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MODELING AND IDENTIFICATION OF HYSTERETIC DYNAMICS OF MR DAMPERS AND APPLICATION TO SEMIACTIVE VIBRATION CONTROL OF SMART STRUCTURES

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ABSTRACT: In this paper a brief study on magnetorheological dampers and a nonlinear control methodology is presented. MR dampers are complex nonlinear devices whose force-velocity response is hysteretic. Several models have been developed that approximately recreate this behavior. In vibration mitigation systems that make use of MR dampers, it is necessary the implementation on nonlinear control techniques to account for the damper nonlinearities. A case study is presented in this paper in which a base-isolated building containing an MR damper is to be stabilized during a seismic motion. For this purpose, a backstepping controller is designed. The controller performance is evaluated by means of numerical simulations. Some observations about backstepping control experimentation are also made.

KEYWORDS: semiactive control, vibration reduction, MR dampers, modeling, identification, smart structures.

1. INTRODUCTION

The protection of civil structures from hazardous phenomena such as earthquakes and strong winds, have raised the interest to find reliable ways to solve this problem. Up to date, several structural control strategies have been presented. Among them, passive dampers were designed to absorb the vibrations of the structure. They do not require sources of energy to operate, but once tuned they cannot adapt to changes in external loading conditions (Johnson et al., 1998). This means that passive damper design is strongly dependent on the knowledge about the characteristics of the environment where it is going to operate. To solve the adaptation problem, active dampers were developed. Adaptation is possible by measuring the structure response and/or disturbances. However, active dampers require large power sources to operate. Moreover, these systems inject energy to the structure and may destabilize it in a bounded-input/bounded-output sense. These concerns about passive and active dampers have led to the development of hybrid and semiactive dampers which are particularly promising in addressing some of these problems (Dyke et al., 1998). Semiactive control devices combine the features of active and passive devices: their properties can be adapted in real time without injecting energy to the system. These devices are also known as controllable passive dampers (Yang et al., 2002). Semiactive devices have shown to perform significantly better than passive devices. And in comparison to active actuators, semiactive dampers perform as well as them but without requiring large power sources.

The magnetorheological (MR) damper is a promising smart device characterized mainly by its rapid response and low power requirement (Spencer and Soong, 1999). Despite the advantages of MR dampers over other devices of its class, design of systems equipped with them can be challenging because MR dampers exhibit a complex nonlinear behavior which is difficult to model. The MR damper force-velocity response describes a hysteresis loop. This in turn, makes imperative the use of nonlinear control techniques for vibration mitigation systems design.

In this paper we explore a control technique based on backstepping for vibration mitigation in a based isolated building. The base isolation system is composed of a frictional passive damper and an MR damper. Before entering into control design details, a review about MR damper modeling is presented in Section 2. Then in Section 3, the model of the building to be controlled is described. In section 4, details of the backstepping control design are outlined. Next, in Section 5 the simulation results are presented. In Section 6, some experimental results are discussed for a similar structure. Section 7 presents the conclusions and future work.

2. MR DAMPER MODELING

The complexity of the MR damper dynamics has resulted in a variety of mathematical models that approximately recreate the force-velocity response. The damping force varies with the velocity and the magnitude of the applied magnetic field. A typical MR damper response is shown in Figure 1.

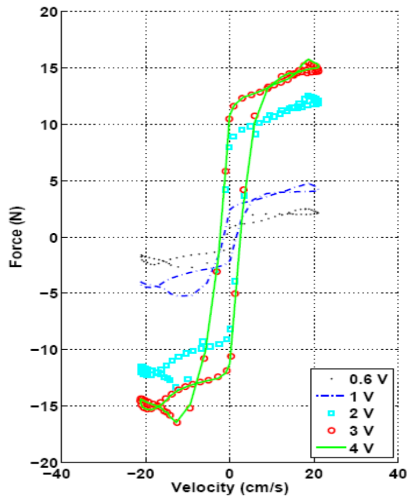


Figure 1. *MR damper force - velocity response at different magnetic field levels.*

