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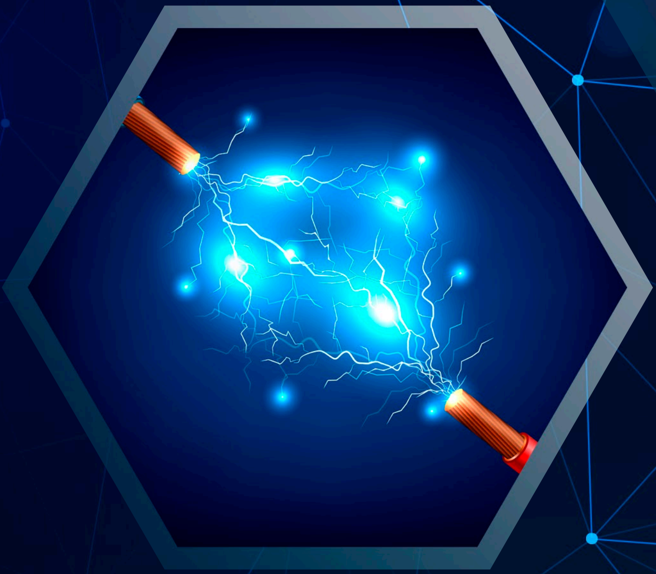

PHY 111A

LABORATORY MANUAL

Department of Physics

IIT Kanpur

PHY 111A

Laboratory Manual

(2025-26-I)

Department of Physics

Indian Institute of Technology

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PHY 111 A (2025-26-I)

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(A1, A2, A3, A9)	Wednesday	**AUG:** 6, 13, 20, 27 **SEP:** 3, 10, 24 **OCT:** 8, 15, 22, 29 **NOV:** 12
(B5, B7, B9, B10)	Thursday	**AUG:** 7, 14, 21, 28 **SEP:** 4, 11, 25 **OCT:** 9, 16, 23, 30 **NOV:** 6,13
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*** Saturday Make-Up Lab**

- *Remember your Lab turn and come prepared. No additional explanations will be provided during your lab turn by the T.A. For your experiment you must come prepared to the lab.*
- *For your experiments consult the notice board outside Physics 111A lab.*
- *Use of mobile phone is strictly prohibited in the lab.*
- *Students aren't allowed in the lab without proper shoes.*
- *Few optional experiments based on error analysis are available. Those who are interested may ask Mr. Arvendra Singh Rathaur (In-charge UG Lab) for the handout and perform the experiments when you are free.*

Instructions

To Students

Objectives of the Laboratory Course:

- ✓ *To provide an opportunity for learning through doing, observing and testing.*
- ✓ *To become familiar with the instruments and gain experience in handling them.*
- ✓ *To acquire the skill of making the optimum use of the given apparatus.*
- ✓ *To establish for yourself the working of physical principles.*
- ✓ *To become aware of limitations of the accuracy with which measurements can be made.*
- ✓ *To relate textbook information to the behavior of the physical world around you.*

As scientists and technologists, you will be dealing with instruments and apparatus of various kinds throughout your career. You will greatly benefit from the beginning if you take a serious and enthusiastic attitude towards experimental work. There is no substitute for the experience that you gain by carrying out even simple laboratory measurements.

➔ **Laboratory work** is the heart of physics and it should be taken seriously. Your grades in this course are based on your performance in the weekly sessions, the reports you write and on how well you do in the examinations.

➔ **Preparation:** Before coming to the laboratory session, you must carefully read the instructions given for performing the experiment of the day. Unless you come fully prepared with this background material you will not be able to complete the required work and, what is more, you will miss the opportunity of learning all aspects of the experiment. Lack of background often makes the experiment uninteresting and much more time has to be spent later for an understanding of the points missed. Thus, for your own benefit, prior study of the instructions sheets is very important. Also, if you are unprepared, you will not do well in the quiz conducted by your instructor during the session. In some cases, you may find it helpful to go to the website of PHYWE, *(www.phywe.de)* who has supplied some of the experiments. Short video clips of some of the experiments are available in the lab. You may take these from ***Mr. A.S. Rathaur***, In-charge UG Lab.

➔ Printed **Report Sheets** are available in the bookstore. All work must be done in ink. *You must get at least one observation of each kind checked and signed by your instructor, failing which your report will not be graded.* You must complete all experimental and report writing work during the session. Every observation made must be recorded directly on the Report Sheet. No ***rough*** record is allowed. The completed Report must be submitted within the 3- hour lab period on the same day.

➔ **Equipment needs your care:** On reaching the laboratory you should check the apparatus provided and ascertain if there are any shortages or malfunctions. Set up the equipment in accordance with the instructions. Proceed carefully and methodically. Remember that scientific equipment is expensive and quite susceptible to damage. So, handle it carefully. If the apparatus is complicated, consult the instructor before you proceed with the actual performance of the experiment. Make the required measurements and record them neatly in tabular form. Double-check to make sure that you have recorded all the necessary data.

➔ **Acceptable results with given apparatus:** It is more important to see what result you get with given apparatus rather than what is the 'correct' result. The apparatus given to you is capable of certain accuracy and your result may be completely acceptable even if it differs from 'correct' results. You must learn to do things on your own even if you might make mistakes some time.

➔ **Graphs:** Each graph should occupy one complete sheet; the information as to quantities plotted, scale chosen and units should be mentioned clearly on the graph.

➔ Following is the **Format of the Report:**

- Your name, roll number, instructor's name, date, title of the experiment.
- Essential diagram of the experiment and the formulae used.
- Templates of the tables and the format are attached at the end of each experiment. You are expected to use this format in your report sheet. You must come with the prepared report sheet on the lab day.
- Relevant substitutions and results expressed with the estimated error and units.
- Conclusions including what you learnt from the experiment.
- Precautions you took to do the experiment. Measure sources of error. Criticism of the design. Suggestions for improving the experiment.

➔ You must bring ***manual, pen, pencil, eraser, graph sheet, calculator, transparent ruler and report sheet*** with you.

➔ **You must keep your workplace neat and clean and leave the lab. Neat and tidy.**

➔ **Punctuality:**

- Each report should be submitted on the day of your experiment.
- Please come to the lab on time, as being late may mean deduction of marks.

Essentials of
Data Analysis

This chapter tries to give you a working knowledge of data analysis which would be essential in all experiments that you do. Specifically, the following topics are covered:

- *Graphical Analysis*
- *Error Analysis*
- *Significant Figures*

Please try and do the home assignments given in this chapter. During regular lab sessions you will perform the experiment, analyze the data, write the report and submit by 1:15 pm before leaving the lab.

GRAPHICAL ANALYSIS

It is said that a picture is worth a thousand words. A graph can succinctly represent an ocean of information. Scientists and engineers use them as visual aids to recognize and communicate patterns ('all knowledge is pattern recognition!'), discover relationships between physical variables, and extract meaning and characteristics from data. In our experiments we are going to rely on them heavily, hence we will begin by mastering the fundamentals of plotting graphs. Follow the guidelines given below for all your graphs:

- Use sharp pencils.
- Draw on full page of graph paper (compressed ones don't help analysis). Use appropriate scale factors.
- Plot the dependent variable on the vertical y-axis and the independent variable on the x-$axis$.
- Graphs are ***meaningless*** unless the axes are labeled. Label them and mention units in parentheses.
- Give your graph an appropriate title.
- If the quantities being plotted are within an error range indicate that in error bars.

Virtually all the patterns that you will encounter in this Laboratory course will have one of the following mathematical forms:

i) $y = mx^n + c$, where n is some exponent and m is some numerical constant,

ii) $y = ae^{bx} + c$, where a, b & c are constants.

The constants will have physical significance, and hence an important part of analyses of laboratory data involves finding the values of n and m in (i) and a, b and c in (ii) which best describe a set of data pairs (x, y).

Can you make qualitative sketches of these functions?

For (i) above, consider the cases for which $n = 1, n > 1, 0 < n < 1$?

