Minimum Capacitance Requirment for Self-Excited Induction Generator

H.P.Tiwari and J.K. Diwedi

Abstract -**In this paper, a simple and direct analytical method is proposed to obtain the minimum requirement of the capacitance for self-excited induction generator under load and no-load conditions for different speeds. A computer software has been developed to compute the minimum capacitance requirement at different load conditions for excitation of the machine for wide range of speeds. Results are compared and discussed in detail.**

I. Introduction

Due to increasing price and depletion of conventional energy sources, interest in renewable energy resources such as wind, solar, tidal, micro-hydel has intensified. The three common electric generators are the dc generator, alternator (field wound/permanent magnet) and capacitor excited induction generator. The d.c. generator and field-wound alternator have maintenance problem associated with commutator and brush-gear. Induction generators are receiving close attention because of the quantities such as small size, low weight, robust construction, low unit cost, manufacturing simplicity, absence of separate source for excitation, better stability, self protection under fault conditions and low maintenance requirements [1,2]. In late 1970s, the efforts concentrated on wider applicability of self-excited induction generator {SEIG) due to oil crisis in early seventies, made exploitation of non-conventional sources inevitable for generating electricity. Wind has been identified as one of the most viable sources for converting its energy into electricity [3,6].

The Application of SCRs in SEIG-Systems has led to an upsurge in the use of electronic controller and regulators in wind energy based schemes[3]. The SEIG-schemes have been used to generate d.c. to a.c. power in isolation for feeding a.c. power to grid through a dc link, by using variety of control strategies like controlled rectifier unit for d c supply, chopper controlled SEIG and ac-dc link schemes [7]

II..Self Excitation of Induction Generator

In isolated applications, an induction generator operates under self-excited mode. If an appropriate capacitor bank is connected across the terminals of an externally driven induction machine, an EMF is induced in the machine windings due to the excitation provided by the capacitor. The magnitude and frequency of this EMF depends upon the prime mover speed, value of capacitance and load impedance. For generator connected to a power grid, the terminal voltage and frequency are known and hence their analysis is straightforward.

Further, owing to the saturation, the magnetizing reactance varies with the operating point. The induced voltage and current will continue to rise until the VAR supplied by the capacitor is balanced by the VAR demanded by the machine, a condition which is essentially decided by the saturation of the magnetic circuit. The machine now operates as a self excited induction generator (SEIG) and can feed a load, at a voltage and frequency dictated by the value of the capacitor.

It is known that in the operation of self-excited induction generators, for a given capacitance value and load, there is a minimum speed below which the excitation cannot be sustained. To extend the speed range, the practice has been to increase the capacitance value particularly for wind driven applications where there is a wide variation in speed.

As soon as induction motor speed exceed its synchronous speed, it starts delivering active power to the three-phase line. However for creating its own magnetic field it absorb reactive power from the line to which it is connected. Reactive power flows in the opposite direction to active power. The active power is directly proportional to the slip above the synchronous speed. The reactive power required can also supplied by a group of capacitors connected across machine terminals. This arrangement can be used to supply a three phase load without using an external source. The frequency generated is slightly less than that load without using an external source. The frequency generated it slightly less than that corresponding to speed of rotation. The terminal voltage increases with capacitance. If capacitance is insufficient, the generator voltage will not build up. Hence capacitor bank must be large enough to supply the reactive power.

Fig.1 shows the capacitor connected to the machine are in delta for economic reasons. If these are connected in star connection, voltage across and reactive power supplied by each capacitor will be 1/√3 and 1/3 times that in delta connection. For the same reactive power, three times as

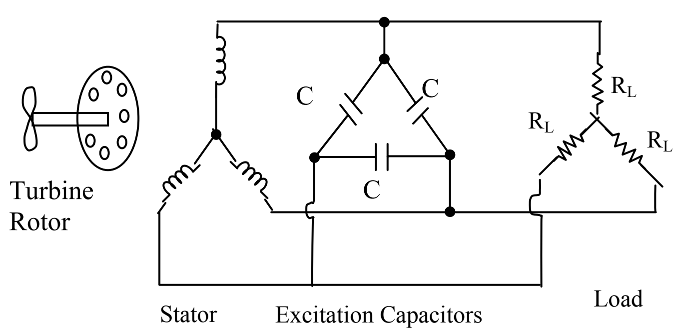

Fig.1: Self-Excited Induction Generator.