One of the first MR damper models was developed following the Bingham plastic model which assumes that a body behaves as a solid until a minimum yield stress is exceeded and then exhibits a linear relationship between the stress and the rate of shear or deformation. To characterize ER dampers, Stanway et al. (1987) proposed a model that consists of a viscous dashpot placed in parallel with a Coulomb friction element, as shown in Figure 2 (a). The force generated by the device is given by:

$$F = f_c \cdot \operatorname{sgn}(\dot{x}) + c_0 \dot{x} \tag{2.1}$$

where x is the piston displacement, c_0 is the damping coefficient and f_c is the frictional force, which is related to the fluid yield stress. This model assumes that the fluid is rigid in the pre-yield condition. However, the Bingham model does not reproduce the hysteretic force-velocity loop although it makes a good estimation of the forces at high velocities. This can be seen in Figure 2 (b): it compares the predicted and experimental responses of an MR damper prototype using the Bingham model (Zapateiro et al., 2007).

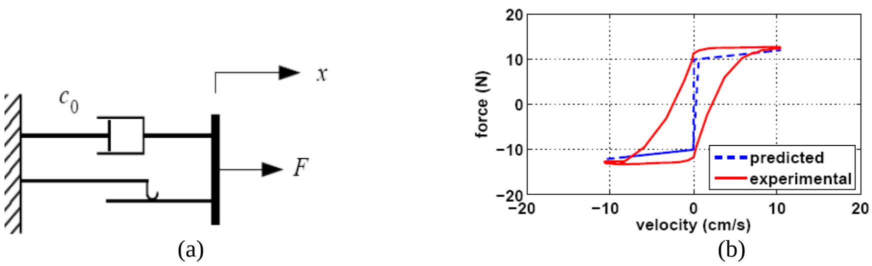


Figure 2. *(a) Bingham model. (b) Comparison between Bingham model and experimental data.*

In 1997, Spencer et al. proposed a phenomenological model based on the hysteresis equations of Bouc and Wen. The damping force can be determined by:

$$\begin{aligned} F &= c_0 \dot{x} + \alpha z \\ \dot{z} &= -\gamma z |\dot{x}| |z|^{n-1} - \beta \dot{x} |z|^n + \delta \dot{x} \end{aligned} \tag{2.2}$$

where c_0 is the damping coefficient and z is an evolutionary variable that accounts for the hysteretic dynamics. The parameters α, β, γ, δ and n are adjusted to control the shape of the force-velocity hysteresis loop. Figure 3(a) compares the predicted and experimental data of the damper prototype using the Bouc-Wen model (Zapateiro et al., 2007).

The Bingham and Bouc-Wen models make use of dynamics concepts that approximately recreate the damper behavior. Another way to find an accurate MR damper model is to collect several experimental data and train a neural network. In this approach, dynamic neural networks are trained so that they predict the response of the damper to several inputs, mainly piston displacement, velocity and acceleration, as well as voltage control signals (that manipulate the magnetic field magnitude) and the fed back damping force.

Figure 3(b) shows a comparison between the behavior predicted by a neural network and the experimental data of the prototype MR damper (Zapateiro and Luo, 2007). One important advantage of neural networks is the computational efficiency despite ignoring further details about the physical behavior of the MR damper. However, neural networks along with the Bouc-Wen models have gained great acceptance to be used for control purposes. A good review on other MR damper models can be found in Butz and von Stryk (2002).

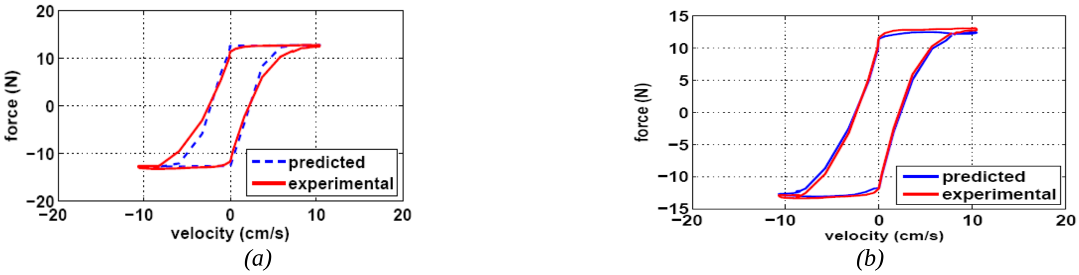


Figure 3. *Comparison between model and experimental data.(a) Bouc-Wen. (b) Neural Networks.*

3. VIBRATION MITIGATION IN A SEMIACTIVELLY CONTROLLED BASED ISOLATED BUILDING

Consider a 10-story building whose base is isolated by means of passive frictional isolator and an MR damper, as illustrated in Figure 4. The overall system can be described with the set of Eqns. 3.1-3.6.

