Investigation of Stability of Fuzzy Logic Based Power System Stabilizers Using Phase – Plane Analysis

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Abstract — **This paper presents a novel approach to investigate the stability and an effective rule base development of Fuzzy Logic based power system stabilizers using phase-plane plots and limit cycle analysis. Small signal stability analysis is generally done by eigen value analysis. The linearised state model and eigen values for the power system equipped with conventional power system stabilizers can be easily obtained for any operating conditions, hence the stability analysis is concerned only with the eigen values thus obtained. When the Fuzzy Logic based stabilizers are used to replace the existing conventional stabilizers, the Eigen value analysis is rather impossible to conduct and there should be an alternative means for examining the small signal stability. Hence in order to investigate the small signal stability of fuzzy logic based power system stabilizers, Lyapunov's direct method is utilized in this paper using the phase-plane plots of the input state variables given to the fuzzy logic controller. This method is attempted for the stability analysis and for the effective rule base development of fuzzy logic based stabilizers. The effect of fired rules on the stability and the effect of rules for the minimization of limit cycles is illustrated and tested for single machine infinite bus system and for multimachine power system.**

Index Terms— **Small signal stability, Power system stabilizer, Fuzzy logic, Phase-plane plots, Limit cycles.**

I. INTRODUCTION

Power system stabilizers are usually provided to damp out the rotor mechanical low frequency oscillations, which are in the range of 0.1 to 2.5Hz. They basically consists of phase – lead compensators along with the wash out circuit and hence produces additional electrical torque component which cancels the negative damping torque produced by the large gain AVR excitation system [11]. The state modeling of the entire system along with these stabilizers can be easily obtained and hence its Eigen value analysis can be easily performed.

Fuzzy Logic control is emerging to provide a versatile and better control methodology. This method is realized simply by mapping the inputs and outputs of this controller by a set of linguistic rules. This method is model independent i.e., when there is no exact mathematical model of the physical system this method can be attempted and hence was attempted only for such cases earlier. In recent years fuzzy logic control is gaining more attention. The power system operating conditions and topologies are time varying and disturbances are unforeseeable. These uncertainties make it very difficult to effectively deal with power system stability problems through conventional controller that is based on linearised system model and for single operating condition. Hence the fuzzy logic control approach is emerging as a complement to the conventional approach. The most important advantage of fuzzy controller is that it is an intelligent controller. The fuzzy controller is a non-linear controller and not so sensitive to system topology, parameter and operating condition change as in the case of conventional linear controller. There is a very little mathematical computation involved in this method and this control method will not increase the order of the system. It is realized that this method of control can perform very effectively when the operating conditions change rapidly and also when the system non-linearities are significant. These features make it very attractive for power system applications. Therefore the applications of fuzzy logic in power system control grow rapidly [2]. However, till now the investigation of fuzzy logic applications in power system control design is mainly in excitation control and for PSS design [6]. Investigation of stability of fuzzy controllers is still a challenge in research area.

M.A.M Hassan et al [2] have given a way to replace the conventional power system stabilizer with fuzzy logic based stabilizer. They have used the standard fuzzy membership functions to compute the stabilizing signal of PSS and made simulations for SMIB system for different operating conditions. M. A. Abido et al [3] introduced a hybrid Neuro-Fuzzy power system stabilizer for multimachine power systems. A. Hariri et al [5] have proposed a fuzzy logic based power system stabilizer with learning ability by introducing a standard subjective membership function and a self-tuned parameter. Y. -Y. Hsu et al [6] has designed fuzzy power system stabilizers for multimachine power systems without model identification. The proposed fuzzy PSS uses two real-time measurements viz., generator speed deviation and acceleration as input signals. The stability of fuzzy controllers has not discussed in these papers.

The basic control actions of fuzzy logic controllers are in the form of linguistic rules and are hence flexible. The rules are generally framed by domain experts or heuristically or by observing the performance of the controller and modifying the rules, which is a cumbersome process. Hence the only draw back of this method is in the rule base development. This can be achieved with the help of phase-plane plots of the input variables, which are given to the fuzzy controller. The rules are modified till a stabilized phase-plane plot is obtained. Lyapunov's stability (direct method) is utilized in this paper to investigate the phase-plane plots. The method is illustrated and tested for single machine infinite bus system (SMIB) and on multimachine power system (MMPS).

