STRENGTH DEGRADATION MODELLING OF STRUCTURES SUBJECTED TO EARTHQUAKES AND FIRE

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ABSTRACT :

Economic considerations dictate that building structures be able to resist extreme events, such as a major earthquake or a fire, without collapse but with some structural damage. This makes it imperative for design to be based on nonlinear analysis that incorporates strength degradation. This study examines the complexities associated with modelling degradation of strength in structures for seismic and high temperature environments. It is shown that both dynamic and high temperature analyses are akin to displacement controlled static analysis. If appropriate numerical procedures are used, strength degradation does not result in dynamic instability often associated with this phenomenon. Inclusion of strength degradation as a material property can lead to results that are sensitive to model discretisation for both dynamic and high temperature loadings.

KEYWORDS: Strain softening, plasticity, post-peak response, elevated temperature

1. INTRODUCTION

The FEMA 440 (2005) document on nonlinear seismic analysis has given significant importance to strength degradation and recognises that the current procedures for addressing this are ambiguous and unclear. Similarly the codes on structural fire design (e.g. Eurocode 2, 1996) incorporate reduction of strength at elevated temperatures for both concrete and steel. Strength degradation at the component or structural level in static load-deformation problems has been shown (e.g. Van Mier, 1984) to be due to a combination of elastic unloading and strain localisation (small regions that demonstrate high strains, e.g. due to cracking). Numerical simulation of localisation using finite element analyses has been previously shown to be mesh sensitive for static problems when strength degradation is taken as a material property (e.g. Pankaj, 1990; Bicanic and Pankaj, 1990). This study considers the issues associated with strength degradation in general and of mesh sensitivity in particular for structural components subjected to earthquake and high temperature loadings.

2. STRENGTH DEGRADATION IN SEISMIC ANALYSIS

Consider a single degree of freedom (SDOF) system idealised using an axial element with a concentrated mass (Fig. 1a). The axial element has Young's modulus *E*, cross-sectional area *A*, and length *L*. Assume that the material of the bar has an elasto-plastic stress-strain material behaviour as shown in Fig. 1b. A normalised ground acceleration history is shown in Fig. 1c. This acceleration history is scaled to a peak ground acceleration of 0.3*g* for subsequent analyses. The acceleration history is a simulated record compatible with the elastic acceleration spectrum of Eurocode 8, though this is not important for the purpose of this paper. It is apparent that if monotonically increasing static loads are applied at the mass point no solution will be obtained once the load exceeds $A\sigma_{y0}$. The displacement response of the SDOF system subjected to the scaled earthquake of Fig. 1c is shown in Fig. 2. In addition to the response obtained from the material property assumed in Fig. 1b, Fig. 2 also shows the response for elastic, hardening ($E_T = 0.1E$) and perfectly plastic ($E_T = 0$) cases. This problem brings into focus a number of issues associated with dynamic response with elements undergoing in-cycle strength degradation. Firstly, this simple example demonstrates that in-cycle strength degradation can be incorporated in a computational model and it does not necessarily lead to dynamic instability, which has been a concern (FEMA 440, 2005). Seismic analysis is akin to a displacement controlled static analysis and does not cause unbounded response. Clearly a large value of *R* or a very steep softening slope may result in solutions not converging. The former implies that the seismic demand is too high in comparison to seismic capacity while the latter indicates