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = [c_1, 0, .., 0]^T \dot{y} + [k_1, 0, ..., 0]^T y \tag{3.1}$$

$$m\ddot{y} + c\dot{y} + ky + f_{bf} = \Phi(\dot{y}, \dot{d}) + f_g + f_c \tag{3.2}$$

$$\Phi(\dot{y}, \dot{d}) = -\mathrm{sgn}(\dot{y} - \dot{d}) \left[\mu_{\max} - \Delta\mu e^{-\nu|\dot{y}-\dot{d}|}\right] Q \tag{3.3}$$

$$f_g = -c\dot{d} - kd \tag{3.4}$$

$$f_{bf} = c_{bf}(\dot{y} - \dot{x}_1) + k_{bf}(y - x_1) \tag{3.5}$$

$$\left.\begin{aligned} f_c &= -\delta\dot{y} - \alpha z \\ \dot{z} &= -\gamma|\dot{y}|z|z|^{n-1} - \beta\dot{y}|z|^n + A\dot{y} \\ \alpha &= \alpha_a + \alpha_b u; \quad \delta = \delta_a + \delta_b u \end{aligned}\right\} \tag{3.6}$$

where $\mathbf{x} = [x_1, x_2, ..., x_n]^T \in \mathbf{R}^n$ is the horizontal floor displacement vector, $y \in \mathbf{R}$ is the horizontal base displacement, d and $\dot{d}$ are the seismic excitation displacement and velocity. The main structure and the base isolation subsystem are given by Eqns. 3.1 and 3.2. **M, C** and **K** represent the mass, damping coefficient and stiffness of the structure, while m, c and k are those of the base. Equation 3.3 describes the passive base isolator dynamics, where μ_{max} is the friction coefficient for high sliding velocity, $\Delta\mu$ is the difference between μ_{max} and the friction coefficient for low sliding velocity, ν is a constant and Q is the force normal to the friction surface. Equation 3.4 describes the incoming earthquake dynamics. Equation 3.5 accounts for the dynamic coupling between the base and the main structure. Equation 3.6 is the Bouc-Wen model for MR dampers where the parameters δ and α are voltage (u) dependent, z is an immeasurable evolutionary variable used to introduce the hysteretic behavior of the damper and n, A, β and γ are hysteresis shape control parameters. In this example, it is assumed that the structural response at the base and the first floor is known at all times and the seismic excitation, although unknown, is bounded by $\left|\dot{d}(t)\right| \leq D_v$, $\left|d(t) \leq D_d\right|$ and $\left|f_g(t)\right| \leq F$.

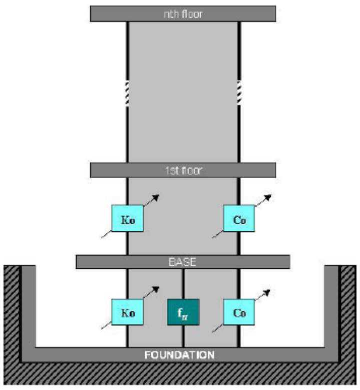


Figure 4. *Case study structure*.

The following propositions about the intrinsic stability of the structure will be used in formulating the control law.

Proposition 1. The unforced main structure subsystem (3.1) i.e., with the coupling term $[c_1,0,...,0]^T \dot{y} + [k_1,0,...,0]^T y \equiv 0,\ t \geq 0$ is globally exponentially stable for any bounded initial conditions.

Proposition 2. If the coordinates $(y, \dot{y})$ of the base and the coupling term $[c_1,0,...,0]^T \dot{y} + [k_1,0,...,0]^T y$ are uniformly bounded, then the main structure subsystem is stable and the coordinates $(\mathbf{x}, \dot{\mathbf{x}})$ of the main structure are uniformly bounded for all $t \geq 0$ and any bounded initial conditions.