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much capacitance is required in star connection which increases the system size and cost. These capacitances have been used to self-excite the generator.

A single phase equivalent circuit of SEIG as shown in Fig.2 is called a negative resistance oscillator. The oscillations occurs at a particular frequency where the capacitive reactance equals the inductive reactance of the generator. The oscillations are more pronounced if the R-L load is disconnected at the time of start and is switched in after normal voltage build up. The induction generator produces as small voltage due to residual magnetism which initiates oscillations. Change in capacitance will change the frequency of oscillation therefore the machine speed. It is found that at a particular shaft power input, decrease in capacitance causes the speed to increase. The change is more for heavy load than for light loads.

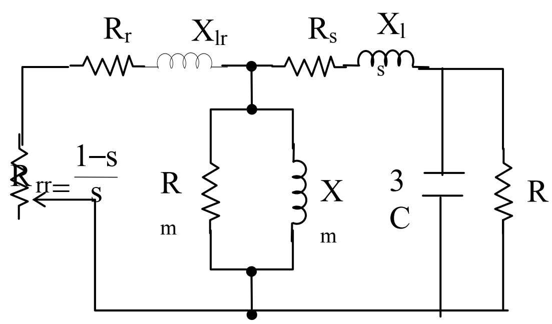

Fig. 2: Single-phase equivalent circuit of self-excited induction generator.

Fig.3 shows magnetisation characteristics of an induction generator. Different capacitive reactance lines obtained by maintaining constant speed and changing the value of capacitance. The volt-ampere characteristics of capacitor will be straight lines with slope as 1/ωc. the resultant voltage of generator will be stable at a value determined by the intersection of the magnetisation curve and the straight line. The capacitance value, below which the generator will fail to build-up voltage called critical capacitance value (C_3).

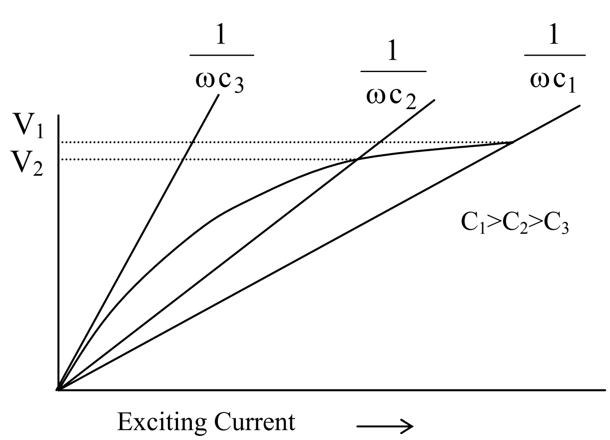

Fig. 3: Magnetisation Characteristics (self - Excitation)

III. STEADY-STATE EQUIVALENT CIRCUIT MODEL

Conventional equivalent circuit is used to analysis the induction generator under steady-state operation. However, as the operating frequency of the generator varies with the driving speed and the load. The per phase equivalent circuit has to be modified as shown in Fig. 4. Where all the parameters are referred to the rated frequency. Also the following assumptions are made.

1. Only the magnetising reactance is assumed to be affected by magnetic saturation, and all other parameters of the equivalent circuit are assumed to be constant.
2. Leakage reactance of stator and rotor, in per unit, are taken to be equal (i.e. $X_{lr} = X_{ls} = X_l$) to simplify the analysis.
3. MMF space harmonics and time harmonics in the induced voltage and current are ignored. This assumption is valid in well designed machines.
4. Core loss in the machine is neglected. Infect, the machine must operate at the threshold of saturation for minimum capacitance requirement. Therefore ignoring such losses will result in no serious errors in estimating C_{min}.

