Particle Swarm Optimization Based Method for Optimal Siting and Sizing of Multiple Distributed Generators

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***Abstract*—This paper presents a method for optimal siting and sizing of multiple distributed generators (DGs) using particle swarm optimization (PSO). A simple and effective cumulative performance index, utilizing voltage profile improvement, loss reduction and voltage stability index (VSI) improvement is considered in this work. The system loss is minimized using PSO considering constant power as well as voltage dependent load models. The effectiveness of the proposed method is demonstrated on three test systems and the results are compared with the analytical and classical optimization methods.**

***Index Terms*- Distributed Generation, optimal sizing and siting, particle swarm optimization, voltage stability index.**

I. INTRODUCTION

Distributed generation, which is known as embedded generation in Anglo-Saxon countries, dispersed generation in North American countries and decentralized generation in Europe and Asian countries [1], is an electric power source connected directly to the distribution network or on the customer side of the meter. DGs are feasible alternative for new capacity addition, especially in the competitive electricity market environment, and offer benefits such as short lead time and low investment risk. It is built in small-capacity modules that can track load variation more closely. DGs can be renewable energy sources (such as wind, solar and biomass), or internal combustion reciprocating engines [1, 24].

Due to considerable costs, the DGs must be allocated suitably with optimal size to improve the system performance such as to reduce the system loss, improve the voltage profile while maintaining the system stability. The problem of DG planning has recently received much attention by power system researchers. Selecting the best places for installing DG units and their preferable sizes in large distribution systems is a complex combinatorial optimization problem. Different formulations have been used based on calculus-based methods, search-based methods and combinations of various approaches, such as, gradient and second-order algorithms [2], Hereford Ranch algorithm [3], heuristic iterative search method [4,10], analytical method [5,9,13,16,23], Tabu search [6], hybrid fuzzy- Genetic Algorithm (GA) method [7], GA [8,17,19,22], linear programming (LP) method [11].A comparison of various methods are presented in [20]. The summary of various works done on DG placement is briefly mentioned in Table I.

In this paper, a Particle Swarm Optimization (PSO) approach has been used to determine the optimal size and location of the DGs by the minimizing power loss while maintaining the voltage profile and stability margin. The effectiveness of the proposed approach is demonstrated on 12-bus, 30-bus and 69-bus test systems [16,20,23]. A cumulative performance index (CPI) is suggested to compare the performance of various methods.

II. PROBLEM FORMULATION

Consider a network with *n* nodes. The effect of DG is considered as negative real power load at the buses. Hence, the real power balance equation has only been modified. In this paper, change in loss, voltage, and a voltage stability index has been used for calculating the cumulative performance index to compare various approaches. The following conditions have been incorporated in the algorithm to obtain the desired result.

(i) Any DG size between zero and the sum of the total load can be randomly generated using (11) in the initial population and, subsequently, in the next generation population, by swarm operation.

(ii) All the bus locations except the slack bus, is tried for optimal location for DG placement one by one as per the algorithm shown in Fig. 2.

The loss minimization formulation utilizes an objective function, *f* given by

$$f = \sum_{x=1}^{n} P_{Loss(x)} \tag{1}$$

Subject to the following constraints,

$$P_{gi} + P_{dgi} - P_{li} = 0, \tag{2}$$

$$Q_{gi} + Q_{dgi} - Q_{li} = 0, \tag{3}$$

$$V_{\min} \le V \le V_{\max}, \tag{4}$$

where *x*, P_{gi} and Q_{gi} are the branch number, the real and reactive power generated at bus-*i*, respectively; P_{dgi} and Q_{dgi} are the real power and reactive power injected by the DG at bus-*i*.