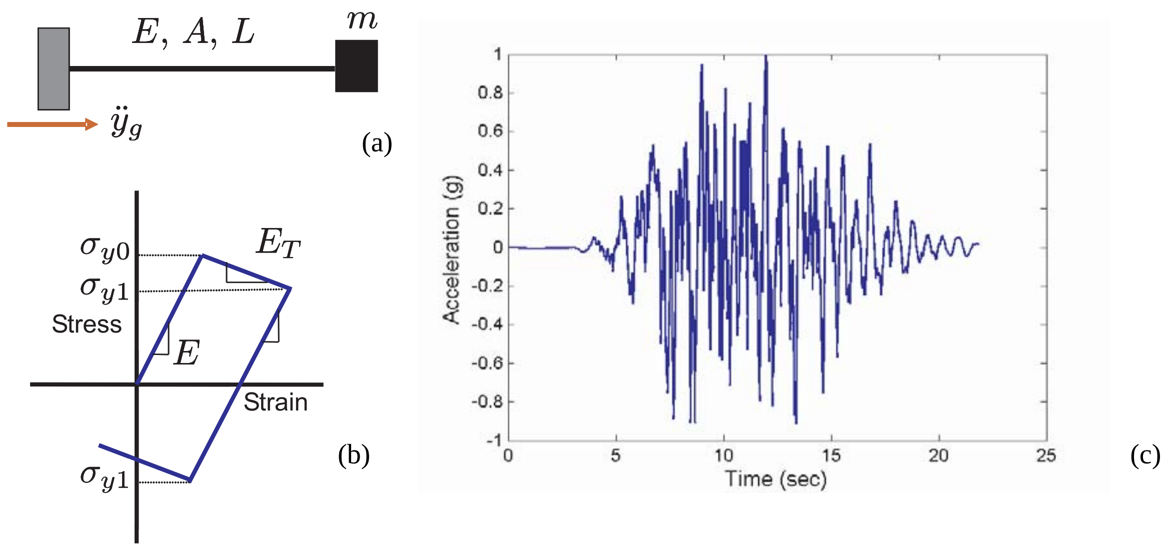


Figure 1: (a) SDOF system with $T = 0.5$ s, $\xi = 0.05$; (b) Stress-strain behaviour of the axial element with $E_T = -0.1E$; $\sigma_{y0} = mS_{a(\max)} / AR$; $R = 1.5$ (c) Normalised ground acceleration history

lack of ductility. The limit of the slope of the strain softening branch has been previously discussed in the literature (Pankaj, 1990). Secondly, plastic deformations are accumulated in a number of discrete steps and for the remainder of the time the frequency response of the system is identical to an elastic system. Fourier analysis of the response shown in Fig. 2a showed that plasticity (including strength degradation) introduces zero frequency content to the response. Thus strength degradation (or other forms of hardening rules) on their own and without stiffness degradation do not change the frequency characteristics of a nonlinear system. Some nonlinear static procedures of seismic analysis assume that equivalent linear parameters of a system undergoing inelastic behaviour comprise increased damping and lengthened period. The latter assumption needs to be considered carefully in the absence of stiffness degradation.