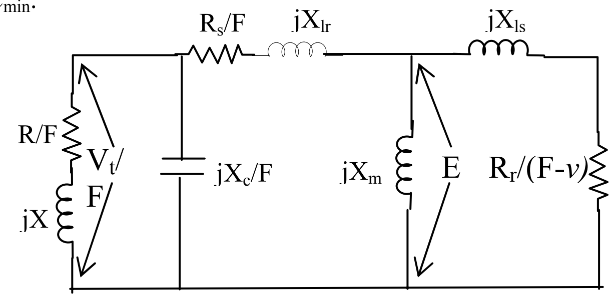

Fig..4: P.u. per phase steady-state equivalent circuit of self-excited induction generator.

IV. MATHEMATICAL MODEL

Equivalent circuit are modified to Fig. 4 to get simple mathematical relationships:

p.u. frequency (F) = f_g / f_r
and p.u. speed (v) = N/N_s

The operating slip of the machine is

$$s = \frac{N'_s - N}{N'_s} = \frac{F - v}{F}$$

The loop equation for the current I can be written as

IZ = 0

Where Z is the net loop impedance given by

$$Z = \left[\left\{ \left(\frac{R_r}{F - v} \right) + jX_{lr} \right\} \| \, jX_m \right] + \frac{R_s}{F} + jX_{ls} + \left[\frac{-jX_C}{F^2} \| \left(\frac{R}{F} + jX \right) \right]$$

Under steady state excitation I ≠ 0 therefore Z =0, equating real and imaginary parts of Z to zeroes.

The real part yields

$$- a_1F^2 + a_2F^2 + (a_3 X_C + a_4) F - a_5 X_C = 0 \qquad (1)$$

and the imaginary parts yields

$$- b_1F^4 + b_2 F^3 + (b_3 X_C + b_4) F^2 - (b_5 X_C + b_6) F - X_C b_7 = 0 \qquad (2)$$

Where a_i, i =1 to 5 and b_i, i = 1 to 7 are positive real constants given in Appendix.

Simplifying Figure 4 to Figure 5, gives

$$R_L = \frac{RX_C^2}{F\left[F^2R^2 + \left(F^2X - XC\right)^2\right]}$$

and

$$X_L = \frac{\left[-XX_C + R^2 + F^2X^2\right]X_C}{\left[F^2R^2 + \left(F^2X - X_C\right)^2\right]}$$

For conservation of real power and reactive power in the circuit of figure 5, the sum of the admittance of its three branches must be zero because it doesn't contain sources of EMF or current i.e. $Y_1 + Y_2 + Y_3 = 0$

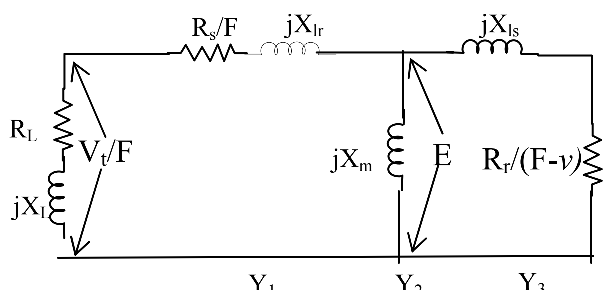

Fig. 5. Equivalent circuit of induction generator with load.