A. Voltage Stability Index for Radial Distribution Networks

In [20, 28-29], the authors presented a voltage sensitivity analysis technique that calculates an index at each node to identify the most sensitive node for the voltage collapse. The index is derived from a bi-quadratic equation, which is generally used for the voltage calculation in the distribution

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TABLE I

DISTRIBUTED GENERATOR PLACEMENT METHODS

References	Objectives	Optimization/solution method
Rau *et al.* [2]	Minimizing the loss, line loading and reactive power requirement in the network	Gradient and second-order method
Kim *et al.* [3]	Minimizing the system loss	Hereford Ranch algorithm
Griffin *et al.* [4]	Minimizing the system loss	Heuristic iterative method
Willis [5]	System loss minimization	Analytical based on 2/3 rule
Nara *et al.* [6]	Minimizing the system loss	Tabu search
Kim *et al.* [7]	Minimizing the system loss	Hybrid fuzzy nonlinear goal programming and GA
Teng *et al.* [8]	Minimizing the system loss and customer interruption costs	GA
Wang *et al.* [9]	Minimizing of system loss	Analytical approach
El-Khattam *et al.* [10]	Minimizing cost of investment and operation of DGs and system loss	Heuristic iterative search method
Keane *et al.* [11]	Maximizing DG capacity	Linear programming
Harrison *et al.* [12]	Maximizing DG capacity	Optimal power flow
Popoviv *et al.* [13]	Maximizing DG capacity	Sensitivity analysis
Carpinelli *et al.* [14]	Minimizing cost of system losses and improvement in voltage quality and harmonic distortions	Hybrid e-constraint-based multi-objective programming and GA
Celli *et al.* [15]	Minimizing cost of network upgrading, power losses, energy not supplied and energy required by the customers	Hybrid e-constraint-based multi-objective programming and GA
N. Acharya *et al.* [16]	Minimizing the system loss	Analytical approach
Borges *et al.* [17]	Minimizing the system loss and acceptable reliability level	GA
Durga *et al.*[18]	social welfare maximization and profit maximization	Optimal power flow
Teng *et al.* [19]	Maximizing the DG benefits in terms as cost, reliability, loss	GA
T. Gözel *et al.*[20]	Minimizing the system loss	Comparison of analytic and classical optimization method for DG placement
Harrison *et al.* [21]	evaluating network capacity for DG placement	GA and Optimal power flow
R. K. Singh *et al.*[22]	Minimizing the system loss	GA
T. Gözel *et al.*[23]	Minimizing the system loss	Analytical approach
A. Kumar *et al.* [24]	Minimization of fuel cost and system loss	Mixed integer non-linear programming
Ghosh *et al.* [25]	Minimization of cost and loss	Iterative search technique with load flow

load flow algorithms. Let us consider a distribution line model as given in Fig.1. The bi-quadratic equation, relating the voltage magnitude at the sending and the receiving ends and power at the receiving end of the branch, can be written as

$$V_j^4 + 2V_j^2(PR+QX) - V_j^2V_i^2 + (P^2+Q^2)\,|\,Z^2\,| = 0 \qquad (5)$$

Where, V_i and V_j stand for the phase voltage magnitudes at bus-*i* and bus-*j*, respectively, and *Z* is the line impedance. *R*, *X*, *P* and Q are the line resistance, reactance, line transferred active and reactive powers, respectively as shown in Fig.1.

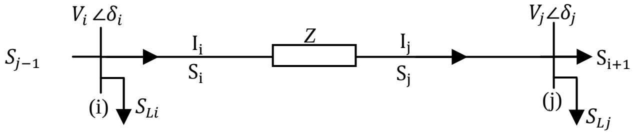

Fig.1. Two bus portion of a radial distributed system.

