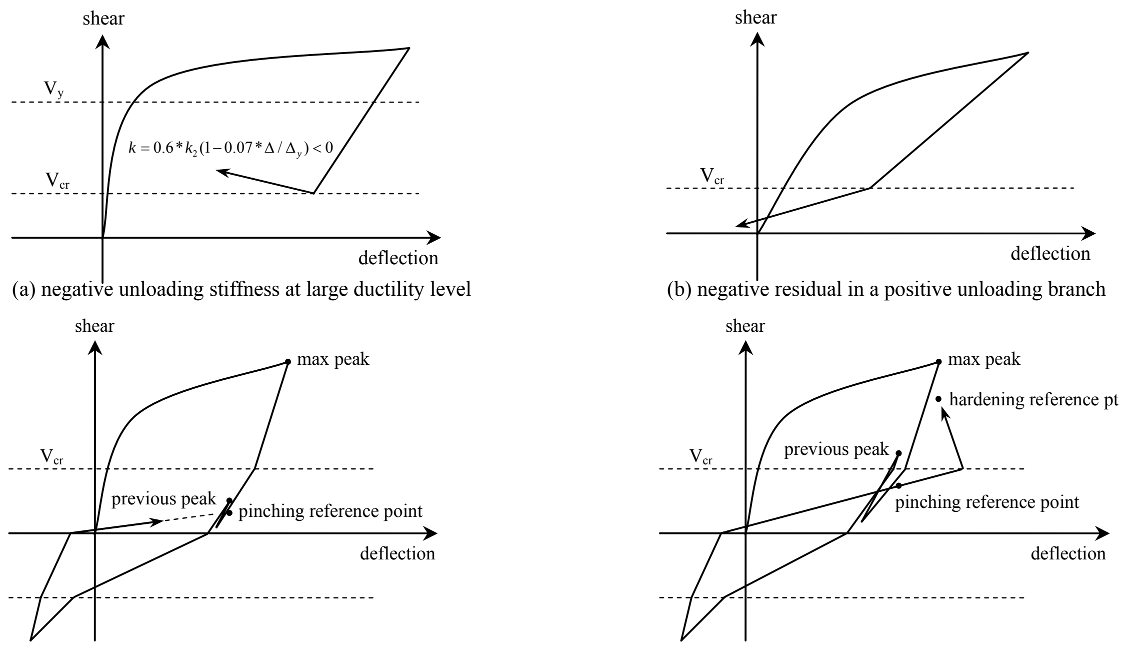
Figure 2 Implementation of the ASFI scheme

3. HYSTERETIC MODELS

3.1. Review of Existing Hysteretic Models

The most well-known flexural hysteretic model is probably the Takeda model (Takeda et al. 1970), whereas Qzcebe and Saatcioglu's model (1989), taking into consideration the axial load variation, the strength deterioration, the stiffness degrading, and the pinching behavior, is pioneering for estimating the shear responses of RC members. To minimize the work of computational coding for the flexural and shear spring subroutines, it is preferred to compose the subroutines for both sub-elements using the same numerical model framework, with the least amount of modification made to each sub-element to account for its specific unloading/reloading characteristics. For this reason, the Ozcebe and Saatcioglu's shear model is selected as the basis, combined with the Takeda model and some improvements, to make up two new hysteretic models for the flexural and shear responses.

The aforementioned models depict the general rules for flexural and shear reversals; however, to put the models into FE applications, the potential discontinuity resulted from some specific loading conditions must be precluded. According to the equations in Qzcebe and Saatcioglu's model, the unloading stiffness will become zero or negative when the ductility level is equal to or greater than 14.29. Those equations for unloading stiffness must be revised first to allow for possible larger ductility levels. In addition, depending on the shape of the primary curve and the locations of crack and yield points on it, it is possible in their model that the residual displacement of a positive unloading branch turns out to be negative, and vice versa, due to an inadequate small unloading stiffness. To fix this problem, a minimum unloading stiffness originated from the idea in Takeda's model is applied in the proposed model. Moreover, when the element is reloaded previously from the opposite side, the pinching stiffness is controlled by a reference point, which is a fraction of the peak shear of previous unloading branch. If this peak shear is very small, the calculated pinching stiffens will be very close to zero and thus not reasonable. Finally, for the cases when the "pinching reference point" falls below the cracking shear level, resulting in a very large crack closing displacement, the hardening stiffness might become negative and cause serious errors. All of these model defects will induce convergence problems in the FE programs and fail all the analyses accounting for ASFI. Fig. 3 depicts the aforementioned model defects graphically.



(a) negative unloading stiffness at large ductility level

(b) negative residual in a positive unloading branch

(c) nearly-zero pinching stiffness

(d) negative hardening stiffness

Figure 3 Examples of model defects in Ozcebe & Saatcioglu's shear hysteretic model.