Both real and imaginary parts of above equation are zeroes, the following expression are obtained:

$$\frac{\left(R_L + F_S / F\right)}{\left(R_L + \dfrac{R_s}{F}\right)^2 + \left(X_{ls} - X_L\right)^2} + \frac{R_r / (F - v)}{\left[R_r / (F - v)\right]^2 + X_{lr}^2} = 0 \qquad (3)$$

$$-\frac{1}{X_m} - \frac{X_{lr}}{\left(R_r / (F - v)\right)^2 + X_{lr}^2} - \frac{\left(X_{ls} - X_L\right)}{\left(R_L + R_s / F\right)^2 + \left(X_{ls} - X_L\right)^2} = 0 \qquad (4)$$

From equation (3), the frequency may be determined for given load impedance, excitation capacitance and rotor speed. This equation considered as a simple quadratic equation in terms of p.u. speed (v) instead of using a 7th order polynomial equation in terms of p.u. frequency (F) which is solved only by a numerical technique.

$$K_2v^2 + K_1v + K_0 = 0 \qquad (5)$$

And the expression for K_2, K_1 and K_0 are function of machine parameter and load parameter in absolute value of quantities. Solving this equation for its roots

$$v = \frac{-K_1 \pm \sqrt{K_1^2 - 4K_2K_0}}{2K_2}$$

V. General Solution For Load Condition

The self-excited induction generator has to operate in the saturated region. Therefore the terminal capacitor should have a value such that X_m always lies in the saturated region for given speed and load. The magnetising reactance X_m decreases with increasing saturation and varies considerably with operating conditions owing to saturation, the assumption of a single value X_{smax} in the analysis is acceptable, because feasible C_{min} value is inevitably associated with the lowest magentising KVAs and magnetising curve at which a stable operating point is just feasible and hence X_{smax} will be very slightly less than the machine's unsaturated magnetising reactance.

Now given machine parameter values, speed and $X_m = X_{smax}$, to find the minimum capacitor value required for self-excitation and the p.u. frequency F that simultaneously satisfy equation (1) and (2).

Now to find exact expressions for C_{min} under no load, inductive and resistive loads. Let $X_m = X_{smax}$ equation (1) can be written as

$$X_C = \frac{A_1F^3 - A_2F^2 - A_4F}{A_3F - A_5} \qquad (6)$$

Similarly equation (2) can be written as :

$$X_C = \frac{B_1F^4 - B_2F^3 - B_4F^2 + B_6F}{B_3F_2 - B_5F - B_7} \qquad (7)$$

Where all coefficients $A_i = a_i$, i = 1, 2, …. 5 and $B_i = b_i$ = 1, 2, ……. 7 are calculated at $X_m = X_{smax}$.

For a particular capacitance value. Simultaneously

satisfies equations [6] and [7], it follows that :

$$\frac{A_1F^3 - A_2F^2 - A_4F}{A_3F - A_5} = \frac{B_1F^4 - B_2F^3 - B_4F^2 + B_6F}{B_3F_2 - B_5F - B_7}$$

Which can simplified to

$$(A_1B_3 - A_3B_1)F^4 - (A_2B_3 + A_1B_5 - A_3B_2 - A_5B_1)F^3 + (A_2B_5 - A_3B_4) - A_4B_3 - A_1B_7 - A_5B_2)F^2 - (A_3B_6 + A_5B_4 - A_4B_5)F + (A_5B_6 + A_4B_7) = 0$$

After a tedious elaboration, above equation can reduced to

$$\alpha_4 F^4 - \alpha_3 F^3 + \alpha_2 F^2 - \alpha_1 F + \alpha_0 = 0 \qquad (8)$$

or

$$\alpha_4 F^4 + \alpha_2 F^2 + \alpha_0 = \alpha_3F^3 + \alpha_1F \qquad (9)$$

Where α_i, i = 0, 1, ……… 4 are positive constants, given in Appendix and capacitance value $C = 1/ 2\pi f_b Z_b X_C$

let { F^i, i ≤ 4} be the set of positive real roots of equation (8) and let { C_i, i ≤4} be the corresponding set of positive capacitor values by substituting roots of equation (8) in equation (6) or equation(7). All these values of capacitance at a particular speed and X_{smax} are sufficient to guarantee self-excitation of induction generator, thus the minimum capacitance required is :