HOME ASSIGNMENT #1

The force F of wind resistance acting on a ball is found to depend on the diameter d of the ball. Plot the following data:

$\boldsymbol{d\ (in\ cm)}$	2.0	4.0	6.0	8.0	10.0
$\boldsymbol{F\ (in\ N)}$	0.11	0.46	0.90	1.83	2.51

Examine the graph you have just constructed. Identify which of the above functions it resembles qualitatively.

Modify the given variables so that the graph can be cast into the form of a straight line and obtain the equation that best represents the relationship.

HOME ASSIGNMENT #2

The time t it takes for a volume of water to drain from a tub is found to depend on the diameter d of the drain hole. Repeat the above exercise for the following data:

$\boldsymbol{d\ (in\ cm)}$	1.0	2.0	3.0	4.0	5.0
$\boldsymbol{t\ (in\ sec)}$	3620	860	420	230	140

HOME ASSIGNMENT #3

The famous astronomer Johannes Kepler took approximately 20 $years$ and 900 $pages$ of calculations to determine that there was a definite relationship between the distance of the planets from the sun and the amount of time it takes to orbit the sun. You know this relationship as Kepler's third law of planetary motion. Based on the data below determine the relationship:

Planet	**Distance (in miles)**	**Period (in years)**
Mercury	36,000,000	0.241
Venus	67,270,000	0.616
Earth	90,000,000	1.000
Mars	141,000,000	1.880
Jupiter	483,900, 000	11.860
Saturn	887,200,000	29.460
Uranus	1,784,000,000	84.3000
Neptune	2,793,000,000	164.070
Pluto	3,675,000,000	248.000

HOME ASSIGNMENT #4

Exponential relations describe many natural phenomena. Among these are growth and decay processes (such as in radioactivity, cooling etc.), and absorption phenomena. Below an unusual case of such an exponential relation is given. The data consists of measurements of width of successive growth spirals of a particular seashell.

$\boldsymbol{N\ (spiral\ no.)}$	1	2	3	4	5	6
$\boldsymbol{Width\ (in\ mm)}$	2.5	4.5	6.5	11.5	20.0	31.0

Obtain a linear formulation of the given data (hint: semi-logarithmic). Determine the mathematical form (including all constants involved) of the relationship.

HOME ASSIGNMENT #5

The voltage decay as a function of time across a capacitor in an RC circuit is given below.

$\boldsymbol{Time\ (in\ sec)}$	$\boldsymbol{Voltage\ (in\ V)}$
6.2	5.53
8.7	4.89
10.0	4.58
12.5	4.04
16.3	3.35
18.4	3.05
22.5	2.45
25.0	2.16
28.5	1.85
32.9	1.44
38.8	1.09
42.0	0.92
47.8	0.70
52.0	0.56
55.4	0.47
62.5	0.33
67.2	0.26

1. Obtain the value of characteristic decay time constant τ by plotting the data in a semi-logarithmic paper.
2. Obtain the initial value of the voltage across the capacitor.

HOME ASSIGNMENT #6

In an experiment, paper rings of different diameter are mounted on a vibrating table to study their resonant frequencies. Depending on the diameter, the rings show resonant vibration for different frequencies of the vibrating table. The data from this experiment is given below.

$\boldsymbol{Diameter\ of\ the\ ring\ (cm)}$	3.4	4.6	6.4	8.7	10.9	13.2
$\boldsymbol{Resonant\ frequency\ (Hz)}$	63.48	30.77	13.38	6.24	3.58	2.19

1. Plot the resonant frequency vs the diameter of the ring in a log-log graph to obtain the mathematical relationship between the two variables.
2. From your graph predict the resonant frequency for a ring of diameter 20 cm.

Errors in graphical analysis

The usual way of indicating errors in quantities plotted on a graph paper is to draw error bars. The curve should then be drawn so as to pass through all or most of the bars.

Straight line fits:

Here is a simple method of obtaining a straight line fit on a graph. Having plotted all the points, locate, visually, the centroid $(\bar{x}, \bar{y})$.

Then consider all straight lines through the centroid (use a transparent ruler) and visually judge which one will represent the data the best. Determine its slope m.

Having drawn the best line, estimate the error in its slope m, as follows. Rotate the ruler about the centroid until its edge passes through the top of the error bars at the 'top right' and through the bottom of the error bars at the 'bottom left'. This new line gives one extreme possibility, let the difference between the slopes of this and the best line be Δm_1. Similarly determine Δm_2 corresponding to the other extreme. The error in the slope may be taken as

$$\Delta m = \frac{\Delta m_1 + \Delta m_2}{2\sqrt{n}}$$

where n is the number of data points. The factor $\sqrt{n}$ comes because evaluating the slope from the graph is essentially an averaging process.

It should be noted that if the scale of the graph is not large enough, the least count of the graph may itself become a limiting factor in the accuracy of the result. Therefore, it is desirable to select the scale so that the least count of the graph paper is much smaller than the experimental error.

ERROR ANALYSIS

To err is human; to evaluate and analyze the error is scientific.

➔ Introduction

Every measured physical quantity has an uncertainty or error associated with it. An experiment, in general, involves (i) direct measurement of various quantities (primary measurements) and (ii) calculation of the physical quantity of interest which is a function of the measured quantities. An uncertainty or error in the final result arises because of the errors in the primary measurements (assuming that there is no approximation involved in the calculation).

Error analysis, therefore, consists of (i) estimating the errors in all primary measurements, and (ii) *propagating* the error at each step of the calculation. This analysis serves two purposes. First, the error in the final result is an indication of the precision of the measurement and, therefore, an important part of the result. Second, the analysis also tells us which primary measurement is causing more error than others and thus indicates the direction for further improvement of the experiment.

For example, in measuring 'g' with a simple pendulum, if the error analysis reveals that the errors in 'g' caused by measurements of $l\,(length\ of\ the\ pendulum)$ and $T\,(time\ period)$ are $0.5\ \mathrm{cm/sec^2}$ and $3.5\ \mathrm{cm/sec^2}$ respectively, then we know that there is no point in trying to devise a more accurate measurement of l. Rather, we should try to reduce the uncertainty in T by counting a larger number of periods or using a better device to measure time. Thus, error analysis *prior to the experiment* is an important aspect of planning the experiment.

Nomenclature of Errors

i) *Discrepancy* denotes the difference between two measured values of the same quantity.

ii) *Systematic errors* occur in every measurement in the same way – often in the same direction and of the same magnitude – for example, length measurement with a faulty scale. These errors can, in principle, be eliminated or corrected by proper analysis and calibration.

iii) *Random errors* cause the result of a measurement to deviate in either direction from its true value. We shall confine our attention to these errors and discuss them under two heads: estimated and statistical errors.

iv) A measurement which has small random errors has high *precision*. A measurement which has small random errors as well as systematic errors has high *accuracy*.

➔ Estimated Errors

An estimated error is an estimate of the maximum extent to which a measured quantity might deviate from its true value. For a primary measurement, the estimated error is often taken to be the least count of the measuring instrument. For example, if the length of a string is to be measured with a meter rod, the limiting factor is the accuracy in the least count, i.e., 0.1 cm. A note of caution is needed here.

What matters really is the *effective least count* and not the nominal least count. For example, in measuring electric current with an ammeter, if the smallest division corresponds to $0.1\ amp.$, but the marks are far enough apart so that you can easily make out a quarter of a division, then the effective least count will be 0.025 amp. On the other hand, if you are reading a Vernier scale where three successive marks on the Vernier scale (say $27^{th}, 28^{th}, 29^{th}$) look equally well in coincidence with the main scale, the effective least count is 3 times the nominal one. Take another example. In a null-point electrical measurement, suppose the deflection in the galvanometer seems to remain zero for all values of resistance R from $351\ \Omega$ to $360\ \Omega$. In that case, the uncertainty in R is $10\ \Omega$, even though the least count of the resistance box may be less. Therefore, *make a judicious estimate of the least count*.