3.2. Proposed Shear and Flexural Hysteretic Models

The problem with Ozcebe & Saatcioglu's shear reversal model of yielding a zero or negative stiffness when ductility level is equal to or greater than 14.29 is fixed by introducing a new set of stiffness degrading equations. For shear model, the revised unloading stiffness above the crack shear level is given by Eqn. 3.1, and below by Eqn. 3.2. The new equations extend the maximum allowable ductility level to 50.0, while maintaining the stiffness very close to Ozcebe and Saatcioglu's original model under low ductility level. Constant k_2 in the revised equations is the same as those defined in the original model and is illustrated in Fig.4. Comparisons of the normalized unloading stiffness between the revised and original equations are displayed in Fig.5.

$$k_{uld1} = k_2 * 1.4 * e^{-0.35*(\Delta/\Delta_y)^{0.01}} * (1 - 0.02 * \Delta / \Delta_y)^{3.5} \qquad (3.1)$$

$$k_{uld2} = 0.6 * k_2 * 1.3 * e^{-0.35*(\Delta/\Delta_y)^{0.01}} * (1 - 0.02 * \Delta / \Delta_y)^{5.5} \qquad (3.2)$$

Similar equations for flexural reversal model is also proposed and calibrated with over 10 static cyclic pushover tests data. For unloading above the crack moment level, the stiffness is given by Eqn. 3.3, and below by Eqn. 3.4. Since there is no significant pinching behavior in the flexural response, the pinching stiffness in the original shear model is replacement by Eqn. 3.5 in the proposed moment reversal model, subjected to a minimum stiffness k_5. As for the stress hardening branch (i.e., reloading above M_{cr}), the hardening reference point, accounting for the strength deterioration due to loading cycles, is a fraction of the maximum peak point, and given by Eqn. 3.6.

$$k_{uld1} = k_2 * 1.2 * e^{-0.125*(\theta/\theta_y)^{0.25}} * (1 - 0.016 * \theta / \theta_y)^{3.5} \qquad (3.3)$$

$$k_{uld2} = 0.70 * k_2 * 1.2 * e^{-0.125*(\theta/\theta_y)^{0.35}} * (1 - 0.020 * \theta / \theta_y)^{4.5} \qquad (3.4)$$

$$k_p = 0.56 * k_2 * 1.2 * e^{-0.125*(\theta/\theta_y)^{0.35}} * (1 - 0.020 * \theta / \theta_y)^{4.5} \qquad (3.5)$$

$$M_p' = M_p * e^{[-0.002\sqrt{\theta_m/\theta_y} * n - 0.010\sqrt{n} * (\theta_m/\theta_y)]} \qquad (3.6)$$

where n = number of cycles in one direction at its maximum rotation level, θ_m
M_p = the bending moment on primary curve corresponding to θ_m

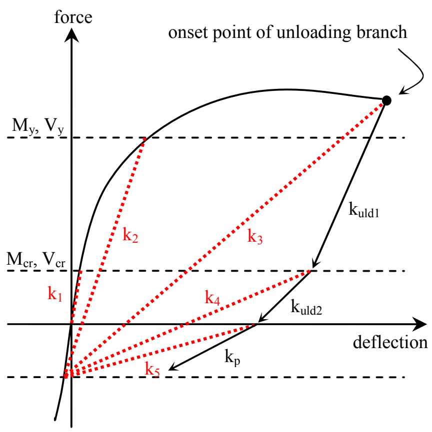


Figure 4 Reference stiffness

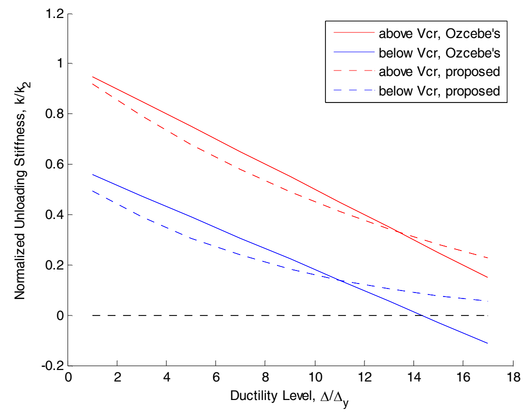
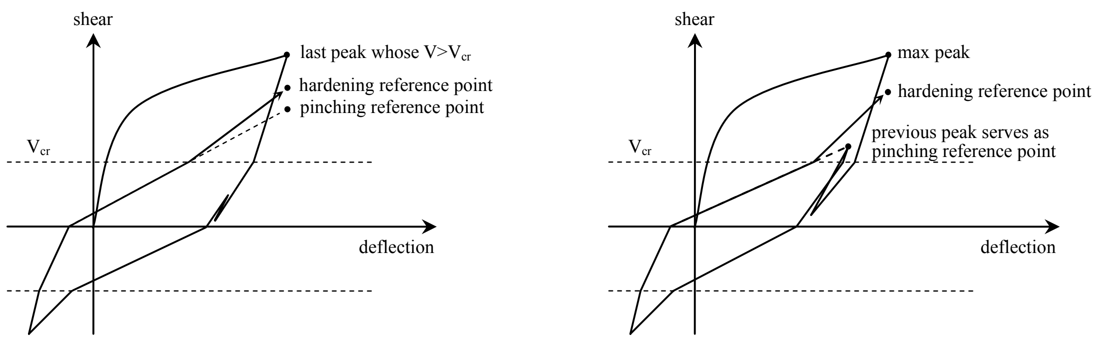