$C_{min} = \min \{ C_i, i \le 4\}$

If equation (8) has no real roots, than no excitation is possible. In fact there is a minimum speed value, below which equation (8) has no real roots. The coefficients α_i, i = 0, 1, … 4 equation (8) are functions of p.u. speed v, load and machine parameters. Decreasing the speed results in a situation where the two polynomials of equation (9) do not intersect, resulting four imaginary roots of equation (8). The value of this speed will be called the cut-off speed v_c. if F were to be greater than v, than from figure R_r / (F-v) is strictly positive, and therefore no excitation is possible. To summarise, if $v \ge v_c$, the roots of equation (8) are positive and bounded by v. Therefore choosing $F_{max} = \max \{F_i, i \le 4\}$ will result in the minimum capacitor required for self-excitation.

Considering quadratic equation (5) of p.u. speed (v). The coefficients K_0, K_1 and K_2 are the functions of machine and load parameters. Solving the equation

$$v = \frac{-K_1 \pm \sqrt{K_1^2 - 4K_2K_0}}{2K_2}$$

A simplification

$$v = \frac{-K_1 - \left[G^2 - 4\frac{R_r^2}{X_{lr}^2}K_i^2\right]^{1/2}}{2K_2}$$

Where $G = -K_1 - 2F K_2$

Only negative sign of discriminant is considered to obtain the smaller of two roots of quadratic equation leading to a small negative value of slip, as the slip remains small despite the wide variation in rotor speed. From above equation p.u. speed (v) can be readily obtained for any given set of p.u. frequency (F) and capacitance (C) which earlier computed. The coefficients K_2 and K_1 are function of machine and load parameters, which are in absolute values of quantities, given in Appendix.

VI. A Special Case Of No Load Condition

The no load condition can be derived from the general case by taking X= 0 and taking the limit as R goes to infinity. In this case the equations 6, 7 and 8 are reduced to:

$$X_C = AF^2 - B\frac{F}{F - v} \tag{10}$$

$$X_C = v + DF(F - v) \tag{11}$$

and

$$F^2 = \left(1 + \frac{D}{E + D - A}\right)vF + \frac{B + v^2 D}{E + D - A} = 0 \tag{12}$$

where $A = X_{ls} + X_{smax} \parallel X_{lr}$

$$B = \frac{R_s R_r}{X_{lr} + X_{s\max}}$$

$$D = \frac{R_s}{R_r}\left(X_{ls} + X_{s\max}\right)$$

$$E = X_{ls} + X_{s\max}$$

Equation (12) solved for F as

$$F = \frac{v}{2}\left\{\frac{1 + \frac{R_s}{R_r}\left(1 + \frac{X_{lr}}{X_{s\max}}\right)^2 \pm \sqrt{\left[1 - \left(\frac{vc}{v}\right)^2\right]}}{1 + \frac{R_s}{R_r}\left(1 + \frac{X_{lr}}{X_{s\max}}\right)^2}\right\} \tag{13}$$

Here

$$v_c = \frac{2R_s}{X_{s\max}}\sqrt{\left[\frac{R_r}{R_s}\left(1 + \frac{X_{lr}}{X_{s\max}}\right)^2\right]}$$

Obviously, both roots are real and positive, provided that $v \ge v_c$

Differentiating equation 10 w.r.t. F

$$\frac{dS_C}{dF} = 2AF + \frac{Bv}{(F - v)^2}$$

which is always positive for any 0<F<v, implying that X_C is an increasing function of p.u. speed (F) in other words capacitance is an decreasing function of F.i.e. it follows that $F_{max} = \max(F_1, F_2)$ minimises C. therefore, from equation (11).

$$C_{\min} = \left[2\pi f_b Z_b\left(X_{lr} + X_{s\max}\right) \times \left(\frac{R_s}{R_r}F_{\max}\left(F_{\max} - v\right) + F_{\max}^2\right)\right]^{-1}$$

where F_{max} is the value of F letting positive sign in equation 13.