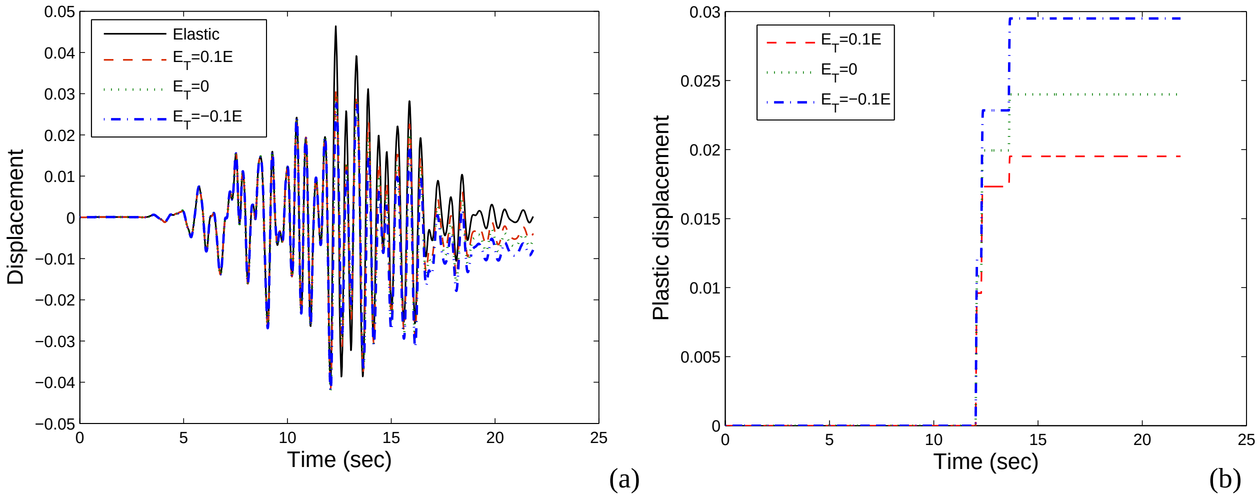
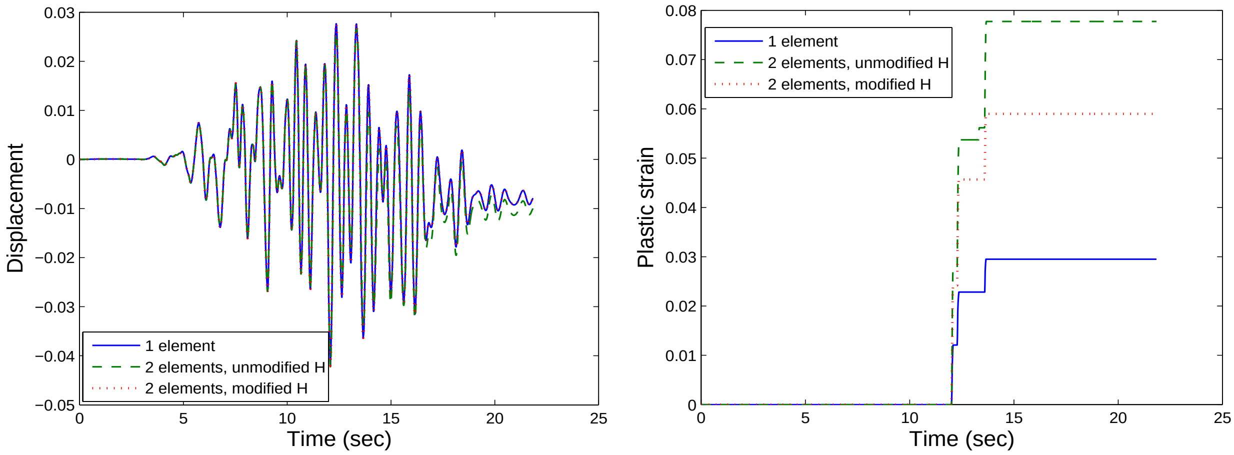


Figure 2: (a) Displacement response; (b) Plastic displacement response

Let us now consider the issue of mesh sensitivity. It has been demonstrated that as a system such as the one shown in Fig. 1a is subjected to increasing displacements or loads, it will undergo strain localisation (e.g. van Mier, 1984). This means that a small region will experience cracking or crushing resulting in reduced load carrying capacity while other regions will experience elastic unloading. Thus strength degradation is a kind of an average of localising and unloading regions. Due to the localised nature of post-peak deformation there is no unique stress-strain relationship for the material itself. Instead it has been suggested (Pankaj, 1990; van Mier, 1984) that a unique relationship between stress and average crack width be adopted. In the context of computational modelling this implies that slope of the line representing strength degradation will be dependent on the element sizes used in modelling. Let us again consider the system of Fig. 1a. At the instant the displacement of the mass exceeds $\sigma_{y0}L/E$ the axial element will exhibit strength degradation. In a computational analysis this degradation will be distributed over the entire element rather than being localised. With a priori knowledge about local behaviour it is possible to include a discontinuity within an element using the partition of unity procedure. However, let us consider a simple smeared crack approach in which we aim to obtain increased strain localisation with mesh refinement. From this point of view consider modelling the single axial element of Fig. 1a using two elements, each of length $L/2$. Since loading in this particular problem will induce uniform internal loads let us increase, fractionally, the yield strength of one of the elements to force strain localisation in the weaker element and possible elastic unloading in the other. Once again stress-strain behaviour including the slope of the softening branch is assumed to be a material property. Two analyses are conducted. In the first the softening slope for both elements is assumed to be identical to the case of a single element. In the second the strain softening modulus H

$$H = \frac{d\sigma}{d\varepsilon^p} = \frac{E_T}{1 - E/E_T} \tag{1}$$

is reduced to half the value used for the single element case. The results are shown in Fig. 3.