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In this paper stability of fuzzy logic controller and an effective rule base development has been investigated even when no expert knowledge is available using phase-plane analysis and applied for single machine infinite bus system and for multimachine power system.

II. Mathematical Modelling

The linearised mathematical modeling of the single machine infinite bus system and multimachine power system is carried out by linearising the equations [13] around the operating point and hence obtained the required state equations. A three-machine nine-bus system is considered for the linearised modeling of multimachine power system, and hence its state equations are obtained [11]. For single machine infinite bus system the standard Philips-Heffron model is considered, which is of forth order state equation (with out PSS). The order of multimachine power system is eleventh order with out power system stabilizer [11].

A. Mathematical modeling of single machine infinite bus system (SMIB) with Fuzzy Logic Based PSS

The state equations obtained for the Philips-Heffron model for SMIB system are as follows: -

$$\Delta\delta^{\cdot} = \omega_B * \Delta\omega \quad (1)$$

$$\Delta\omega^{\cdot} = -\ (K1/2H)\ \Delta\delta - (D/2H)\ \Delta\omega - (K2/2H)\ \Delta Eq' + \Delta Tm/2H \quad (2)$$

$$\Delta Eq'^{\cdot} = -\ K4/Tdo'\ \Delta\delta - 1/(Tdo'^{*}K3)\ \Delta Eq' + (1/Tdo)\Delta E_{FD} \quad (3)$$

$$\Delta E_{FD}^{\cdot} = -\ K5\ (K_E/T_E)\Delta\delta - K6\ (K_E/T_E)\Delta Eq' - (1/TE)\Delta E_{FD} + (K_E/T_E)\Delta Vref \quad (4)$$

The input to the fuzzy controller are $\Delta\omega$ and $\Delta\omega^{\cdot}$ and the fuzzy controller output is the stabilizing signal ΔVs. The constants K1 to K6 are calculated for different operating conditions [13].

B. Modeling of Multi-machine power system (MMPS)

Modeling of multimachine power system is done by taking the three machine nine-bus system, Generator 1 is taken as reference and hence is modeled as classical model, Generators 2 & 3 are modeled as two-axis models [11]. The excitation system on machines 2 & 3 is modeled as one time lag transfer function. The rotor dynamics of machines 2 & 3 are studied with respect to machine 1[13].

Thus the state equation of the form $X^{\bullet} = AX + BU$ and $Y = CX + DU$ are obtained for the 3 machine 9 bus system. The state vector X and the input vector U are as follows:

$$X^T = [\Delta\omega_1\ \ \Delta E'_{q2}\ \ \Delta E'_{d2}\ \ \Delta\omega_2\ \ \Delta E'_{q3}\ \ \Delta E'_{d3}\ \ \Delta\omega_3\ \ \Delta\delta_{12}\ \ \Delta\delta_{13}\ \ \Delta E_{FD2}\ \ \Delta E_{FD3}]$$

$$U^T = [\Delta T_{m1}\ \ \Delta T_{m2}\ \ \Delta V_{ref2}\ \ \Delta T_{m3}\ \ \Delta V_{ref3}]$$

Fuzzy logic based stabilizers are placed on machine2 and 3. The inputs for fuzzy logic based stabilizer placed on machine 2 are $\Delta\omega 2$ & $\Delta\omega 2^*$ and $\Delta Vs2$ is the output. For machine 3 the inputs are $\Delta\omega 3$ & $\Delta\omega 3^*$ and $\Delta Vs3$ is the output.

III. Stability Of Fuzzy Control Systems

Fuzzy control system has been proven to be powerful when applied to the control of processes, which are not amenable to conventional analysis and design techniques. The designs of most of the existing fuzzy control systems have relied mainly on the process operator's or control engineer's experience based heuristic knowledge. Hence the controller's performance is very much dependent on how good this experience is. Thus from the control engineering point of view, the major effort in fuzzy knowledge based control has been devoted to the development of particular knowledge base for specific applications rather than to general analysis and design methodologies for coping the dynamic behavior of control loops. In particular, stability analysis is of extreme importance and the lack of satisfactory formal techniques for studying the stability of control system involve knowledge base has been considered a major drawback of fuzzy control systems.