➔ Statistical errors, Mean and Standard Deviation

If we make a measurement x_1 of a quantity x, we expect our observation to be close to the quantity but not exact. If we make another measurement, we expect a difference in the observed value due to random errors. As we make more and more measurements, we expect them to be distributed around the correct value, assuming that we can neglect or correct for systematic errors. If we make a very large number of measurements, we can determine how the data points are distributed in the so-called *parent distribution*. In any practical case, one makes a finite number of measurements and one tries to derive the true value of the quantity as best as possible.

Consider N measurements of quantity x, yielding values $x_1, x_2, \ldots, x_N$. One defines the arithmetic mean, which is an average value of the quantity x as,

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

The deviation d_i of any measurement x_i from the mean $\bar{x}$ of the parent distribution is defined as $|x_i - \bar{x}|$. Note that if $\bar{x}$ is the true value of the quantity being measured, d_i is the true error in x_i .

There are several indices one can use to indicate the spread (dispersion) of the measurements about the central value, e.g., the mean value. One can define the average deviation $avgdev$ as the mean of the *magnitudes* of the deviations (absolute values of the deviations from the mean).

$$avgdev = \frac{1}{N}\sum_{i=1}^{N}(|x_i - \bar{x}|)$$

This can be used as a measure of the dispersion in the observations about the mean. However, a more popular measure of the dispersion is the standard deviation σ, defined as

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 = \left(\frac{1}{N}\sum x_i^2\right) - (\bar{x})^2$$

σ^2 is known as *variance* and the *standard deviation* σ is the square root of the variance. In other words, it is the root mean square (rms) of the deviations.

The above expression underestimates $\sigma$for small N. Statisticians tell us that a better expression to use is the following:

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

where the denominator is $N - 1$ instead of N. In practice, for large N the distinction between these formulae is unimportant.

The mean $\bar{x}$ is a parameter which characterizes the information we are seeking when we perform an experiment. The mean is, of course, not only the parameter is used to characterize a distribution, but it is the most popular one and also the best when an experiment is performed under near ideal conditions*. It can be proved that if we use the mean of the measured values for calculating the deviation, the sum of the square of the deviations is a minimum. The standard deviation is simply related to this minimum value of the square of the deviations and is used for specifying error quantitatively.

The standard deviation characterizes the uncertainties associated with our experimental attempts to determine the "true" value. σ, for a given finite number of observations, is the uncertainty in determining the mean of the parent distribution. Thus, it is an appropriate measure of the uncertainty in the observations.

Repeated measurements

Suppose a quantity x is measured n times. The best estimate for the actual value of x is the average $\bar{x}$ of all the measurements. If errors are assumed to be randomly distributed, the error in *the mean value* is given by,

$$\delta_{\bar{x}} = \frac{\delta_x}{N}$$

where δ_x is the error associated with a single measurement. You may use the estimated error in each primary measurement as δ_x. Hence one way of *minimizing random errors* is to repeat the measurement many times. Note that repeating a measurement has no effect on a systematic error.

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* In place of mean, one can characterize a distribution by the median (the middle value) which is a robust measure of central tendency. Median would give a better estimate than the mean if the data set is contaminated by too much noise and contains outlier points. Mean gives all the data points equal weight and hence can be easily affected by an outlier while the median would automatically reject an outlier. The appropriate width to use with the median is the mean deviation which is the average absolute deviation calculated from the median. One can show that the average absolute deviation is a minimum if it is calculated about the median.

➔ Propagation of errors

Calculation of the error associated with f, which is a function of measured quantities x, y and z

Let

$$f = f(x, y, z) \qquad \text{.......................} \; (1)$$

Taking the differential

$$\mathrm{d}f = \frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial y}\mathrm{d}y + \frac{\partial f}{\partial z}\mathrm{d}z \qquad \text{.......................} \; (2)$$

Eq. (2) relates the differential increment in f resulting from differential increments in x, y, z. Thus, if the errors in x, y, z (denoted as $\delta x, \delta y, \delta z$) are small compared to x, y, z respectively, then we may say

$$\delta f = \left|\frac{\partial f}{\partial x}\right| \delta x + \left|\frac{\partial f}{\partial y}\right| \delta y + \left|\frac{\partial f}{\partial z}\right| \delta z \qquad \text{.......................} \; (3)$$

where the modulus signs have been put because errors in x, y and z are independent of each other and may be either positive or negative. Therefore, the maximum possible error will be obtained only by adding absolute values of all the independent contributions. (All the δ's are considered positive by definition). The use of (3) is especially simple in some simple cases.

i) For addition or subtraction, the absolute errors are added, e.g., if $f = x + y - z$, then

$$\delta f = \delta x + \delta y + \delta z \qquad \text{.......................} \; (4)$$

ii) For multiplication and division, the fractional (or percent) errors are added, e.g., if $f = \frac{xy}{z}$, then

$$\left|\frac{1}{f}\right| \delta f = \left|\frac{1}{x}\right| \delta x + \left|\frac{1}{y}\right| \delta y + \left|\frac{1}{z}\right| \delta z \qquad \text{.......................} \; (5)$$

iii) For raising to constant powers, including fractional powers, the fractional error is multiplied by the power, e.g., if $f = x^{3.6}$, then

$$\left|\frac{1}{f}\right| \delta f = 3.6 \left|\frac{1}{x}\right| \delta x \qquad \text{.......................} \; (6)$$

Equation (3) is far too conservative. It totally ignores the cancellation of errors which happens when errors occur with opposite signs and thus overestimates the final error. It gives us an upper limit for the error in an experiment and thus we have

$$\delta f \le \left|\frac{\partial f}{\partial x}\right| \delta x + \left|\frac{\partial f}{\partial y}\right| \delta y + \left|\frac{\partial f}{\partial z}\right| \delta z$$

SIGNIFICANT FIGURES

A statement of result such as $f = 123.4678 \pm 0.2331$ cm contains many superfluous digits. Firstly, the digits 678 in quantity f do not mean anything because they represent something much smaller than the uncertainty δf. Secondly, δf is an *approximate* estimate for error and should not need more than one significant figure. An acceptable expression would be 123.4 ± 0.2 cm.

No physical measurement is exact. For example, consider a meter scale, which has a least count of 1 mm used for measuring the length of a pencil. The reading, if we keep one end at the origin of the scale, might lie between say 8.5 cm and 8.6 cm. One might estimate the length as 8.52 cm, the final digit 2 being an estimate of a part of a millimeter division on the scale. Perhaps the estimate of final digit could have been 3 or 1 instead of 2. In any case, we find that the length expressed as 8.52 tells us that the length lies between 8.53 and 8.51 – the measurement has 3 significant figures. A significant figure is reasonably trustworthy. The number of significant figures does not depend upon the decimal point. The length could have been written as .0852 m or as 85.2 mm which mean exactly the same thing. Whenever we make a measurement, our reading should indicate the number of significant figures. Suppose we want to measure the area of a square whose sides are 8.52 cm with the final digit indicating the precision or accuracy of the measurement. In this case, the number has 3 significant figures. Then to what precision or accuracy do we know the area? We know

$$A = l^2$$

$$\Rightarrow \; \Delta A = 2l \times \Delta l \quad (\Delta l \text{ is the possible error in } l = 0.01 \text{ cm})$$

$$\Rightarrow \Delta A = 2 \times 8.52 \times 0.01 = 17.04 \times 0.01 \approx 0.17 \text{ cm}^2$$

Hence the area is known to an accuracy of 0.2 cm^2 which means

$$A = (8.52)^2 \text{ cm}^2 = 72.6 \text{ cm}^2 \text{ (keeping only the first digit after the decimal point)}$$

In addition and subtraction carry the operation only as far as the first column of doubtful figure.

(1) Addition of two lengths

If $l_1 = 2.5\underline{4}$ cm (known to 3 significant figures)

and $l_2 = 10.2\underline{9}$ cm (known to 4 significant figures)

$$l_1 + l_2 = \overline{12.8\underline{3}} \text{ cm}$$

The doubtful figure is underlined.