It is seen from the equation (5) that the receiving end voltage V_j has four roots and the maximum positive value of these roots, is a feasible solution and gives the line receiving end node voltage magnitude.

$$V_j = 0.707[b + (b^2 - 4c))^{1/2}]^{1/2} \qquad (6a)$$

$$b = V_i^2 - 2PR - 2QX \qquad (6b)$$

$$c = ((P^2+Q^2)(R^2+X^2)) \qquad (6c)$$

From (6a), it is clearly seen that the real value of the receiving end voltage magnitude will exist and a critical loading point is reached, when (6d) related the any line is zero.

$$b^2 - 4ac \geq 0 \qquad (6d)$$

This equation is defined as the Voltage Stability Indicator of the line's receiving end bus as follows:

$$VSI(j) = V_i^4 - 4(PX+QR) - 4V_i^2(PR+QX) \qquad (7)$$

After the load flow study, the node voltages and the branch currents are known, Therefore, *P* and *Q* at the receiving end of each line can easily be calculated and, hence, using (7), the voltage stability index of each bus can be calculated. The node, at which the value of the stability index *VSI*(*j*) is minimum, is the most sensitive to the voltage collapse [31].

B. *Load Modeling*

In conventional load flow studies, it is assumed that the active and the reactive power demands are constant regardless the voltage at that bus. In actual power systems, different categories and types of loads, such as domestic, industrial and commercial loads, might be present. The nature of these types of loads is such that their active and reactive powers are dependent on the voltage and frequency of the system. Moreover, load characteristics have significant effects on load flow solutions and convergence ability. Common static load models for active and reactive powers are expressed in a polynomial or an exponential form. The exponential load models can be given as:

$$P_{li}(i) = P_{l0}(i)(V/V_0)^p \qquad (8)$$

$$Q_{li}(i) = Q_{l0}(i)(V/V_0)^q \qquad (9)$$

Where, *p* and *q* stand for load exponents, $P_{li}(i)$ and $Q_{li}(i)$ stand for the values of the active and reactive powers at i^{th} bus at the nominal voltages, V and V_0 stand for the load bus voltage and the load nominal voltage, respectively [30].

C. Cumulative Performance Index (CPI)

The CPI is an index proposed to quantify the overall benefits to place DG and its normalized value has been used in the simulation results. Among the several benefits offered by the DG, only three major ones are considered in this paper: voltage profile improvement, line-loss reduction, and voltage stability improvement. Therefore, CPI is formulated as:

$$CPI = \begin{bmatrix} \alpha_1\left(\dfrac{\Delta S_L}{S_{L0}}\right) + \alpha_2\left(\dfrac{\Delta V_{Min}}{V_{Min0}}\right) + \alpha_3\left(\dfrac{\Delta VSI_{Min}}{VSI_{Min0}}\right) + \\ \alpha_4\left(\dfrac{\Delta V_{Max}}{V_{Max0}}\right) + \alpha_5\left(\dfrac{\Delta VSI_{Max}}{VSI_{Max0}}\right) \end{bmatrix} \tag{10}$$

where, $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5 are the constants and their values lie between 0 to 1. In this paper, these are taken as unity. ΔS_L, ΔV_{Min}, ΔVSI_{Min}, ΔV_{Max} and VSI_{Max} are the change in the total power loss, change in minimum voltage, change in minimum VSI and change in maximum voltage and change in maximum VSI with the DG plant. S_{L0}, V_{Min0}, VSI_{Min0}, V_{Max0} and VSI_{Max0} are the total power loss, minimum voltage, minimum VSI and maximum voltage and maximum VSI, respectively, without DG.

III. PROPOSED ALGORITHM

In the test systems, bus 1 has been considered the slack bus and is not considered for the DG placement. A simple PSO based method to solve for optimal location and size of DGs simultaneously has been proposed as shown in Fig. 2. The initial values (sizes) of the DGs are randomized for all definite particles of the PSO. Moreover, the PSO algorithm is executed to optimize the fitness function, which is defined in (1). $\Delta Loss$ is change in loss of the system and $\in$ is the desired tolerance in loss, here 04 % of base case is considered.