Fuzzy control systems are essentially non-linear systems. For this reason it is difficult to obtain general results on the analysis and design of fuzzy control systems. Furthermore, the knowledge of the dynamic behavior of the process to be controlled is normally poor. Therefore, the robustness of the fuzzy control system should be incorporated to guarantee stability in spite of variations in system dynamics.

There are two main directions of general stability theory [12].

1) Stability in the sense of Lyapunov which refers to internal representations (the state vector tends towards zero), and
2) Input-output stability, which refers to the external representations (relatively small outputs with regard to the input).

The fuzzy control system can be represented by means of a non-linear function $u=\phi(x)$. This can be analyzed by the dynamic behavior of closed-loop system consisting of (i) Fuzzification, (ii) Inference engine and (iii) Defuzzification.

Stability analysis of a fuzzy control system requires characterization of the relation between the rules and the state-space associated with the dynamic system under control. This relationship is based on the relative influence of each rule of the, rule base on the control action produced by fuzzy inference engine.

A closed-loop system trajectory can be mapped on the position space shown in Fig. 1. A sequence of rules obtained according to the order in which they are fired forms the so-called linguistic trajectory, which corresponds to a certain system trajectory. From the design point of view, this method provides interesting guidelines for the analysis of fuzzy control system. Non-cooperative rules (rules not fired) can be easily modified.

Let us consider the closed-loop system represented as $dx/dt = f(x) + bu$, where $u = \phi(x)$, f(x) is a non-linear function which represents the plant dynamics with $f(0) = 0$, x and b are vectors of fuzzified input of the controller obtained from the crisp input x*; then $\phi(x) = \text{Defuzzification}\ (\mu_x * 0\ \mu_R\ (x,u))$, closed-loop behavior will depend on the nature of f(x) and $\phi(x)$.

The dynamic behavior of a stable feedback system can be designed by modifying with opposite sign of rules, in the limit cycle area.

IV. LYAPUNOV'S DIRECT METHOD FOR THE STABILITY OF NON-LINEAR SYSTEMS

Lyapunov's direct method is concerned with assessing the stability of a dynamic system described by a set of nonlinear equations of the form $X^* = F(X)$. The point $X^\wedge$ is the equilibrium point if it satisfies the equation $F(X^\wedge) = 0$. Lyapunov's direct method is based on finding a suitable scalar function V (X) defined in the state – space of the dynamic system that is positive definite and has a stationary minimum value at the equilibrium point $X^\wedge$ (i.e. for any $\Delta X \neq 0$ it holds that $V(X^\wedge + \Delta X) > V(X^\wedge)$). The point $X^\wedge$ is stable if the derivative $V^* = dV/dt$ is negative semi-definite along the trajectory X (t) of the equation $X^* = F(X)$ (i.e. when $V^* \leq 0$). The point $X^\wedge$ is asymptotically stable if the derivative V^* is negative definite along the trajectory X (t) (i.e. when $V^* < 0$).

A negative value of the derivative V^* Means that function V (X) decreases with time tending towards its minimum value. As the minimum value of V (X) is at the equilibrium point, the trajectory X (t) tends towards the equilibrium point $X^\wedge$. Where the point $X_0 = X(t=0^+)$ denotes a nonzero initial condition lying beyond the equilibrium point $X^\wedge$. It is important to note that the higher negative value of V^*, the faster the trajectory X (t) tends towards the equilibrium point $X^\wedge$. Consequently, a given control strategy is optimal in the sense of Lyapunov if it maximizes the negative value of V^* at each instant of time.

V. LIMIT CYCLE INVESTIGATION

Limit cycle is the phenomenon that can be observed in the system composed of non-linear element, i.e. a system in which output is not varying in proportion to the input of system. Normally, non-linear system consists of energy storing elements due to which energy at the output of the system does not change according to the change of energy at the input side. As by the law of energy conservation, total energy of the system is constant through out the time. This can neither dissipated in loss element nor increased by source. A more general and practical case is one in which dissipation and/or increase of energy can occur. When the above effects are included, due to which an entirely new mode of system operation can occur and is termed as Limit cycle. There are self - excited oscillations are present in the system even in the absence of input to the system, these oscillations are of fixed amplitude and fixed frequency type. A system can have more than one limit cycle, but for investigation the limit cycle having the highest frequency of oscillation need to be considered. Limit cycles are of two types, stable and unstable. Limit cycles having fixed amplitude and frequency of oscillations are called stable limit cycles, whereas if the frequency of oscillations are fixed and with growing amplitudes with time are referred as unstable limit cycles.