On the other hand,

if $l_1 = 2.\underline{5}$ cm

and $l_2 = 10.2\underline{2}$ cm

$$l_1 + l_2 = 12.\underline{72} = 12.\underline{7} \text{ cm}$$

(2) Multiplication of two lengths

If $l_1 = 2.5\underline{4}$ cm and $l_2 = 10.2\underline{9}$ cm

$$l_1 \times l_2 = 2.5\underline{4} \times 10.2\underline{9} = 26.\underline{1366} = 26.1 \text{ cm}^2$$

where we have rounded off keeping only the first doubtful figure. In this case the second doubtful figure being 3 does not change the first doubtful figure in rounding off.

For rounding off the insignificant figures (doubtful figures after the first doubtful figure) are dropped, but the first doubtful figure (or the last significant figure) is unchanged if the figure dropped is less than 5. It is increased by 1 if it is greater than 5. There are different practices for rounding off if the figure dropped is 5. You can take any of them. You can increase the digit by 1 in this case, if you so like. Alternatively, you may keep the digits after 5 also, keeping 2 significant figures in the error bar.

Rules for Computation

1. In addition and subtraction take only the first column that contains the doubtful figure. Use the rule of rounding off mentioned earlier.
2. In multiplication and division, carry the result to the same number of significant figures that are in the factor with the least number of significant figures.

An Example

» Consider the measurement of acceleration due to gravity g using the simple pendulum. We have,

$$\mathrm{g} = \frac{4\pi^2 l}{T^2}$$

where $l =$ length and $T =$ time period of the pendulum. Suppose, $l = 95.2\ \mathrm{cm}$ and $T = 1.95\ \mathrm{sec}$

$$\mathrm{g} = \frac{4\pi^2 l}{T^2} = 988.388\ \mathrm{cm/sec^2} \qquad \text{(obtained by calculator)}$$

However, not all figures are significant. To determine which figures are significant, we note

$$\frac{\Delta \mathrm{g}}{\mathrm{g}} = \left(\frac{\Delta l}{l}\right) + 2\left(\frac{\Delta T}{T}\right)$$

$$\Rightarrow \frac{\Delta \mathrm{g}}{\mathrm{g}} = \left(\frac{0.1}{95.2}\right) + 2\left(\frac{0.01}{1.95}\right) \approx 0.01$$

which shows that the number of significant figures in this case is only 2. Indeed, $\Delta \mathrm{g} \approx 0.01 \times \mathrm{g} = 10\ \mathrm{cm/sec^2}$ which means that the second digit is uncertain and we should write the answer as $\mathrm{g} = 990\,\mathrm{cm/sec^2}$. Note that Δl and Δr are not necessarily the least counts of the scale and the clock repeatedly used to measure the quantities. Depending on the circumstances they may be larger or even smaller than the least count. If the pendulum's ends cannot be defined (located) precisely Δl could be limited by this rather than least count of the scale used. Similarly, T can read far more accurately than the clock's least count by taking readings for the time taken for more than one oscillation.

HOME ASSIGNMENT #7

In each of the following measured quantities and their errors are given. A quantity to be calculated is defined in terms of the measured quantities. Calculate: (a) the defined quantity, (b) the error in the defined quantity, and (c) the percentage error in the defined quantity.

1) The quantity to be calculated is average velocity $v = x/t$ and the measured values are:

$$x = 1.748 \pm 0.10 \times 10^{-2}\ \mathrm{m}\,, \qquad t = (5.41 \pm 0.05) \times 10^{-3}\ \mathrm{s}$$

2) The quantity to be calculated is K, the kinetic energy. The measured values and errors are:

$$M = 1.25 \pm 0.05\ \mathrm{kg}\,, \qquad v = 0.87 \pm 0.01\ \mathrm{m/s}$$

Reference: Introduction to Error Analysis by J.R. Taylor (University science books, CA) 2nd edition, 1997.

Experiment 01

Prism Spectrometer

Aim:

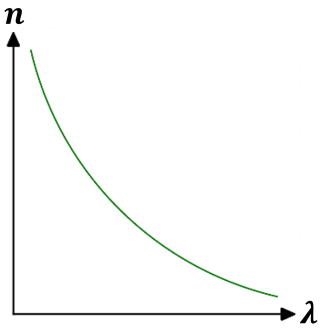
- *To determine the refractive indices of a glass prism at various wavelengths of mercury light.*
- *To plot the dispersion curve for the given prism and calculate the dispersive power of the prism.*
- *To obtain the coefficients in Cauchy's equation from the graph of* $(\boldsymbol{n-1})$ *vs* $\left(\boldsymbol{1/\lambda^2}\right)$.

PRINCIPLE

A prism splits white light into different colors because the refractive index of the material of the prism depends on the wavelength of light. This dependence can be written as

$$n = n_0 + \left(\frac{k}{\lambda^2}\right) \qquad \text{..............} \ (1)$$

This means as the wavelength is increased, the refractive index decreases. $Eq.(1)$ can be rewritten as

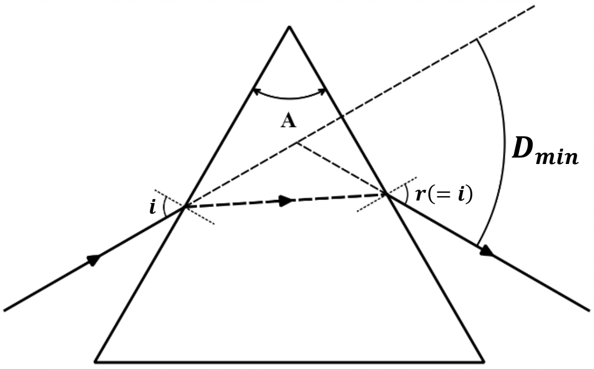


$\boldsymbol{Fig.1}$

$$\mathrm{n} = 1 + \mathrm{A}\left(1 + \frac{B}{\lambda^2}\right) \qquad \text{........................} \ (2)$$

This is known as Cauchy's equation; the constant A is called the coefficient of refraction and B is known as the coefficient of dispersion. Note that the coefficient of refraction is different from the index of refraction. Equation (2) is an empirical expression known as Cauchy’s Equation that is justified by a detailed model of interaction of the molecules of the material with light and is valid at wavelengths where the material doesn’t absorb light strongly or resonantly.

When a parallel beam goes through the prism, getting refracted twice, the emergent beam bends through some angle with respect to the incident beam. This angle is called the angle of deviation. It changes with the angle of incidence and is minimum when the incident and emergent beams make equal angles with the corresponding refracting surfaces. The angle of minimum deviation D_{min} is related to the angle of prism A and the refractive index n of the material of the prism as



$(angle\ of\ minimum\ deviation)$

$\boldsymbol{Fig.2}$

$$n = \frac{\sin\left(\frac{A+D_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad \text{........................} \ (3)$$

A narrow beam of light from a spectral line source (for example, a mercury source) which emits visible radiation of characteristic and known wavelengths is made incident on the prism. By measuring the minimum deviation corresponding to each wavelength we may establish the dependence of n upon λ. Estimation of n by the method of minimum deviation is robust and simple as only the angle of minimum deviation has to be measured.

The dispersive power of a material is defined by,

$$\omega = \frac{n_B - n_R}{n_Y - 1}$$

where n_B, n_R and n_Y are refractive indices of material for blue, red, and yellow lights respectively. The reciprocal of the dispersive power is called the dispersive index and it lies between 20 and 60 for most optical glasses. It is a very important property of the material that is used for the design of optical devices like microscopes, cameras etc. with minimal chromatic aberrations.

APPARATUS - SPECTROMETER

$Fig. 3$

→ The essential parts of a spectrometer ($see\ fig. 4$) are

- a telescope (T)
- a collimator (C) which rigidly attached to the base
- a prism table (B)
- a platform (P) which carries two markings and their associated verniers V, 180° apart.