A. PSO Algorithm

The Particle Swarm Optimization (PSO) algorithm is one of the Evolutionary Computation (EC) techniques. PSO is a population-based and self-adaptive technique introduced originally by Kennedy and Eberhart in 1995 [32-33]. This stochastic-based algorithm handles a population of individuals, in parallel, to probe capable areas of a multidimensional space where the optimal solution is searched. The individuals are called particles and the population is called a swarm. Each particle in the swarm moves towards the optimal point with adaptive velocity. Each particle in the population is treated as a mass-less and volume-less point in a n-dimensional space. Mathematically, the position of particle in a n–dimensional vector is represented as:

$$X_m = (x_{m,1}, x_{m,2}, x_{m,3}, \ldots x_{m,n}) \tag{11}$$

The velocity of this particle is also an n-dimensional vector,

$$V_m = (v_{m,1}, v_{m,2}, v_{m,3}, \ldots v_{m,n}) \tag{12}$$

Alternatively, the best position related to the lowest value (for minimization objective) of the objective function for each particle is

$$Pbest_m = (pbest_{m,1}, pbest_{m,2}, pbest_{m,3}, \ldots pbest_{m,n}),$$

and the global best position among all the particles or best *pbest* is denoted as:

$$Gbest_m = (gbest_{m,1}, gbest_{m,2}, gbest_{m,3}, \ldots gbest_{m,n}),$$

During the iteration procedure, the velocity and position of the particles are updated.

It should be noticed that the value of DG size varies between 0 to the sum of the loads. This is regarded as the position of a particle during the optimization process. The steps used in the proposed algorithm are given below.

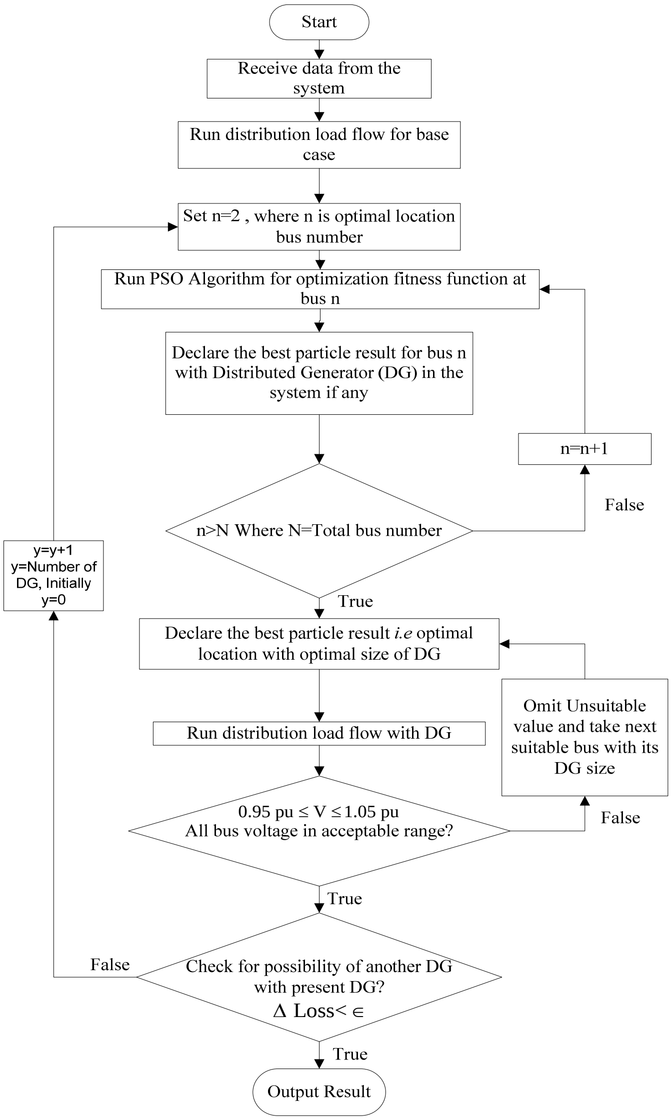

Fig.2. Algorithm for distributed generator placement

Step 1: (Input System Data and Initialize): In this step, the distribution system configuration data, with constraints, such as maximum-minimum allowed voltages and DG size range are specified. The population size of swarms and iterations are set. The population of m^{th} particles X_m (consisting of only real part of DG size varies between 0 to sum of loads) as well as their velocity V_m in the search space is initialized in this step. Vectors X and V are described as shown in (11-12). The PSO weight factors are also set in this step.