The proofs of these propositions are detailed in Luo et al. (2000).

4. CONTROLLER FORMULATION

The goal of the semiactive control is to reduce the absolute response in the base level in such a way that the base isolator works in its elastic region. In order to design the backstepping controller, Equation 3.2 is rewritten into a state space form with $y_1 = y$ and $y_2 = \dot{y}$:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= -\frac{1}{m}\left[c y_2 + k y_1 + f_{bf} - \Phi - f_g - f_c\right] \end{aligned} \tag{4.1}$$

c, k, c_{bf} and k_{bf} are parameters that represent the damping coefficients and stiffness. The following control law asymptotically attenuates the vibrations and stabilizes the main structure:

$$u = \frac{-(c + \delta_a - m h_1) y_2 - k y_1 - f_{bf} + \Phi + f_g - \alpha_a z + m e_1 + m h_2 e_2}{\alpha_a z + \delta_b y_2} \tag{4.2}$$

for all $\alpha_a z + \delta_b y_2 \neq 0$, otherwise $u = 0$. h_1 and h_2 are positive constants and e_1 and e_2 are standard backstepping variables given by:

$$e_1 = y_1; \quad \alpha_1 = -h_1 e_1; \quad e_2 = y_2 - \alpha_1 \tag{4.3}$$

To prove the stability of the control law of Eqn. 4.2, define the following Lyapunov function candidate:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2\right) \tag{4.4}$$

The derivative of V is given by:

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 \tag{4.5}$$

Deriving Eqn. 4.3 and substitution of Eqns. 3.2, 3.6 and 4.2 into Eqn. 4.5 yields:

$$\dot{V} = -h_1 e_1^2 - h_2 e_2^2 < 0 \tag{4.6}$$

According to Lyapunov's stability theory, $e_1 \to 0$ and $e_2 \to 0$. Consequently, $y = y_1 = e_1 \to 0$ and $\dot{y} = y_2 = e_2 + h_1 e_1 \to 0$. According to Propositions 1 and 2, the vibration of the base is asymptotically attenuated and the asymptotic stability of the main structure is guaranteed.

The controller of Eqn. 4.2 contains some immeasurable variables so, in order to implement it, some constraints on the variables Φ, z and f_g are made. First, consider the assumptions about the earthquake dynamics. In general, the passive control force generated by the frictional base isolator can make small the relative movements of the structure during the seismic excitation. Thus, $\nu\left|y_2 - \dot{d}\right| < 1$ as can be verified for some standard earthquakes, as shown in Figure 5.

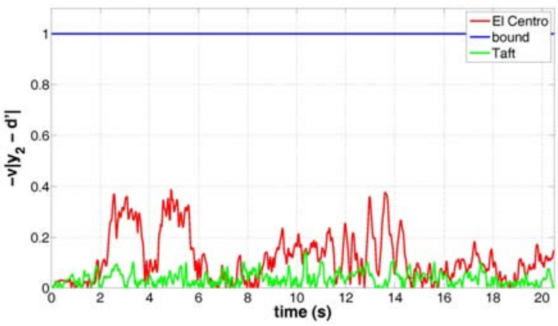


Figure 5. *Dynamics of the base isolator in the presence of the Taft and the El Centro earthquakes.*

A third order approximation of $e^{-\nu\left|y_2-\dot{d}\right|}$ in Eq. 3.3 can be made and denoting $\left|y_2 - \dot{d}\right|_0$ as the maximum value of $\left|y_2 - \dot{d}\right|$, the base isolator dynamics can be approximated by:

$$\begin{aligned}
&\overline{\Phi} \le \Delta_0 + \Delta_1 D_v - \Delta_1 y_2; \quad \Delta_0 = \left(\mu_{\max} - \Delta\mu\right)\left(1 + \nu\left|y_2 - \dot{d}\right|_0 + \frac{\nu^2}{2}\left|y_2 - \dot{d}\right|_0^2 + \frac{\nu^3}{6}\left|y_2 - \dot{d}\right|_0^3\right)^{-1} \\
&\Delta_1 = \mu_{\max}\left(\nu + \frac{\nu^2}{2}\left|y_2 - \dot{d}\right|_0 + \frac{\nu^3}{6}\left|y_2 - \dot{d}\right|_0^2\right)\left(1 + \nu\left|y_2 - \dot{d}\right|_0 + \frac{\nu^2}{2}\left|y_2 - \dot{d}\right|_0^2 + \frac{\nu^3}{6}\left|y_2 - \dot{d}\right|_0^3\right)^{-1} Q
\end{aligned} \tag{4.7}$$