Now again considering quadratic equation (5) of p.u. speed (v) the coefficients K_0, K_1 and K_2 are function of machine and load parameters which are in absolute values of quantities. Solving this equation

$$v = \frac{-K_1 \pm \sqrt{K_1^2 - 4K_2K_0}}{2K_2}$$

Further simplified

$$v = \frac{-K_1 - \left[G^2 - 2\frac{R_r^2}{X_{lr}^2}K_1^2\right]^{1/2}}{2K_2}$$

Where $G = -K_1 - 2FK_2$

Only negative sign of discriminant is considered to obtain the smaller of two roots of quadratic equation leading to a small negative value of slip, as the slip remain small despite the wide variation in rotor speed. From above equation 14 p.u. speed (v) can be readily obtained for any give set of p.u. frequency (F) and capacitance value (C) which are earlier calculated.

At no load condition load resistance (R) is infinite and load reactance (X) is zero, to that in Fig. 5 we have R_L =0 and $X_L = X_C/F^2$ and expression for K_2 and K_1 become.

$$K_2 = F^3\left(R_s X_{lr}^2\right)$$

$$K_1 = F^4\left(-R_r X_{ls}^2 - 2R_s X_{lr}^2\right) + F^2\left(-R_s^2 R_r + 2R_r X_{ls} X_c\right) + \left(-R_r X_c^2\right)$$

The above relationships are used in the subsequent sections for predicting minimum capacitance requirement, maximum p.u. frequency and p.u. speed for particular cases.

VII. Algorithm And Steps

The computer algorithm for predicting the value of minimum capacitance is comprised of two parts, one is for load condition and another is for no-load condition. Each part consists first calculating maximum per unit frequency and minimum capacitance requirement for a particular per unit speed. Similarly algorithm for predicting per unit speed for the given value of minimum capacitance (In this case the two values are kept same to verify the algorithm).

Suitable loops are embodies to draw the characteristics (graphs) between minimum capacitance and per unit speed.

The computation process to obtain minimum capacitance requirement consist of the following steps:

Reading the values

Step 1 – Read machine parameters and base quantities R_s, R_r, X_{ls}, X_{lr}, F_b, Z_b.

Load condition

Step 2 – Read p.u. speed v.
Step 3 – Read load parameters R and X.
Step 4- Calculate coefficients A_1 to A_5 and B_1 to B_7.
Step 5 – Evaluate coefficient (α_0 to α_4) of linear equation
Step 6- Read high value of p.u. frequency (F) for iteration.
Step 7 – Compute function F_x and F_y.
Step 8 - Compare F_x and F_y. If the difference in the two values is greater than a small quantity ε (say 0.000001)_ then go to step 7 to modify the value F by a decrement δF (say 0.00001). Repeat till the difference is less than ε .
Step 9 – Check F : if F = 0. There is no positive real roots or check initial value of F. STOP.
Step 10 – Calculate X_C by computed F_{max}.
Step 11 – Compute C_{min} and log C_{min}.
Step 12 – Limit : log (C_{min}) and *v*.
Step 13 – Assign F_{max} to A and C_{min} to C.
Step 14 – Compute X_C
Step 15 – Compute coefficients K_1 and K_2.
Step 16 – Compute discriminant (disc).
Step 17 – Check disc : if disc < 0 then STOP.
Step 18 – compute p.u. speed v.
Step 19 – limit : log (C_{min}) and v.
Step 20 – Print the results (Input p.u. speed, computed minimum capacitance and computed p.u. speed).