Step 2: (Calculate the Objective Function): The calculation of the objective function (1) is carried out by "Forward-Backward Sweep Method" of distribution load flow [30].

Step 3: (Calculate Pbest): The objective function related to each particle in the population of the current iteration is compared with it in the previous iteration and the position of the particle enjoying a lower objective function as *Pbest* for the current iteration is recorded,

$$pbest_m^{k+1} = \begin{cases} pbest_m^k \text{ if } f_m^{k+1} \geq f_m^k \\ x_m^{k+1} \text{ if } f_m^{k+1} \leq f_m^k \end{cases} \tag{13}$$

Where, k is the number of iterations, and f is the objective function evaluated for the particle.

Step 4: (Calculate Gbest): In this step, the best objective function associated with the *Pbests* among all particles in the current iteration is compared with that in the previous iteration and the lower value is chosen as the current overall *Gbest*

$$Gbest_m^{k+1} = \begin{cases} Gbest_m^k \text{ if } f_m^{k+1} \geq f_m^k \\ pbest_m^{k+1} \text{ if } f_m^{k+1} \leq f_m^k \end{cases} \tag{14}$$

Step 5: (Update Velocity): After calculation of the *Pbest* and *Gbest*, the velocity of particles for the next iteration should be modified by using

$$V_m^{k+1} = \omega V_m^k + C_1 rand(pbest_m^k - X_m^k) + C_2 rand(Gbest^k - X_m^k) \tag{15}$$

where, V_m^k, ω, C_1, C_2, X_m^k, $pbest_m^k$, and $Gbest^k$ are the velocity of particle m at iteration k, inertia weight factor, acceleration coefficients, position of particle m at iteration k, best position of particle m at iteration k and best position among all the particles at iteration k, respectively.

In the velocity updating process, ω, the inertia weight, and C_1, C_2 the acceleration coefficients, should be determined in advance. The acceleration coefficients have two values in the range of (1,2), and represent the weighting of the stochastic acceleration terms that pull each particle towards the individual best position and the overall best position, rand is random functions, generating separate random values in the range [0, 1], and ω is the inertia weight factor, defined as follows:

$$\omega = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{iter_{max}} \times iter \tag{16}$$

where, ω_{max}, ω_{min}, $iter$, $iter_{max}$ - Initial inertia weight factor, final inertia weight factor, current iteration number, maximum iteration number, respectively.

Step 6: (Update Position): The position of each particle at the next iteration *(k+1)* is modified as

$$X_j^{k+1} = X_j^k + V_j^{k+1} \tag{17}$$

Step 7: (Check Convergence Criterion): If $X_j^{k+1} - X_j^k < \varepsilon$, or $iter = iter_{max}$, the program is terminated and the results are printed. Otherwise, the program goes to the Step 2. From (15), one can find that the current flying velocity of a particle comprises of three terms. The first term is the particle's previous velocity revealing that a PSO system has memory. The second term and the third term represent a cognition-only model and a social-only model, respectively. The cognition-only model treats individuals as isolated and reflects private thinking. Whereas, the social-only model implies that the individuals compare the effectiveness of neighbors' beliefs and change towards those which are relatively successful [26].

IV. SIMULATION RESULTS

The proposed method for the DG placement and sizing is demonstrated on 12-bus, 30-bus and 69-bus systems. These systems have LV feeder. The line and bus data with load modeled as constant power type are given in [16,20,23]. The constant and voltage dependent loads with p and q equal to one are considered for simulations [27,30].