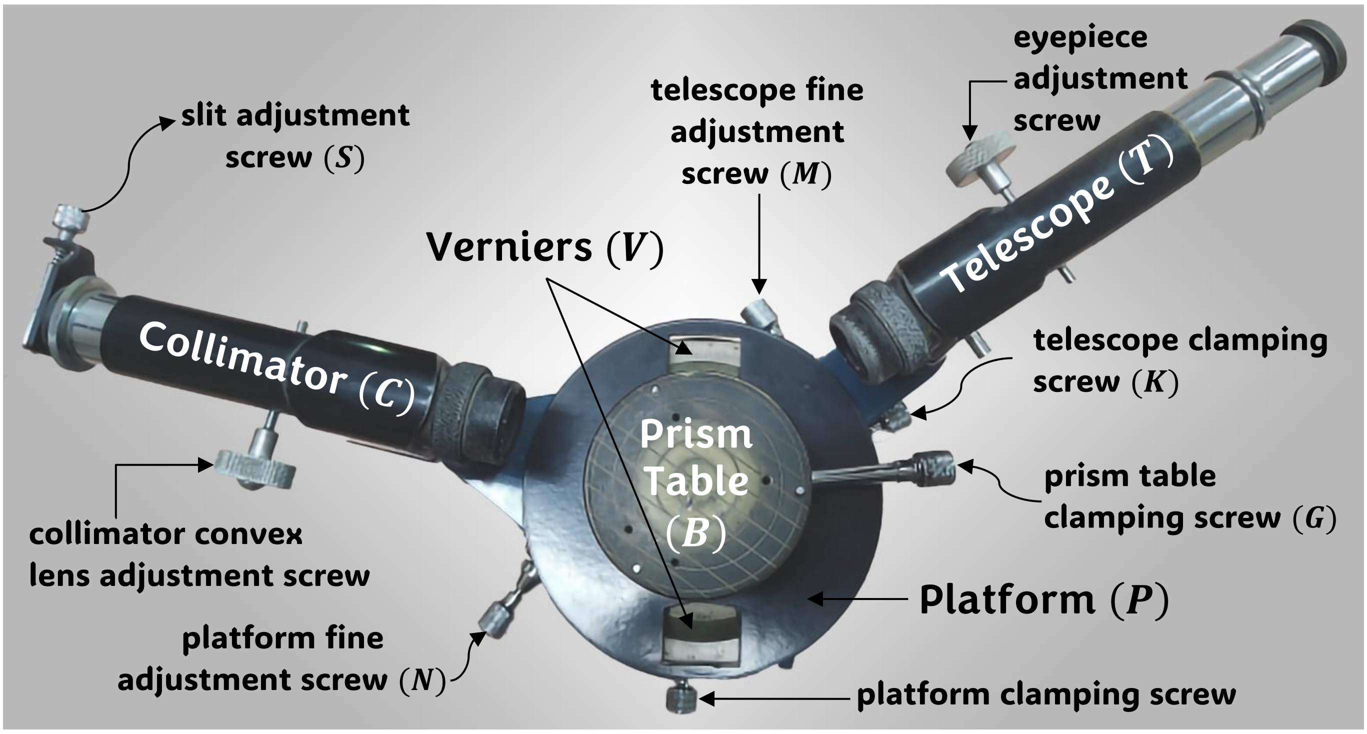
→ The telescope and the platform can be independently rotated around a common vertical axis, their relative orientation being indicated by the reading of the markings on the divided circular scale which rotates integrally with the telescope.

→ The prism table (B) can also be independently rotated. It may be locked to the platform by means of the clamping screw G.

→ By means of the clamping screw K, the telescope (T) may be locked, when subsequent fine angular adjustment may be made by turning the screw M.

→ The platform (P) may be locked by the clamping screw L. When locked, the subsequent fine angular adjustment may be made by turning the screw N.

→ The collimator is provided with a slit aperture the width of which can be adjusted by turning the slit adjustment screw S.



$Fig. 4$

Procedure

Adjustment of the spectrometer:

1) Level the prism table as accurately as possible using a spirit level.

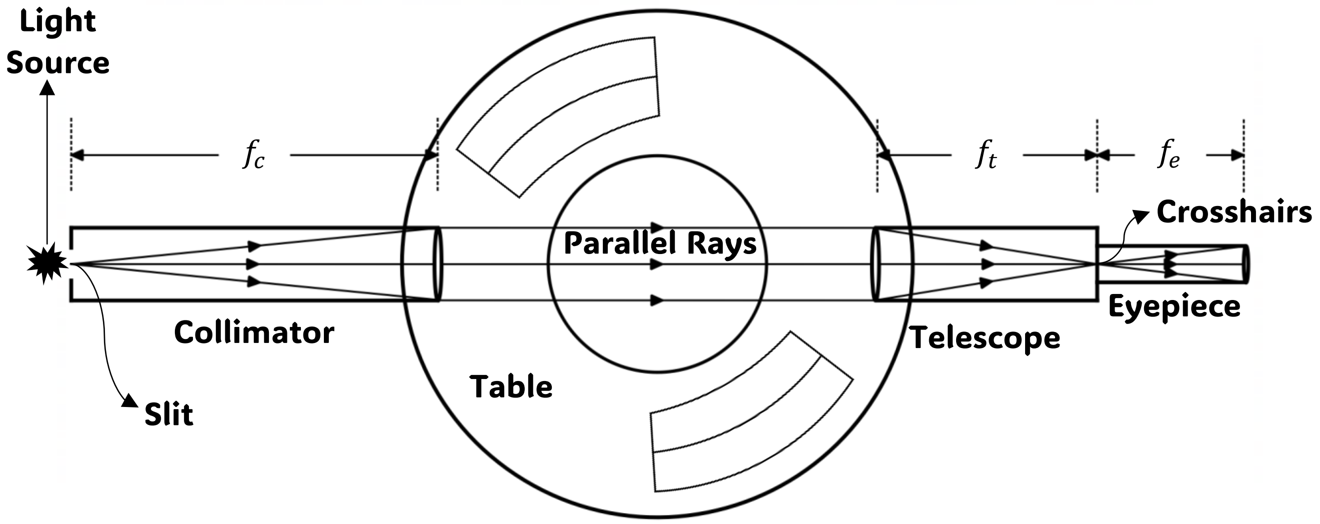
2) Analyze $figs. 4$ & 5 carefully. Locate and identify the collimator which has a slit on one end and the telescope, which has the eyepiece at its end.

3) Point the telescope towards an object as far away as possible and focus the telescope on a far-away object, which is taken to be at infinity. This ensures that parallel rays will focus on to a point as on the right-hand side of the prism table in $fig. 5$.

4) Now look directly into the collimator with the telescope and obtain as sharp an image as possible of the slit by adjusting the focus of the collimator. This ensures that light emerging from the source/slit becomes set of parallel rays as in the left side of the prism table in $fig. 5$.

5) ***Now the focus of the telescope and the collimator should no longer be changed during the rest of the experiment. If accidentally any changes are made, repeat steps 3 & 4 before proceeding further.***

6) $Fig. 5$ shows the path of the light rays as they pass through a correctly adjusted spectrometer. The instrument scale must be carefully read at the desired setting and for work of the highest precision both verniers are used. In the present case adequate accuracy will be obtained by using only one of them.