The non-linear system, is said to be stable when after disturbance it comes back to its equilibrium position or at least staying within the tolerable limit and may exhibit a special behavior of following a close trajectory or limit cycle. The limit cycle describes the oscillation of non-linear system that is why; it is most crucial factor in the design and for the stability analysis of non-linear systems.

There are number of graphical ways of finding out the existence of limit cycles for non-linear systems. The most adopted methods used in the literature are

1) Phase – plane method and
2) Describing – function method.

The describing function method applications are for system with low degree of non-linearity. The use of describing-function in the analysis of high degree of non-linear systems may lead to seriously erroneous results. Whereas the phase-plane method is not restricted to small non-linearities. In this sense, the phase-plane method is quite useful for analyzing non-linear control systems.

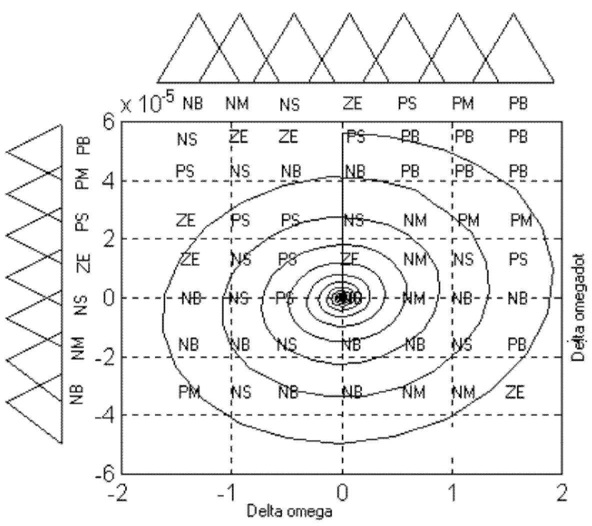

Fig. 1 Phase-plane plot of $\Delta\omega$ & $\Delta\omega^*$ for uncontrolled case of SMIB System.

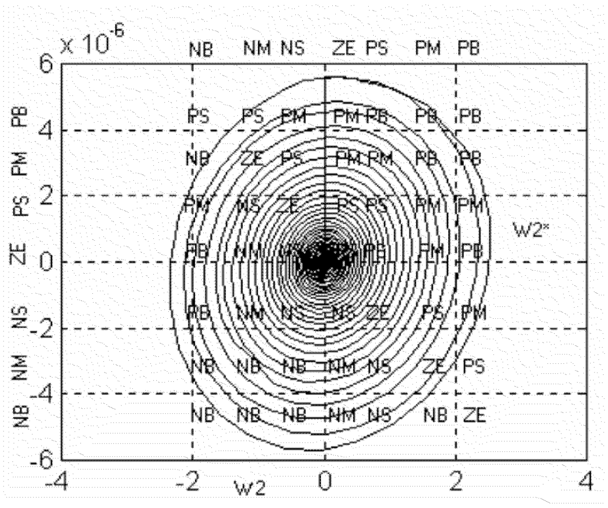

Fig. 2 Phase-plane plot of MMPS uncontrolled W2 Vs W2*.

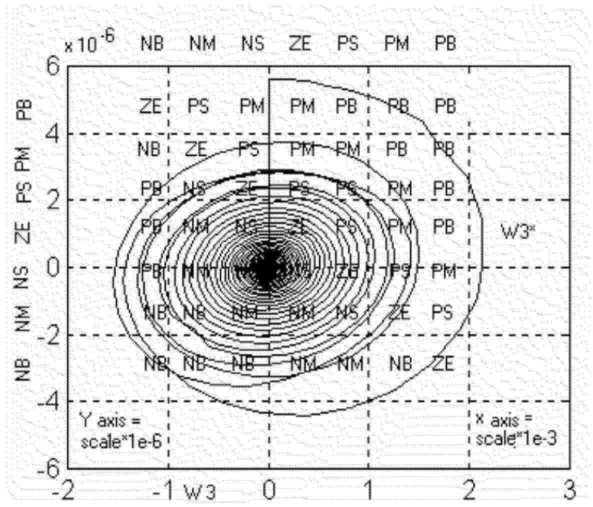

Fig. 3 Phase-plane plot of MMPS uncontrolled case W3 Vs W3*.