(a) (b)

Figure 3: (a) Displacement; and (b) plastic strain response of 1 and 2 element systems with strength degradation

Figure 3a shows that if strain softening modulus is treated as a local material property the displacement response is sensitive to mesh discretisation i.e. 1 and 2 element systems result in a different response. However, when the softening modulus is treated as a nonlocal (or structural) parameter dependent on the element size mesh insensitive solutions are obtained. Figure 3b demonstrates that plastic strain increases as the localisation region is made smaller. In this example the plastic strain for the 2 element discretisation (with modified H) is exactly double that obtained for the 1 element discretisation. Clearly the plastic strain with unmodified H is not compatible to the 1 element case.

3. STRENGTH DEGRADATION IN HIGH TEMPERATURE ANALYSIS

Let us now consider the effect of strength degradation in the analysis of structures subjected to fire. Eurocode 2 (1996) prescribes stress-strain relationships for steel and concrete at elevated temperatures and for both materials the descending branch of the stress-strain curve is treated as a material property. Typically the Young's modulus, the peak strength and the slope of the descending branch (strain softening) all reduce as temperature increases. In this study we examine the issues associated with strain softening using a few simple examples. To maintain transparency in the solutions obtained, for all cases linear elasticity followed by a linear strain softening branch is assumed to represent material behaviour i.e. changes in peak strength and elastic modulus are not incorporated. The examples focus on the effect of mechanical forces emanating from thermal expansion.

3.1 Uniform temperature increase

Consider a simply supported beam with pinned ends as shown in Fig. 4. Typical properties, chosen arbitrarily for numerical simulation, are also shown in the figure. If the beam is subjected to uniform temperature increase the thermal expansion is cancelled out by equal and opposite contraction caused the forces at the restrained ends.

Figure 4: An axially restrained beam with rectangular cross-section of span $L = 6000$ mm; depth $d = 100$ mm; width $w = 50$ mm; Young's modulus $E = 2.05\times10^5$ N/mm^2; yield stress $\sigma_y = 300$ N/mm^2; and coefficient of thermal expansion $\alpha = 12\times10^{-6}/^\circ$C.

It is easy to see that in the elastic regime the compressive stress in the beam will be uniform and given by

$$\sigma = E\alpha\Delta T \qquad (2)$$

where α is the coefficient of thermal expansion and ΔT is the temperature increase. If we go on increasing the temperature and assume the beam is stocky (i.e. we ignore possibility of buckling), the compressive stress will at a certain temperature reach the yield stress σ_y. With continued increase in temperature, strain softening will cause

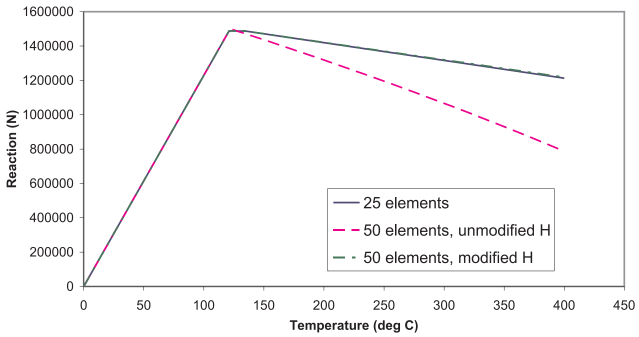


Figure 5: Reaction due to uniform temperature increase with different mesh discretisations and softening modulus

the stress to decrease. It is apparent that the decrease in stress results from some kind of crushing which will be localised i.e. failure will not be distributed over the entire span. With increasing temperature the crushed zone will get crushed further and to maintain force compatibility the remainder of the beam will undergo elastic unloading. In order to numerically simulate this consider the beam idealised using 25 and 50 element discretisations. Localisation is forced by making one element fractionally weaker than the rest. With increase in temperature localisation does occur in the weaker element with elastic unloading in the others. As a result the reaction force at each of the pinned ends decreases with an increase in temperature as shown in Fig. 5. If softening modulus is treated as a material property (in this case we assumed $H = -635$ N/mm^2) then the post-peak response is different for the two idealisations. If, however, the softening modulus for the 50 element discretisation is reduced to half its previous value identical post peak response is obtained as shown in Fig. 5. This illustrates that treating the softening modulus as a material property leads to mesh sensitive results. Another obvious feature of high temperature analysis is that this problem is akin to a displacement controlled static analysis i.e. there is no problem in convergence necessitating any form of indirect displacement control.