$\boldsymbol{Fig. 5}$

The angle of minimum deviation and its measurement:

- **Put the prism on the prism table such that the center of the base of the prism is at the center of the prism table**. Rotate the prism table so that the beam from the collimator falls on one of the refracting surfaces and emerges through the other $(fig. 2)$. The spectral lines should be visible with the unaided eye. Subsequently locate the spectrum separate colored images of the slits with the telescope. Each colored slit emerges at a different angle and is focused by the telescope to a different spot. Select a particular line in the spectrum for observation and note its color and the corresponding wavelength in your observation table. Lock the platform. If necessary, adjust the entrance slit width of the collimator to sharpen the spectrum so that it is as bright as possible. Now rotate the prism table slowly in steps and each time rotate the telescope to receive the selected line at the cross wire. The telescope should come closer to the direct path. If it goes away from the direct path, rotate the prism table in the opposite direction. You cannot bring the line closer to the direct path beyond a point. As you rotate the prism table in either direction at this stage, the image will move away from the direct path. This turning point is the position of minimum deviation for that particular wavelength.
- If you lock the telescope in this position, disturb the prism table a little bit, and gradually rotate it to bring it back to the original position and continue in the same sense, the successive view that you will see in the telescope are like that shown in $Fig. 6$.

- Determine this position very carefully by using the fine adjustment screws on the telescope. Record the reading at the vernier. Re-determine this position for the same line several times and take the average minimum deviation position D_i.
- Repeat the same procedure for all the spectral lines that can be seen clearly. Record all observations carefully.

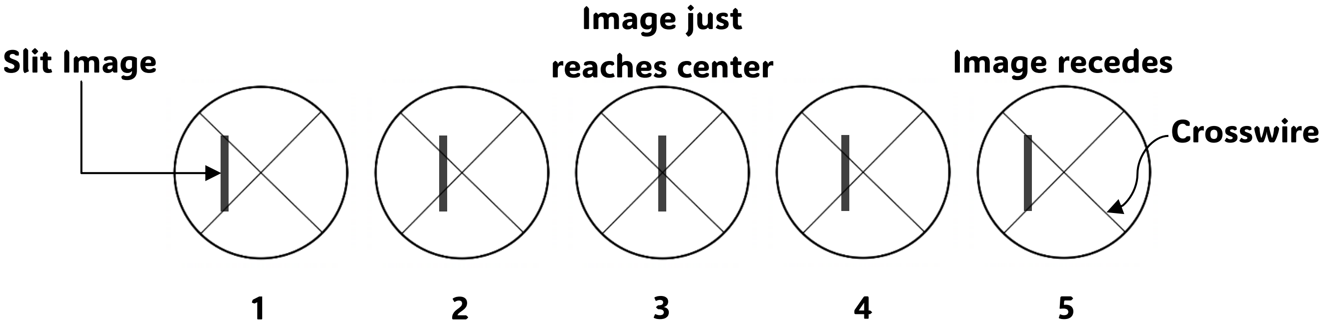


Fig. 6

- Remove the prism and rotate the telescope to bring it directly opposite to the collimator in a straight line. Center the slit image in the crosswire and record this position using the vernier. You may take several readings to get an average value of this zero-point position D_0. The angle of minimum deviation (D_{min}) for the i^{th} spectral line is $|D_i - D_0|$.
- Make necessary calculations for the refractive index n and plot

  (i) $n \; vs \; \lambda$ and

  (ii) $(n-1) \; vs \; \frac{1}{\lambda^2}$

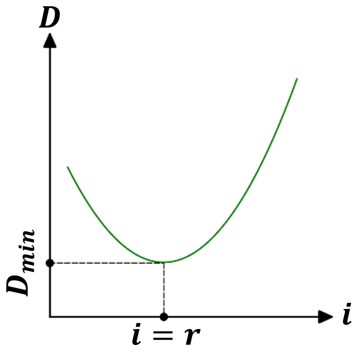


Fig. 7

- Calculate the desired quantities.

CALCULATIONS

Calculation of least count ($l.c$) or vernier constant ($v.c$):

From the image of vernier scale ($fig. 7.1$) you can see that length of 30 $lines \; of \; vernier \; scale$ matches with the length of 29 $lines \; of \; main \; scale$.

$\therefore$ 30 $vernier \; division \; (v.s.d)$

$= 29 \; main \; scale \; division \; (m.s.d)$

$$\Rightarrow 1 \; v.s.d = \left(\frac{29}{30}\right) m.s.d$$

Now, $Vernier \; constant \; (v.c) = 1 \, m.s.d - 1 \, v.s.d$

$$\Rightarrow v.c = 1 \, m.s.d - \left(\frac{29}{30}\right) m.s.d$$

$$\Rightarrow v.c = \left(1 - \frac{29}{30}\right) m.s.d = \left(\frac{1}{30}\right) m.s.d = \left(\frac{1}{30}\right) \times 30' = 1'$$

So, $least \; count \; (l.c) \; or \; vernier \; constant \; (v.c) = 1'$

Fig. 7.1

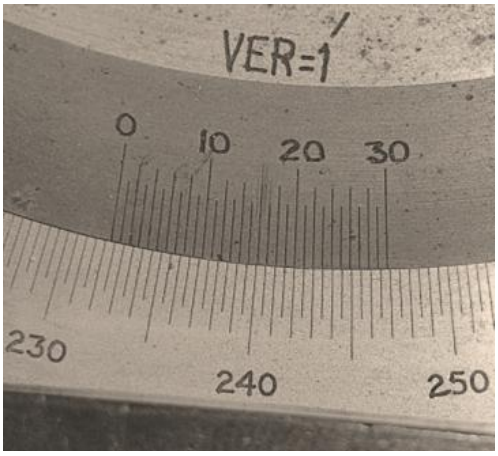
How to note the readings:

Vernier reading 0 is your reference point. First determine at what point vernier 0 cross the main scale reading. Then you will look for the verner reading and that is the reading where a particular line from vernier scale perfectly coincides with any of the main scale reading. Let's see an example.

In $fig.7.2$, $M.S.R = 232°$ and $V.S.R = 20$.

$\therefore Total\ reading = M.S.R + (V.S.R \times V.C)$

$\Rightarrow Reading = 232° + (20 \times 1') = 232°20'$



$\boldsymbol{Fig.7.2}$

Plotting your results:

From Cauchy's equation, we have

$$\mathrm{n} = 1 + \mathrm{A}\left(1 + \frac{B}{\lambda^2}\right)$$

By rearranging the above equation, we get

$$(n-1) = AB\left(\frac{1}{\lambda^2}\right) + A$$

Now if we plot $(n-1)\ vs\ (1/\lambda^2)$ graph, it will be a straight line. By comparing the above form with the standard equation of straight line $[y = mx + c]$, we can observe that AB is the slope of the straight line and A is the $y\text{-}intercept$.

Calculation of A, B and ω:

Since we have started $x\text{-}axis$ from 0.025×10^{-6} (not from 0), so we will not have $y\text{-}intercept$ at $x = 0$

point. To calculate the $y\text{-}intercept$ (A), we put the value of slope m *(calculated from the graph)* and the centroid value $\left[x_c = \left(\sum_{i=1}^{6}\left(\frac{1}{\lambda_i^2}\right)\big/_6\right), y_c = \left(\sum_{i=1}^{6}(n_i - 1)\big/_6\right)\right]$ in the straight line equation $y = mx + c$.

By doing this, we get the value of A as

$$A = c = y_c - mx_c$$

Once we get the value of A, we can calculate B from the relation $AB = m$ $\left[\Rightarrow B = {}^{m}/_{A}\right]$.

To calculate ω, put the calculated values of refractive indices in $eq.(4)$.