The evolutionary variable z is estimated by:

$$z = \hat{z} + \lambda e_2; \quad \dot{\hat{z}} = -\gamma |y_2| \hat{z} |\hat{z}|^{n-1} - \beta y_2 |\hat{z}|^n + A y_2 \tag{4.8}$$

Consider the assumptions made on the unknown disturbance force $|f_g(t)| \le F$, the approximation of the frictional actuator force in Eqn. 4.7 and the estimation of the evolutionary variable in Eqn. 4.8. Finally, consider the following restrictions that for the MR damper used in this problem apply: n=1 and $\gamma \geq \beta$ >0 and let $\tilde{z} = \lambda e_2$ and $h_2 = \lambda \alpha_a / m$. Under these conditions, the following implementable control law stabilizes the closed loop system:

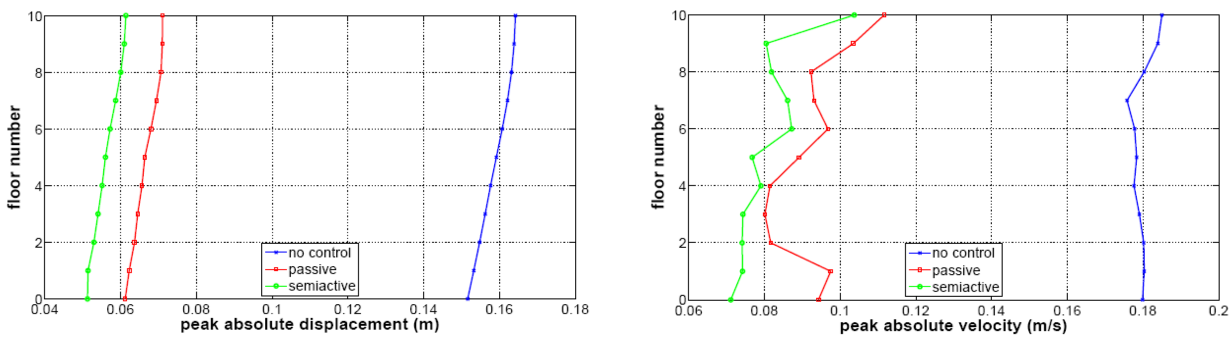
$$u = -\frac{(c + \Delta_1 + \delta_a - m h_1) y_2 - (\Delta_0 + \Delta_1 D_v + F)\mathrm{sgn}(e_2) - f_{bf} - k y_1 - \alpha_a \hat{z} + m e_1 + m h_2 e_2}{\alpha_b \hat{z} + \delta_b y_2 + \alpha_b \lambda e_2} \tag{4.9}$$

provided that $\alpha_b \hat{z} + \delta_b y_2 + \alpha_b \lambda e_2 \neq 0$; otherwise, u = 0. The proof is similar to that of Eqn. 4.2 and not shown here due to space reasons. Details can be found in Zapateiro et al. (2008).

5. NUMERICAL EXAMPLE

The controller is implemented with the following numerical values: the base mass, stiffness and damping coefficients are m=6x10^5 kg, k=1.184x10^7 N/m and c=0.1 respectively; the main structure stiffness varies linearly from the first floor k_1=5x10^7 N/m to the top floor k_{10}=4.5x10^8 N/m, the damping coefficient is c=0.05 and the mass of each floor is m=6x10^5 kg. The base isolator has the following values: $Q = \sum_{i=1}^{10} m_i$, μ_{max}=0.1, $\Delta\mu$=0.09, and ν=2.0. The parameters of the MR damper model are: γ=β=3x10^2 m^{-1}, A=120, α_a=4.5x10^4, α_b=3.6x10^4, δ_a=3x10^2 kNs/m, δ_b=1.8x10^2 kNs/m, Δ_0=2.87x10^2 kN, Δ_1=1.63x10^3 kN, D_v=0.32, F=1.45x10^3 kN, h_1=1.5 and h_2=86.3 and λ=1.