3.2 Uniform thermal gradient without average temperature increase

Now consider the beam of Fig. 4 subjected to a uniform thermal gradient across the depth with temperature decreasing from bottom to top and no average temperature increase. This will cause thermal bowing and discussion on deflections and axial forces emanation has been discussed by Usmani et al. (2001). If one of the beam supports was a roller, then due to the curvature of the beam, the horizontal distance between the ends of the beam will reduce. In computational simulation this feature can only be included through a geometrically nonlinear analysis. In fact if geometrical nonlinearity is ignored then the results for results for a beam with restrained pin supports and one with one roller support will be identical and no stresses will be induced in the beam in any of the two cases.

Comparisons of results for an elastic beam of Fig. 4 with and without inclusion of geometric nonlinearity are shown in Fig. 6 where the temperature refers to its value at the bottom face. In Fig. 6a shows the central deflection of the beam when geometric nonlinearity is included (NLGEOM) and when it is not (no NLGEOM). It can be seen that inclusion of nonlinear geometry causes axial forces which limit the central deflection whereas when this is excluded the deflection goes on increasing linearly. Figure 6b shows the increase in horizontal reaction which remains zero when nonlinear geometry is excluded.

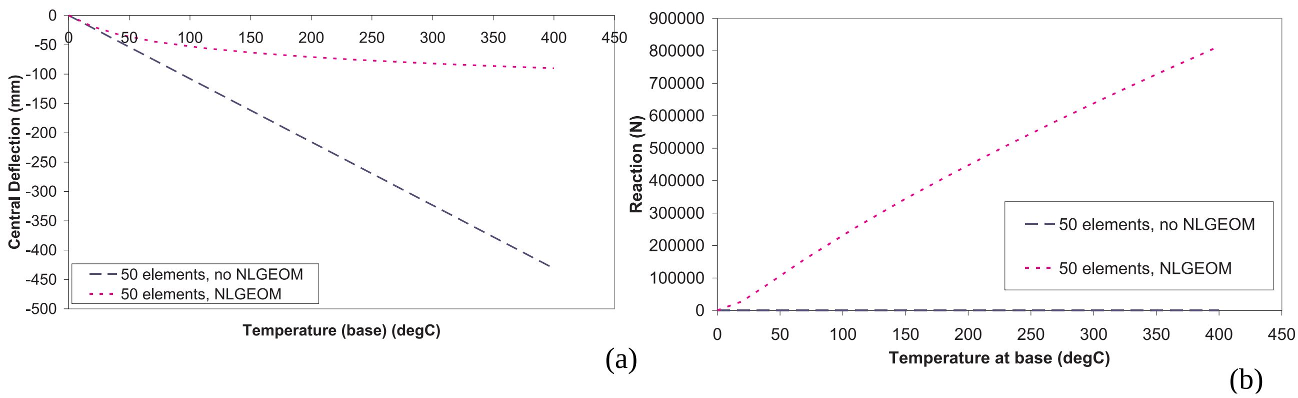


Figure 6: (a) Deflection and (b) reaction in an elastic beam with and without nonlinear geometry

Now consider that both nonlinear geometry and strain softening are included in the analysis and once again a uniform thermal gradient without an average increase in temperature is applied. It is apparent that in addition to axial forces bending moments will be induced in the beam and the largest moment will be at the centre of the span. As a result with increasing temperature the central region will undergo inelastic deformation accompanied by strain softening. The question arises whether with continued increase in temperature the remainder of the beam (away from the central region) would elastically unload with strain localisation in the centre. Interestingly this does not occur in this case and the strain softened region spreads outwards. To explain this phenomenon consider the beam to have two different material – a small central region with an elasto-strain softening material and remainder of the beam comprising a purely elastic material. With increasing temperature gradient the central region becomes inelastic and with continued increase of temperature the thermal bowing proceeds in the form of two elastic beams with a hinge in the middle as shown in Fig. 7a. It is apparent from this configuration that the stresses in the some of the elastic regions will be higher than the central region to maintain equilibrium. The consequence of the above discussion is that for a uniform beam (with no variation in properties) softening will