Uncertainty in n:

We will calculate the error of n considering Snell's formula because we used it for calculating n for different light colors.

$$n = \frac{\sin\left(\frac{A+D_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Taking log on both sides and then differentiating, we get

$$\frac{\Delta n}{n} = \cot\left(\frac{A + D_{min}}{2}\right) \cdot \left(\frac{\Delta A + \Delta D_{min}}{2}\right) + \cot\left(\frac{A}{2}\right) \cdot \left(\frac{\Delta A}{2}\right)$$

Since A is given as known quantity, we can take its error $\Delta A = 0$. So, the final expression becomes

$$\frac{\Delta n}{n} = \cot\left(\frac{A + D_{min}}{2}\right) \cdot \left(\frac{\Delta D_{min}}{2}\right)$$

Here, ΔD_{min} is the difference between two readings. Which means we get two errors from two different readings.

$$\therefore \Delta D_{min} = 2 \times least\ count = 2 \times 1' = \left(\frac{2}{60}\right) \times \left(\frac{\pi}{180}\right) rad = \frac{\pi}{5400} rad$$

Find the error in the refractive index for red color only i.e.,

$$\Delta n_R = \left[\cot\left(\frac{A + D_{min}}{2}\right) \cdot \left(\frac{\Delta D_{min}}{2}\right)\right] \times n_R$$

N.B. - *Remember that for error in ω, you also need to calculate Δn_B and Δn_Y. So if you are not running out of time and think that you can do all the calculation then find the error for them, if not then skip this part for the time being.*

Uncertainty in A:

Since A is the $y\text{-}intercept$ of $(n-1)\ vs\ (1/\lambda^2)$ graph, the error of A can be taken as the smallest division in $y\text{-}axis$.

$$\Delta A = (s.d)_{y\text{-}axis}$$

Uncertainty in B:

$$\frac{\Delta B}{B} = \frac{\Delta m}{m} + \frac{\Delta A}{A} \qquad \text{where,}\ \ \Delta m = \frac{|m - m_1| + |m - m_2|}{2\sqrt{n}} \ [n \text{ is the no. of data points}]$$

Uncertainty in ω:

$$\omega = \frac{n_B - n_R}{n_Y - 1}$$

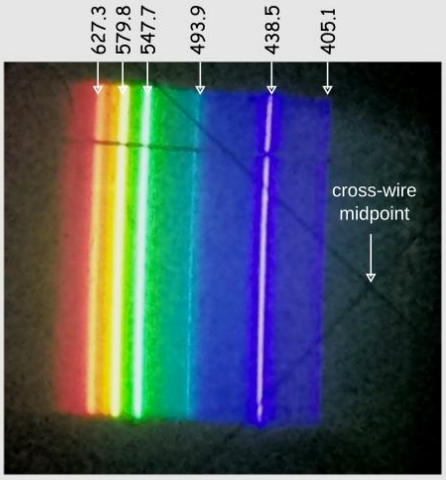
$$\Rightarrow \ln\omega = \ln(n_B - n_R) - \ln(n_Y - 1)$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = \frac{\Delta(n_B - n_R)}{n_B - n_R} - \frac{\Delta n_Y}{n_Y - 1} = \frac{\Delta n_B - \Delta n_R}{n_B - n_R} - \frac{\Delta n_Y}{n_Y - 1}$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = \frac{\Delta n_B + \Delta n_R}{n_B - n_R} + \frac{\Delta n_Y}{n_Y - 1}$$

Spectral lines of the mercury (Hg) source:

You may look at the table below for the visible wavelength spectrum of the mercury lamp provided to you. It is usually measured in research laboratories using a well-defined diffraction grating based spectrometer.



WAVELENGTHS OF DIFFERENT SPECTRAL LINES

Red	627.3 nm	Turquoise	493.9 nm
Yellow	579.8 nm	Blue	438.5 nm
Green	547.7 nm	Violet	405.1 nm

QUESTIONS

1. Why does refractive index depend on the wavelength of light?
2. On what factors do the angle of deviation depend when a beam of light passes through a prism?
3. Why is it necessary to make the beam parallel by passing through the collimator?
4. Why do you use a mercury vapor lamp in this experiment?
5. Derive equation (3).

Reference: Fundamentals of Optics, Fourth edition, F.A. Jenkins and H.E. White, McGraw Hill

REPORT SHEET FORMAT

Prism Spectrometer

Aim:

Working Formulae:

Observation/Table:

Angle of Prism $(A) = 60°$

Vernier constant $(v.c)$ of vernier scale = $[Show\ Calculation]$

Table I: Measurement of the Angle of Minimum Deviation -

Sl. No	λ $(in\ \text{nm})$	Reading with Prism (D)				Reading without Prism (D_0)				Angle of Minimum Deviation $(D_{min} = D_0 \sim D)$
		Main Scale Reading $(M.S.R)$	Vernier Coincidence (x)	Vernier Scale Reading $(V.S.R = x \times v.c)$	Total Reading $(M.S.R + V.S.R)$	Main Scale Reading $(M.S.R)$	Vernier Coincidence (x)	Vernier Scale Reading $(V.S.R = x \times v.c)$	Total Reading $(M.S.R + V.S.R)$	
1										
2										
3										
4										
5										
6										

Table II: Calculation of Refractive Index (n) -

Sl. No	λ $(in\ \text{nm})$	$\frac{1}{\lambda^2}$ $(in\ \text{nm}^{-2})$	$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$	$(n-1)$
1				
2				
3				
4				
5				
6				

Graph: Plot $(n-1)\ vs\ {}^{1}/_{\lambda^2}$.

Calculations: Calculate Cauchy's $A\ and\ B$ coefficients from the Graph, and the dispersive power ω.

Error Analysis: Determine the error in n (for one λ) and Cauchy's $A\ and\ B$ coefficients.

Final Result: Write your final results in this format - $Result \pm Error$. [All units should be mentioned]

Experiment 02

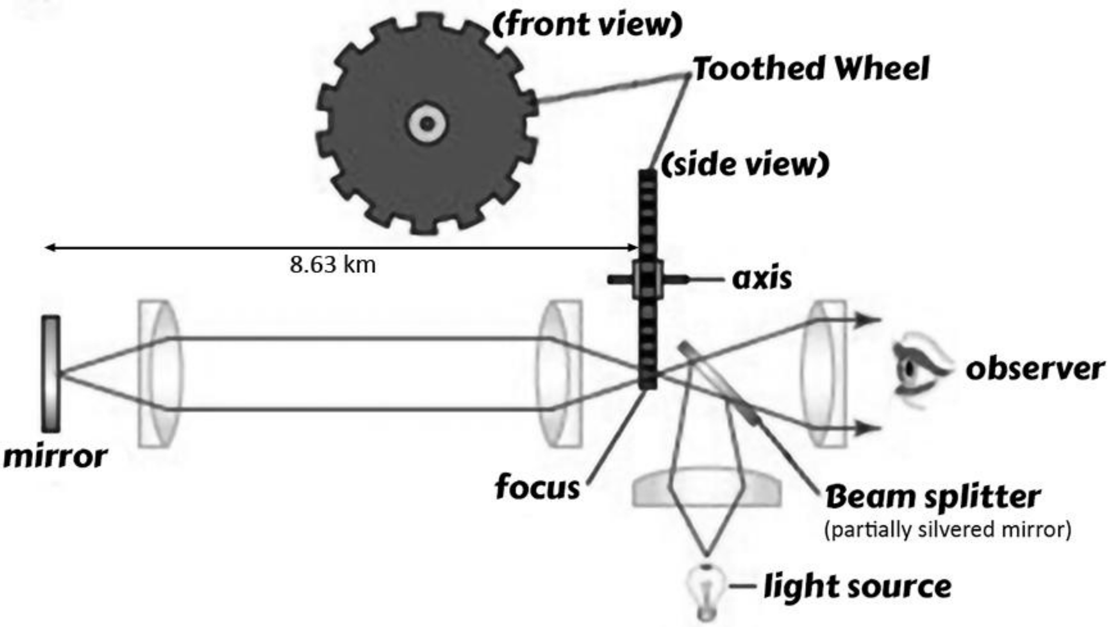
Velocity of Light

Aim:

- *To determine the velocity of light in air.*
- *To determine the velocity of light in a synthetic resin block.*
- *To calculate the refractive index of synthetic resin.*

A BRIEF HISTORY

In 1849, Armand Hippolyte Louis Fizeau was the first to measure the velocity of light. He used a simple apparatus which included a rotating toothed wheel and a remote mirror shown below in $Fig. a$.