No load condition:

Step 21 – Read p.u speed *v*.
Step 22 – Compute cut off speed v_c
Step 23 – Check *v* : if $v < v_c$ STOP
Step 24 – Compute F_{max}
Step 25 – Calculate C_{min} and log (C_{min})
Step 26 – limit : log (Cmin), v_c/v and *v*.
Step 27 – assign F_{max} to a and C_{min} to C.
Step 28 – Compute X_C.
Step 29 – Compute coefficients K_1 and K_2.
Step 30 – Compute discriminant (disc).
Step 31 – check disc : If disc < 0 then STOP.
Step 32 – Compute p.u. speed *v*
Step 33 – limit C_{min} and *v*.
Step 34 – Print the results (input p.u. speed, computed minimum capacitances and computed p.u. speed)

VIII. Result And Discussion

Fig. 6 shows the requirement of minimum capacitance of SEIG under load and no load condition for different set of speed. This figure contains 6 curves (numbered 1 to 6 from top to bottom), the various values related to these curves are as follows:

Curve-1
R_s= 0.071 R_s=0.0881 $X_{ls} = X_{ls}$ 0.01813
R = 0.0p.u. X = 1.0 p.u.
Curve-2
R_s= 0.071 R_s= 0.0881 $X_{ls} = X_{ls}$ 0.01813
R = 0.0 p.u. X = 2.0 p.u .
Curve-3
R_s= 0.053 R_s=0.042 $X_{ls} = X_{ls}$ 0.0670
R = 1.0 p.u. X = 2.0 p.u.
Curve-4
R_s= 0.053 R_s=0.042 $X_{ls} = X_{ls}$ 0.0670
R = 1.5 p.u. X = 1.5 p.u.
Curve-5
R_s= 0.053 R_s=0.042 $X_{ls} = X_{ls}$ 0.0670
R = 2.0 p.u. X = 2.0 p.u.
Curve-6
R_s= 0.053 R_s=0.042 $X_{ls} = X_{ls}$ 0.0670
R = 0.0 p.u. X = 0.0 p.u.

These curve (Fig.6) show that

(i) Minimum capacitance requirement for load is more than no load condition.
(ii) There exit a trend between capacitance requirement at a given speed and the nature of load
(iii) Results also show the requirement of capacitance is minimum for inductive load than resistive load (curve 1 and 2).
(iv) The larger the value of impedance, smaller the value of minimum capacitance required.

IX. Conclusions

The analysis used to determine the minimum value of capacitance has been found equally suitable for different load conditions. This fact can be utilised for making a Self-Excited Induction Generator of smaller size and a economical one for isolated purpose. An automatic switching scheme using power electronic devices can be incorporated to make suitable isolated self-excited induction generator for different load requirements.