Figure 5 Revised vs original unloading stiffness

Minimum unloading stiffness, k_3, k_4 and k_5, applied to both hysteretic models to prevent the extraordinary flat unloading slopes are demonstrated in Fig.4. In Fig.4, k_3 is the slope connecting the onset point of unloading branch to the crack point on the opposite side; k_4 is the slope connecting the point on the current unloading branch at the crack force level to the crack point on the opposite side; and k_5 is the slope connecting the point on the current unloading branch at the zero force level to the crack point on the opposite side. Unlike k_1 and k_2, which are constants, k_3, k_4 and k_5 are variables depending on the location of the onset of current unloading branch.



(a) fix for nearly-zero pinching stiffness (b) fix for negative hardening stiffness

Figure 6 Remedies for the defects in shear hysteretic model

To prevent the error of nearly-zero pinching stiffness in the shear hysteretic model from occurring, an IF-statement is added into the subroutines to check that when the peak shear of previous unloading branch is smaller than the crack shear, the last peak point whose shear is larger than the crack shear level should be used as the "previous peak" instead. Regarding the fourth source of aforementioned model defects, a make-up rule is applied that should the "pinching reference point" fall below the crack shear level, the "previous peak" is enforced to serve as the "pinching reference point". This additional make-up rule is appropriated, because more than 95% of possibility this specific case takes place at the locations where the unloading shear forces barely exceed the crack load. For the rest 5% of cases in which the column is seriously damaged or it undergoes considerably softening so that the pinching reference point falls below the crack shear level, the make-up rule should not be applied and can be opted out by putting in another IF-statement. The FE program, however, in such cases will fail very soon due to convergence difficulty, and it can be perceived as the failure of the RC column. Remedies for these two shear model defects are illustrated in Fig.6.

4. MODEL VERIFICATION

The proposed hysteretic flexural and shear models are implemented as a single user element in ABAQUS. Although the axial force, shear forces, and bending moments seem to be coupled only in the primary curves, they do have interaction with each other in the hysteretic responses because the global and local equilibrium must be hold at any time. In this research, to focus on the accuracy of proposed hysteretic models, the axial force is kept constant for simplification purpose. The effects of axial load variation can later be included in again, through the idea introduced by Lee and Elnashai (2002) with some modifications.

The analytical results obtained from ABAQUS adopting the ASFI scheme and the developed user element is compared with the static cyclic pushover tests and a dynamic shake table test listed in Table 1. Fig. 7 compares the simulated and experimental results of cyclic tests of columns TP-031 and TP-032 (Sakai and Kawashima, 2000). These two tests are essentially identical except different axial loads applied. Specimens TP-031 and TP-032 were subjected to a compression of 12.8% and a tension of 4.6% of gross compressive strength of the section respectively. The experiment results showed that the axial load variation in the column has a significant influence on the lateral hysteretic responses. Dynamic validation of the models is demonstrated in Fig.8 by comparing the simulated and experimental results of column UNR-9F1 (Laplace et al., 1999) under the intensity of 2.5 times the 1941 El Centro earthquake record. This is the sixth stage of the multi-event of earthquake simulation (9 earthquakes in a series) with increasing motion intensity ranging from 0.33 to 4.0 times the ground motion of the 1941 El Centro earthquake record. The excellent agreement between the predicted and experimental results shown in Fig. 7 and 8 validate that the developed user element is capable of modeling the axial-shear-flexural interaction behavior of columns under either static cyclic or dynamic loadings.