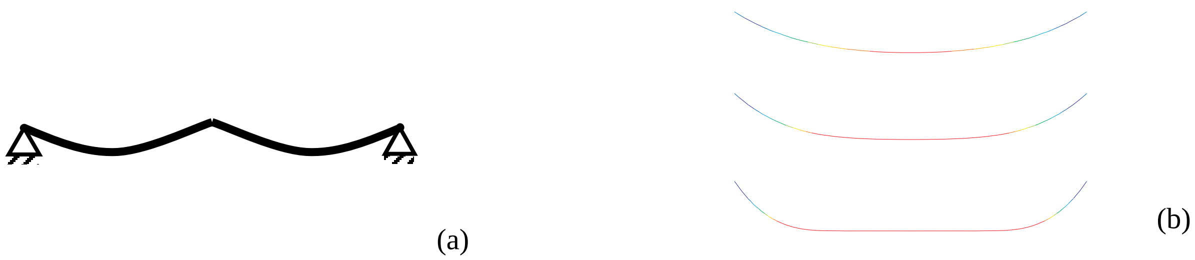


Figure 7: (a) Deflected shape of the beam with softening limited to the central region; (b) Progressive deflection with increasing thermal loading

start in the centre and extend outwards with deflected shapes as shown in Fig. 7b.

This indicates that for the "thermal gradient only" problem strain softening does not result in strain localisation. So numerically results would not be mesh sensitive if the softening modulus were taken to be a material property. This is indeed found to be the case. Figure 8 shows central deflections and horizontal reactions with different mesh discretisations. It can be seen that the results are almost identical.

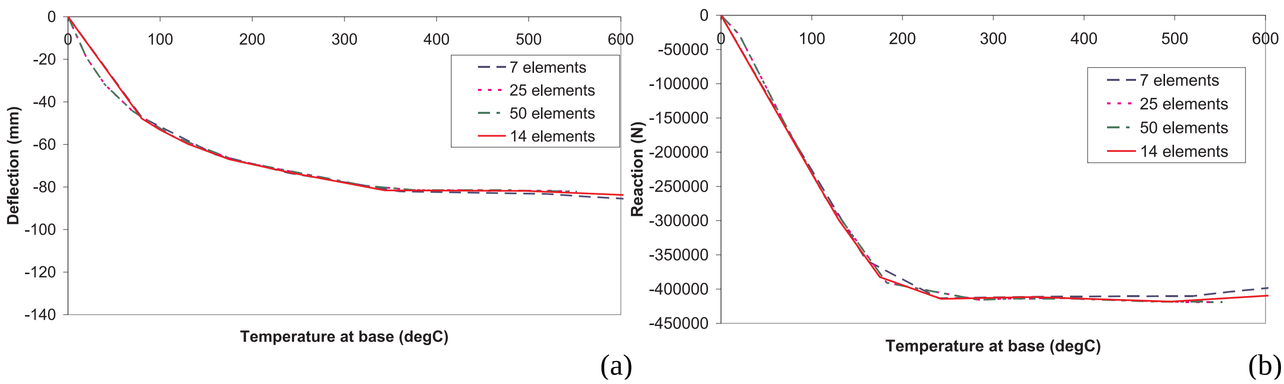


Figure 8: (a) Mid-span deflection and (b) horizontal reaction in an elasto-strain softening beam with mesh discretisations

In the event of a fire structural elements will experience both an average temperature increase and a thermal gradient (the hottest being the surface exposed to fire). The above simple examples show that while the former causes strain localisation accompanied by elastic unloading the latter does not. Therefore for fire analysis inclusion strength degradation in numerical simulation needs considerable further research. More complex situations that need to be included are reduction of the peak strength, elastic modulus and softening slopes with temperature.

4. CONCLUSIONS

Both dynamic and high temperature analyses are similar to displacement controlled static analysis and, if appropriate numerical procedures are used, strength degradation does not result in dynamic instability often associated with this phenomenon. Analysis of the frequency response of structural components subjected to dynamic loads shows that strength degradation introduces a zero frequency component to the response but does not cause a change in the frequency content. Strength degradation when treated as a material property leads to mesh sensitive results for dynamic loading. The simple example considered in this study also indicates that this is also the case for constant temperature increase problems. Thermal gradient alone without average temperature increase does not lead to localisation. Therefore inclusion strength degradation elevated temperatures needs considerably more research before it can be used with confidence.

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