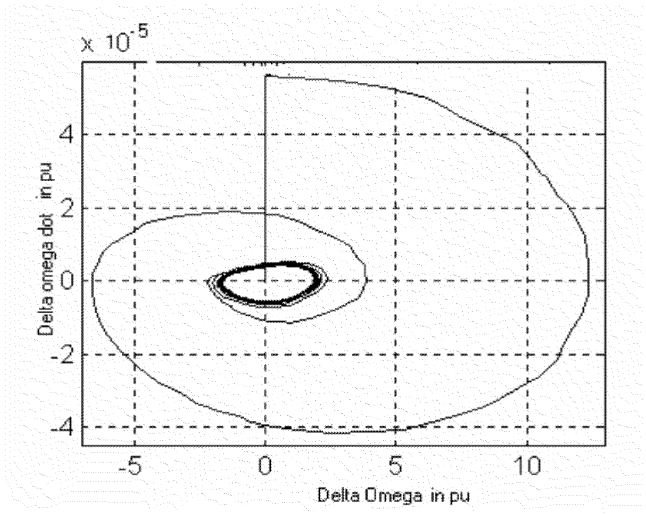

Fig. 4 Phase-plane plot of SMIB with fuzzy PSS Limit cycle 1.

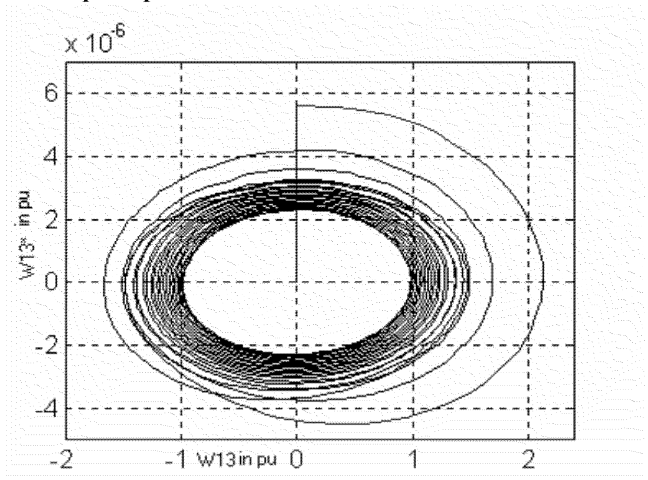

Fig. 5 Phase-plane plot of MMPS with fuzzy PSS Limit cycle 1.

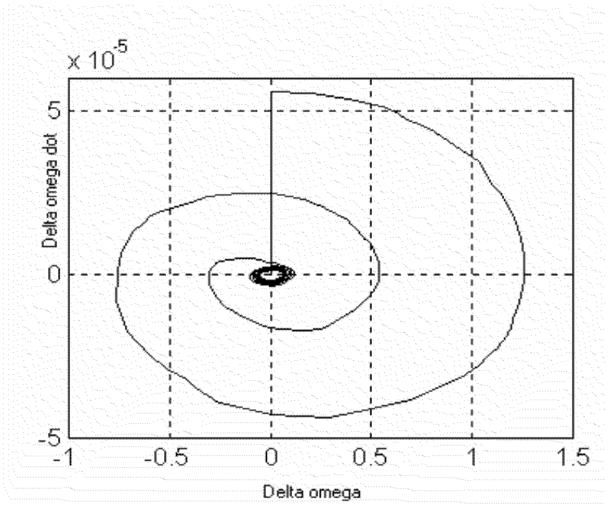

Fig. 6 Phase-plane plot of SMIB with fuzzy PSS Limit cycle 2.