The simulation is run by exciting the structure with the records of the Taft earthquake. Figure 6 shows the results of both passive and passive + semiactive control action compared to that of the system without dampers. In both cases, a reduction in absolute displacement and velocity is achieved with better results when the semiactive device is integrated.



(a) (b)

Figure 6. *Peak displacement (a) and peak velocity (b) under Taft earthquake.*

Finally, Figure 7 shows the MR damper control effort which is within the limits of practical devices.

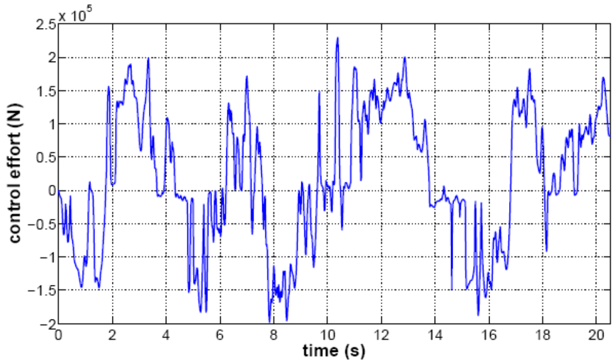


Figure 7. *MR damper control effort.*

7. CONTROLLER EXPERIMENTATION

The performance of the proposed backstepping controller for vibration reduction has already been experimentally verified in a similar structure. Recently, a study was conducted at the Smart Structures Technology Laboratory, University of Illinois at Urbana – Champaign (USA) to test different semiactive controllers in structures with MR dampers. The structure is a 3-story building with an MR damper attached to the first floor. The system was subject to a scaled replica of El Centro earthquake. The experiments were run using a novel real-time hybrid testing method developed in the aforementioned laboratory. In Figure 8 it is possible to see the main result in the case of the backstepping controller. It was possible to verify the vibration reduction in the structure when subject to the seismic motion as can be seen in the absolute acceleration and relative displacement plots.

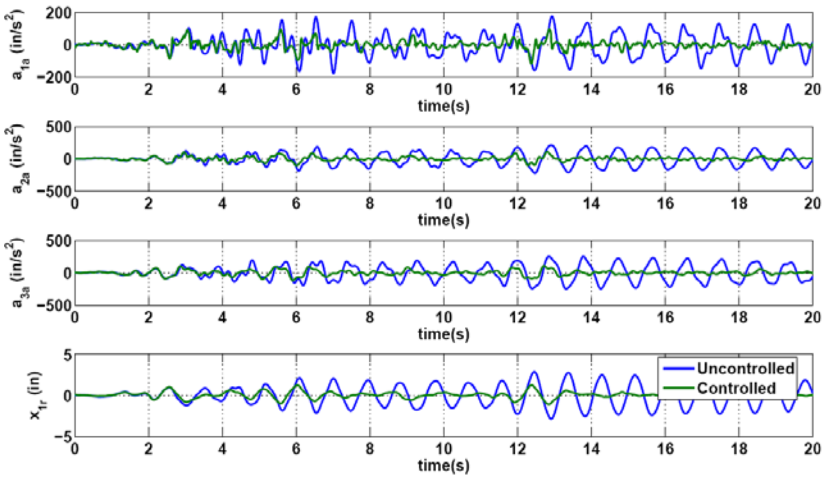


Figure 8. *Experimental results via backstepping controller for a 3-story building with MR damper.*

6. CONCLUSIONS

In this paper some MR damper models for use in seismic motion vibration mitigation were reviewed. An example about how MR dampers can be used for vibration mitigation was studied. Due to the complex nonlinear behavior of MR dampers, a backstepping controller was proposed to account for the nonlinearities. As a result, the simulations showed that the controller performs satisfactorily at reducing the structural response of a base-isolated building subject to a seismic motion.

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