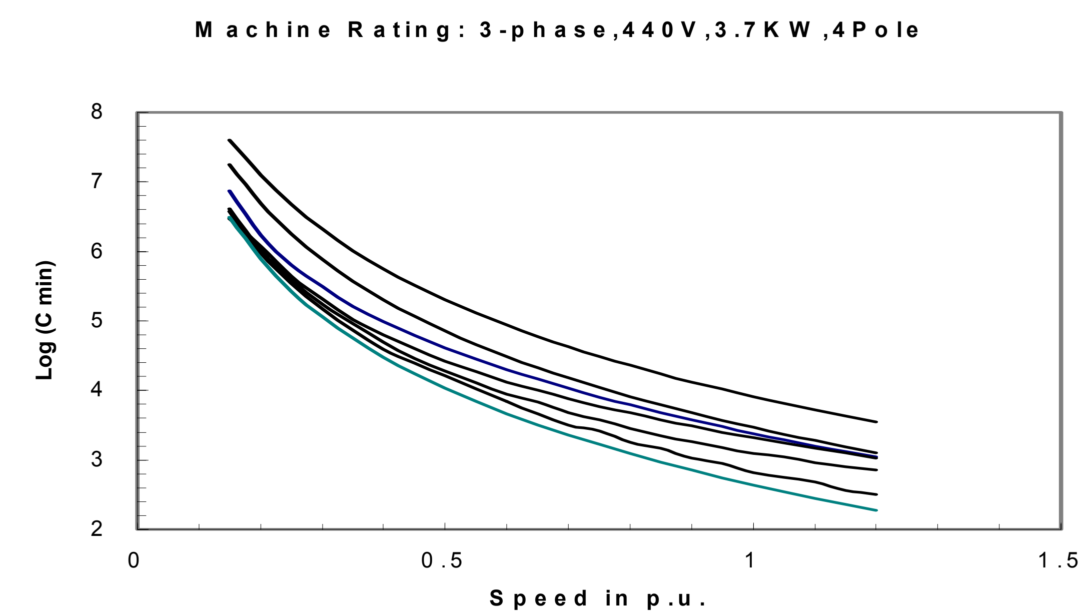

Fig. 6

X. List Of Symbols

R_s, R_r = p.u. per phase stator and rotor resistance (referred to stator) respectively.

X_{lr}, X_{lr} = p.u. per phase stator and rotor leakage reactance (referred to stator) respectively.

X_m = p.u. per phase magnetising reactance (at base frequency.

X_{smax} = p.u. maximum saturated magnetising reactance.

C = per phase terminal-excitation capacitance.

X_c = per phase p.u. capacitive reactance of the terminal-excitation capacitance.

F, v = p.u. frequency and speed respectively.

Z_b, f_b = per-phase base impedance and base frequency respectively.

V_g,V_t =per-phase air-gap and terminal voltages respectively in volts.

f_g , f_r = generated and rated frequency.

N_s $N_{s'}$ = synchronous speed corresponding to rated and generated frequency respectively.

R, X = p.u. load resistance and load reactance per phase.

XI. Appendix

$T = X_{ls} + X_m = X_{lr} + X_m$ $W = X_{ls} + X_l \,||\, X_m$

$A_1 = RTW + XT\ (R_s + R_r)$ $A_2 = vT\,(RW + R_s + R)$

$A_3 = R_r\,(X + T\,) + T\,(R_s + R)$ $A_4 = R\,R_s\,R_r$

$A_5 = vT\,(R_s + R)$ $B_1 = XTW$ $B_2 = v\,B_1$

$B_3 = T\,(X + W)$ $B_4 = RT(R_s + R_r) + R_s\,R_rX$

$B_5 = v\,B_3$ $B_6 = v\,R\,R_s\,T$ $B_7 = R_r\,(\,R + R_s)$

$\alpha = A_5\,B_6 + A_4\,B_7$ $\alpha_1 = A_3B_6 + A_5\,B_4 - A_4\,B_5 - A_2B_7$

$\alpha_2 = A_2B_5 + A_3B_4 - A_4\,B_3 - A_1B_7 - A_5B_2$

$\alpha_3 = A_2B_3 + A_1B_5 - A_3\,B_2 - A_5B_1$ $\alpha_4 = A_1B_3 - A_3B_1$

$$K_2 = F^3(R_sX^2\,X_{lr}^2) + F^3(R^2\,R_sX_{lr}^2 - 2R_sXX_{lr}^2\,X_c) + F(R_sX_{lr}^2\,X_c^2 + RX_{lr}^2X_c)$$

$$K_1 = F^6(-2R_sX^2X_{lr}^2 - R_rX^2X_{ls}^2) + F^4(-2R^2R_sX_{lr}^2 + R_sXX_{lr}^2\,X_c - R^2R_rX_{ls}^2 + 2R_rXX_{ls}^2X_c - R_s^2R_rX^2 + 2R_rX^2X_{ls}\,X_c) + F^2(-{}_sX_{lr}^2X_c^2 - 2RX_{lr}^2X_c^2 - R_rX_{ls}^2X_c^2 - R^2R_s^2R_r + 2R_s^2R_rXX_c - R_rX^2X_c^2 + 2R^2R_rX_{ls}X_c - 2R_rXX_{ls}X_c^2 + (-R_s^2R_rX_c^2 - R^2R_rX_c^2 - 2RR_sR_rX_c^2)$$

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