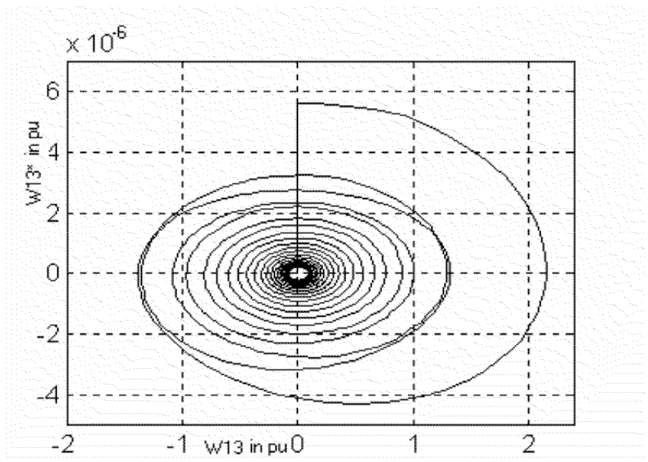

Fig. 7 Phase-plane plot of MMPS with fuzzy PSS Limit cycle 2.

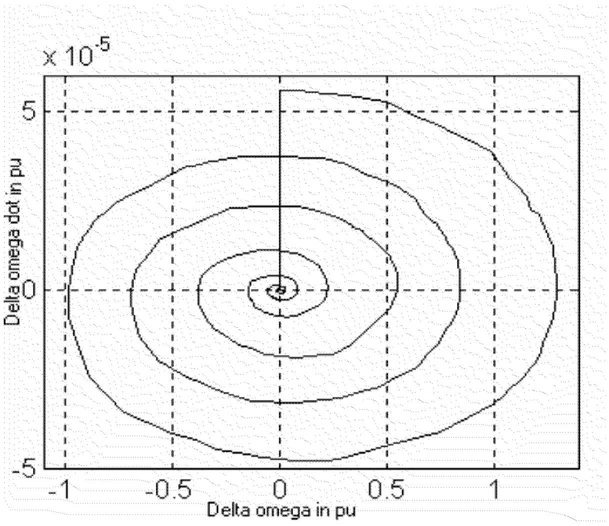

Fig. 8 Phase-plane plot of SMIB with fuzzy PSS Limit cycle 3.

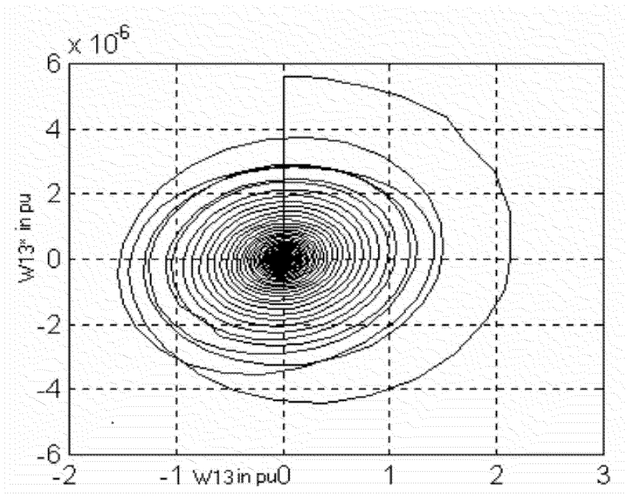

Fig. 9 Phase-plane plot of MMPS with fuzzy PSS Limit cycle 3.

VI. Simulation And Results

The phase plane plots of $\Delta\omega$ Vs $\Delta\omega^{\cdot}$ and $\Delta\delta$ Vs $\Delta\delta^{\cdot}$ for investigating the stability in the sense of Lyapunov were observed for SMIB system and for Multi-machine power system. The stability of linear systems can be studied by simply eigen value analysis. Since the equations for single machine infinite bus system (SMIB) and multimachine power system (MMPS) (i.e. 3-machine 9-bus system) were linearised and simulated. Fuzzy logic controller is highly nonlinear and the eigen values of the system cannot be obtained for this controller. Hence phase-plane and limit cycle analyses are carried out by using the Lyapunov stability studies and were attempted for investigating the stability. Another advantage of exploiting the phase-plane plots is that, it provides information of fuzzy rule base formation and for the rule base stabilization. Stability of fuzzy logic controllers is discussed in detail in this paper using this concept.