A. Radial Feeder with Time Invariant Constant Power Loads and Single DG

In this case, only one DG is considered for the placement. The base case results of the load flow on 12-bus and the 30-bus systems are shown in Tables II. With the placement of one DG, the results are presented in Tables III-IV. For the 12-bus and 30-bus systems, the proposed method has maximum CPI. In the 30-bus system, the Golden section search method fails to converge.

TABLE II

BASE CASE RESULTS OF 12 AND 30- BUS SYSTEMS

Test system	12 bus	30 bus
Total load (MVA)	0.5943	10.2553
Real Power loss (MW)	0.0207	0.8819
React. Power loss (MVAr)	0.0081	0.2581
Min. Volt. (pu)(at bus)	0.9434 (12)	0.8825 (27)
Max. Volt. (pu) (at bus)	1 (1)	
Min.VSI(at bus)	0.7920 (12)	0.6065 (27)
Max.VSI(at bus)	1 (1)	

TABLE.III

RESULTS WITH ONE DG IN THE 12 BUS SYSTEM

Method	Proposed Method	Analytic[16]	Analytic[23]	Golden[20]
Optimal bus location	9	9	9	9
Opt. DG size (MW)	0.24	0.23	0.23	0.24
Real Power loss (MW)	0.0108	0.0108	0.0108	0.0108
React. Power loss(MVAr)	0.0041	.0042	0.0042	.0042
Min. Volt (pu) (at bus)	0.9835 (7)	0.9823 (12)	0.9823 (12)	0.9835 (7)
Max. Volt.(pu) (at bus)	1(1)			
Min.VSI (at bus)	0.9357 (7)	0.9311 (12)	0.9311 (12)	0.9357 (7)
Max.VSI (at bus)	1(1)			
CPI (norm.)	1	0.97	0.97	0.99

B. Radial Feeder with Time Invariant Constant Power Loads and Multiple DGs

In this case, two DGs are considered for placement in the system of case A and the results are presented in Tables V&VI. For the 12 -bus system, the proposed method and the Golden section search methods give solution and their CPI values are found as one. For the 30-bus system, the proposed method is only able to provide the solution.

C. *Radial Feeder with Time Invariant Voltage Dependant Loads and Single DG*

In this case, the base case results are shown in Table VII and one DG is considered for the placement considering the voltage dependant loads (*p* and *q* are taken unity [27,30]) and results are presented in Table VIII. For this system, the analytic methods and the proposed method give almost same values of CPI.

TABLE IV

RESULTS WITH ONE IN THE 30 BUS SYSTEM

Method	Proposed Method	Analytic [16]	Analytic [23]	Golden [20]
Optimal bus location	21	21	21	Does not converge
Opt. DG size (MW)	5.7983	5.4901	5.4901	
Real Power loss(MW)	0.3454	0.3467	0.3467	
React. Power loss(MVAr)	0.0930	0.0941	0.0941	
Min. Volt (pu) (at bus)	0.9589 (12)	0.9571 (12)	0.9571 (12)	
Max. Volt.(pu) (at bus)	1(1)			
Min.VSI (at bus)	0.8453 (12)	0.8390 (12)	0.8390 (12)	
Max.VSI (at bus)	1(1)			
CPI (norm.)	1	0.99	0.99	

TABLE.V

RESULTS WITH TWO DGS IN THE 12 BUS SYSTEM

Method	Proposed Method	Analytic [16,23]	Golden[20]
II Optimal bus location	4	Does not give solution	4
II Opt. DG size (MW)	0.1501		0.1501
Real Power loss (MW)	0.0099		0.0099
React. Power loss (MVAr)	0.0038		0.0038
Min. Volt.(pu) (at bus)	0.9890 (7)		0.9890 (7)
Max. Volt.(pu) (at bus)	1 (1)		1 (1)
Min.VSI (at bus)	0.9568 (7)		0.9568 (7)
Max.VSI (at bus)	1 (1)		1 (1)
CPI (norm.)	1		1