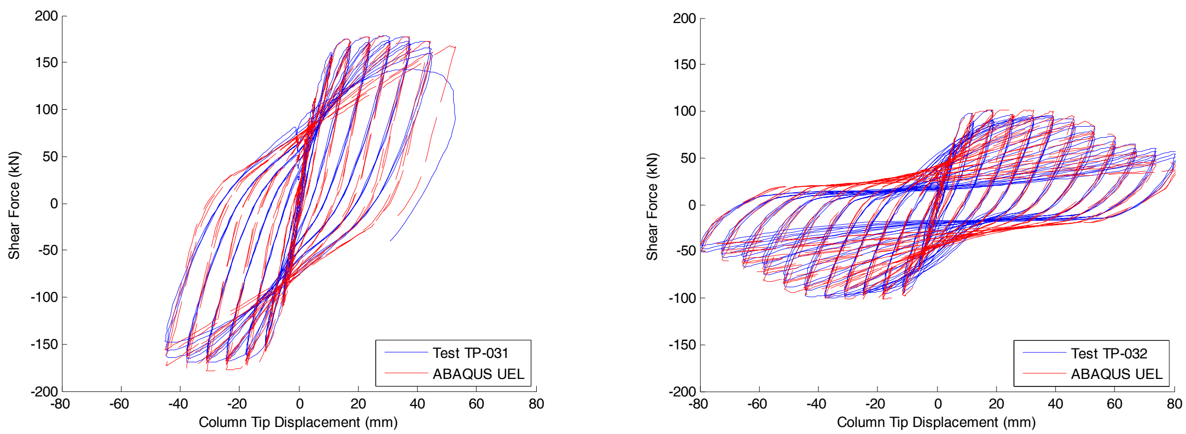


Figure 7 Hysteretic responses of column tests TP-031 (left) and TP-032 (right).

5. CONCLUSIONS

In this paper an ASFI scheme is proposed and two hysteretic models are developed to represent the shear and flexural responses of columns. The proposed ASFI scheme and hysteretic models have been implemented as a user element in the FE program, ABAQUS. Comparison of the analytical results with the experimental data shows that the proposed models and the developed user element are capable of modeling the responses of RC columns very well under either static or dynamic loadings. The comparisons also demonstrate the significant effects of axial load variation on the lateral responses of RC columns, which support the argument that ASFI is very important in the nonlinear analysis of RC columns.

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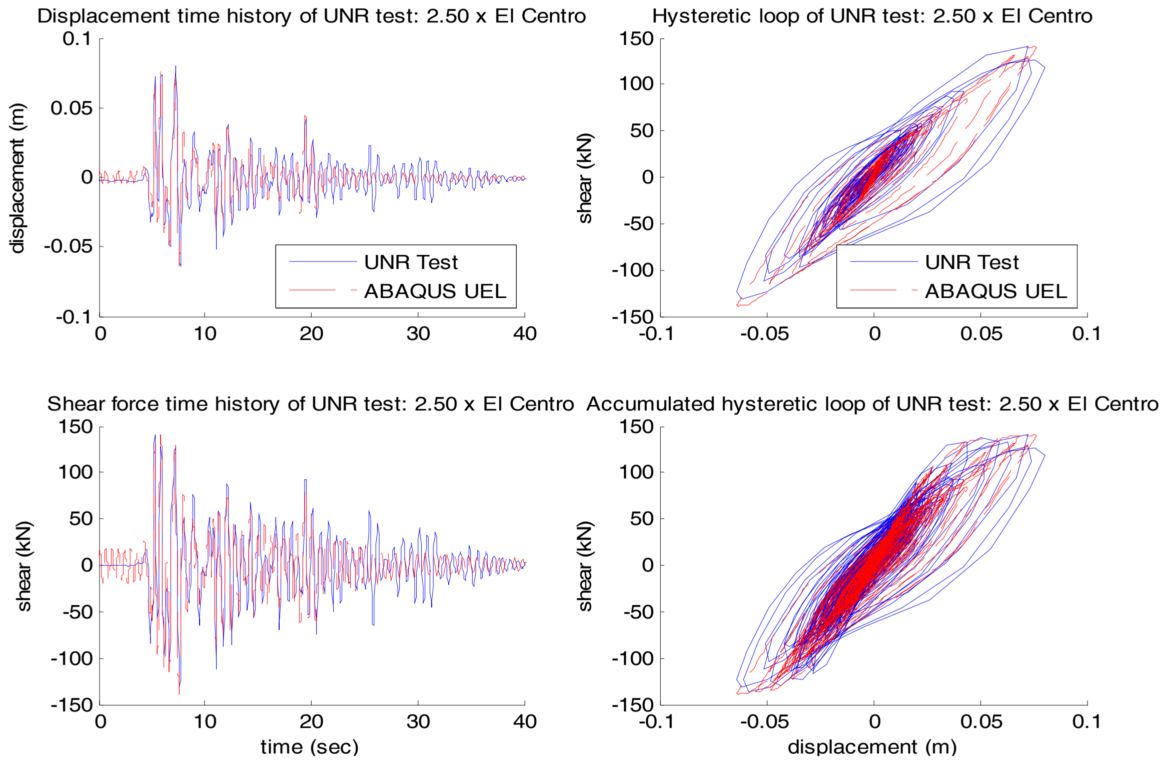

Figure 8 Comparison of the analytical and experimental results of UNR-9F1 shake table test, Stage VI

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