A. Cases study I-Single machine infinite bus (SMIB) system

To demonstrate the effectiveness of rule base development for SMIB system, a case study of limit cycles were performed. The rules were framed heuristically in the area of the phase-plane plot which is shown in Fig. 1, thus obtained for uncontrolled (with out stabilizer) case. The resultant phase plane plot (with fuzzy logic based stabilizer) is shown in Fig. 4. The limit cycle shown in Fig. 4 is with a definite area and thus the trajectory in the phase-plane plot is not converging towards the origin, hence the rules falling in this area needs to be modified. Each rule in this area is modified exactly by an opposite inference; i.e., NS (negative small) is an opposite inference to PS (positive small) etc. Hence NB, NM and NS are the opposite inference rules to PB, PM and PS respectively. If the rule to be modified found to be ZE (zero) then it is modified as NS or PS i.e. one linguistic step to the left or to the right of ZE. Hence after modifying the rules, which are in the area of limit cycles of Fig. 4, the resultant phase-plane plot shown in Fig. 6, is observed with these modified rules. This method of rule base modification is repeated until the resultant phase-plane plot is stabilized, that is shown in the Fig. 8. Graphical results are shown for the rule base mapping and for the elimination of limit cycles to obtain stabilized plots. The stabilized plots were obtained within three steps of such rule base modifications for SMIB system.

B. Case study – II: - Multi-machine power system (MMPS)

The effectiveness of rule base development for MMP system is analyzed by considering, a case study of limit cycles. The rules were framed heuristically in the areas of the phase-plane plots Fig. 2 and Fig. 3, thus obtained for uncontrolled (with out any stabilizer) case. The resultant phase-plane plot (with fuzzy logic based stabilizer) is shown Fig. 5. This figure has limit cycles with definite area and thus the trajectory in this phase-plane plot is not converging towards the origin, hence the rules falling in this non converging area needs to be modified. Each rule in these areas was modified exactly by an opposite inference. Hence, after modifying the rules, which are in these areas of limit cycles, the resultant phase-plane plot Fig. 7 is observed with these modifications. This method of rule base modification is repeated until the resultant phase-plane plot is stabilized shown in Fig. 9. Graphical results are shown for the rule base mapping and for the elimination of limit cycles to obtain stabilized plots. The stabilized plots were obtained within three stages of such rule base modifications are shown for MMPS.

VII. Conclusions

In this paper phase-plane and the limit cycle, analysis has been utilized to investigate the stability of fuzzy controlled power system stabilizer and the fuzzy controller inference engine has been designed even when no expert knowledge is available. The effectiveness of the phase-plane analysis has been demonstrated on single machine infinite bus system and on multimachine power system. The conclusions are:

1. Asymptotic stability for the fuzzy controlled power system stabilizer has been inferred using phase-plane analysis this also assures small signal stability.
2. The rule base stabilization (fuzzy inference engine) can be obtained in two to three steps of rule base modifications.
3. The rules need to be modified are the rules falling in the areas of limit cycles and not the entire rule base.
4. With out the expert knowledge rule base can be designed.

Hence, fuzzy controlled power system stabilizers can be utilized for the real-time control of power systems.

Appendix

Nomenclature:

K_1 = Change in Electrical Power for a change in rotor angle with constant flux linkage.
K_2 = Change in Electrical Power for a change in the direct axis flux linkage with constant rotor angle.
τ'_{do} = Direct axis open circuit time constant of the machine.
K_3 = An Impedance factor, and K_4 = Demagnetizing effect of a change in rotor angle (At steady state).
K_e = Regulator Gain, T_e = Regulator Time constant.
$V_{t\Delta}$ = Change in Synchronous machine terminal Voltage.
$K_5 = V_{t\Delta}/\delta_\Delta$ = Change in the terminal Voltage with change in rotor angle for constant E'_Δ.
$K_6 = V_{t\Delta}/E'_\Delta$ = Change in the terminal voltage with change in E' for constant δ.

Data for 3-machine 9-bus system**:** - (All flows are in MW & MVAR)

Generator 1 (G1): - 71.6 + j27, Generator 2 (G2): - 163 + j6.7 and Generator 3 (G3): - 85 – j10.9.

Load A: - 125 + j50, Load B: - 90 + j30 & Load C: - 100 + j35.

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