TABLE.VI

RESULTS WITH TWO DGS IN THE 30 BUS SYSTEM

Method	Proposed Method	Analytic [16,23]	Golden [20]
II Optimal bus location	10	Does not give solution	Does not converge
II Opt. DG size (MW)	2.0456		
Real Power loss (MW)	0.2804		
React. Power loss(MVAr)	0.0761		
Min. Volt (pu) (at bus)	0.9811 (27)		
Max. Volt. (pu) (at bus)	1 (1 &21)		
Min.VSI (at bus)	0.9264 (27)		
Max.VSI(at bus)	1 (1 &21)		
CPI (norm.)	1		

TABLE VII

BASE CASES RESULTS IN THE 69 BUS SYSTEM

Total load (MVA)	4.5881
Real Power loss (MW)	0.1915
React. Power loss(MVAr)	0.0878
Min. Volt. (pu) (at bus)	0.9167 (65)
Max. Volt.(pu) (at bus)	1 (1)
Min.VSI (at bus)	0.7062 (65)
Max.VSI(at bus)	1 (1)

D. *Radial Feeder with Time Invariant Voltage Dependent Loads and Multiple DGs*

In this case, two DGs are considered for placement on the 69-bus system with voltage dependent load, as in the case C and results are presented in Table IX. For considered case, analytic methods do not give solution and both proposed method and Golden section search methods give same result.

TABLE. VIII

RESULTS WITH ONE DG IN THE 69 BUS SYSTEM

Method	Proposed Method	Analytic [16]	Analytic [23]	Golden [20]
Optimal bus location	61	61	61	61
Opt. DG size (MW)	1.7766	1.8090	1.8090	1.7768
Real Power loss (MW)	0.0798	0.0799	0.0799	0.0798
React. Power loss (MVAr)	0.0391	0.0391	0.0391	0.0391
Min. Volt. (pu) (at bus)	0.9688 (27)	0.9690 (27)	0.9690 (27)	0.9688 (27)
Max. Volt.(pu) (at bus)	1 (1)			
Min.VSI (at bus)	0.8809 (27)	0.8816 (27)	0.8816 (27)	0.8809 (27)
Max.VSI (at bus)	1 (1)			
CPI (norm.)	0.99	1	1	0.99

TABLE.IX

RESULTS WITH TWO DGS IN THE 69 BUS SYSTEM

Method	Proposed Method	Analytic [16,23]	Golden [20]
II Optimal bus location	17	Does not give solution	17
II Opt. DG size (MW)	0.5067		0.5071
Real Power loss (MW)	0.0694		0.0694
React. Power loss(MVAr)	0.0349		0.0349
Min. Volt. (pu) (at bus)	0.9801 (65)		0.9801 (65)
Max. Volt.(pu) (at bus)	1 (1)		1 (1)
Min.VSI (at bus)	0.9227 (65)		0.9227(65)
Max.VSI (at bus)	1 (1)		1 (1)
CPI (norm.)	1		1

V. CONCLUSION

This paper has presented the results of a PSO based algorithm to the optimal allocation of multiple DGs in the distribution networks. The effectiveness of the proposed algorithm in solving DG allocation problem has been demonstrated on distribution test feeders having 12-buses, 30-buses and 69-buses. The results on various test cases reveal that the proposed method performs better or at least similar in comparison with the other classical and analytical Methods for the single DG placement problem.

However, In case of multiple DG placement, only the proposed algorithm provides solution for all systems. The placement of third DG in all the three systems is not recommended as it violates the constraint of 04% benefit in the real power loss by adding another DG. Hence, the optimal numbers of DG for the considered systems are two with the constant power as well as voltage dependent load models.

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