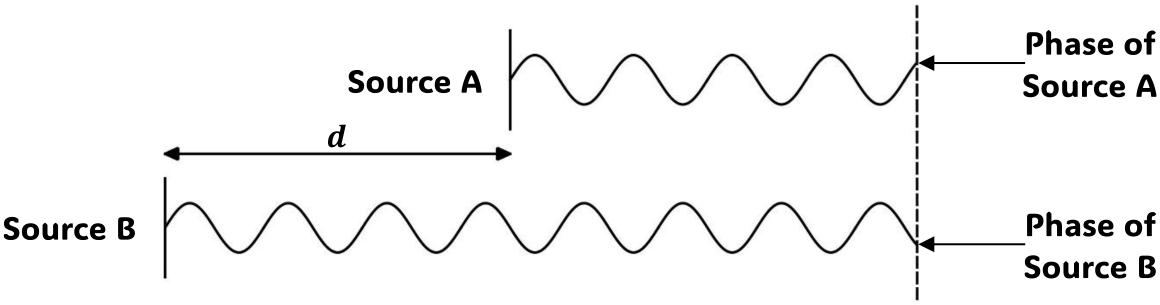


$Fig. a$

In this apparatus ($Fig. a$) a pulse of light with the help of a beam splitter passes through an opening at the bottom of the toothed wheel and strikes a slivered reflecting mirror. By adjusting the velocity of the rotating wheel, the returning pulse of light could be made to either pass through or be obstructed by the tooth on the wheel. Fizeau found the velocity of light to be $315300 \text{ km} \cdot \text{s}^{-1}$. More precise experiments based on rotating mirror apparatus to measure the velocity of light were made by famous names like Jean Bernard Leon Focault (1850) and Albert Abraham Michelson ($between\, 1852 - 1931$). These experiments reached a level of precision that the standard meter was defined based on the velocity of light measured (namely, the distance traversed by light in vacuum during the time interval of $1/299792458\ of\ a\ second$). With the advent of solid state lasers, a more accurate way to measure the velocity of light is based on measuring the phase difference between an incident and reflected path of light.

Outline of the concept of phase difference between waves due to path difference

We briefly describe the concept of the phase difference between two waves. Consider two monochromatic light sources, one emits a monochromatic wave $A(x,t) = A_0 \sin(kx - \omega t)$ and another source which is placed shifted to the left by a distance d and placed below the first source (see the schematic shown in $Fig. b$). The second source emits a wave $B(x,t) = B_0 \sin[k(x+d) - \omega t]$, note the frequency (f) is same for both the sources $(\omega = 2\pi f)$.



$Fig. b$

This distance $\boldsymbol{d}$ is called **path difference**. We say: *These two waves have the path difference of* $\boldsymbol{d}$. What is the phase difference between the two waves? See the illustration above $(Fig. b)$representing the phase difference between the two waves with a path difference of d. One can note in $Fig. 2$, the difference in phases of the two waves arriving at the plane where the phases of the waves from sources A and B are measured. Quantitatively what is the phase difference between the two waves is related to the path difference by:

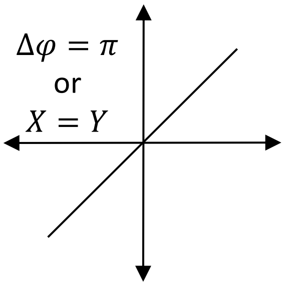
Phase of wave A is, $\varphi_A = kx - \omega t$ .... (1)

Phase of wave B is, $\varphi_B = k(x+d) - \omega t$ .... (2)

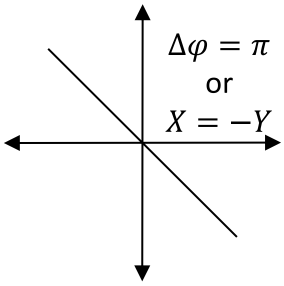
∴ The phase difference is $\Delta\varphi = \varphi_B - \varphi_A = kd$ .... (3)

where, $k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{c}$ $[c = velocity\ of\ the\ wave\ and\ \omega = angular\ frequency = 2\pi f]$

The experiment for measuring the velocity of light, involves the setting up of a controlled phase difference of either 0 or π between two light beams and measuring the velocity of light in air or in a medium based on this principle. Note if we have two waves say $X = A \sin \omega t$ and $Y = A \sin(\omega t + \Delta\varphi)$ and the phase difference $\Delta\varphi$ between these two waves is 0, then $X = Y$ and if $\Delta\varphi$ between the two waves is π then $X = -Y$. If we plot the X wave on the x-$axis$ and the Y wave on the x-$axis$ of an x-y plot, then the x-y plot of $\Delta\varphi = 0$ looks like $Fig. c1$ and $\Delta\varphi = \pi$ looks like $Fig. c2$.



$Fig. c1$



$Fig. c2$

If the phase shift $\Delta\varphi$ between the waves X and Y is arbitrary then in general one gets an ellipse if X is plotted on the x-$axis$ and Y wave is plotted on the y-$axis$. (The above procedure of plotting one wave along the x-$axis$ and another along the y-$axis$ leads to Lissajous figure).

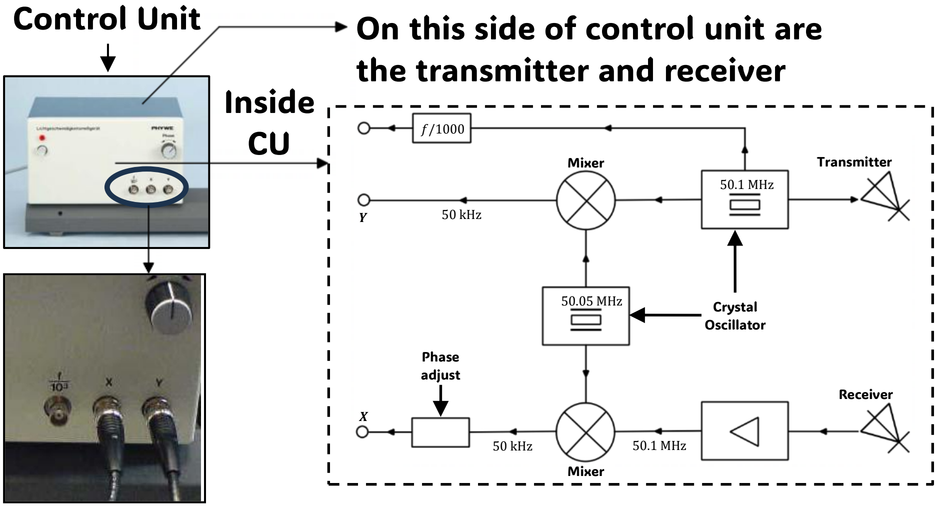
THEORY OF OPERATION OF THE EXPERIMENT

The theory of operation is straight forward, illustrated in $Fig.d$. A high intensity, high frequency Light Emitting Diode (LED) is modulated at $50.1\ \mathrm{MHz}$ by a crystal – controlled oscillator. The transmitter in $fig.d$ shows emitted the light, which is the LED with modulations. The emitted light is received after travelling through some distance and being reflected back to the receiving diode (light sensor). The returning light will be phase shifted w.r.t to the emitted light due to the distance travelled. The modulation signal and the reflected signal (received at the receiver) are independently mixed with a $50.05\ \mathrm{MHz}$ signal, that results in two $50\ \mathrm{kHz}$ outputs $(X\ and\ Y)$ that can be observed on the user's two channel oscilloscope. Furthermore, the relative phase difference between the emitted and the reflected signals are preserved during the mixing process so that the phase difference can be measured. In practice the reflecting mirrors are set close to the source and the initial phase is adjusted to zero. The mirrors are then moved until an appropriate phase shift appears. The phase difference is determined by using a dual channel oscilloscope and using the Lissajous figures described above for getting phase difference of $0\ and\ \pi$. Note the mixing is done by a Frequency mixer (see circuit below), which is a electrical circuit that creates new frequencies from two signals applied to it. A typical example is, say two signals applied to the input of a mixer, $V_1 = C_1 \sin(2\pi f_1 t)$ and $V_2 = C_2 \sin(2\pi f_2 t)$, then the mixer takes the product of V_1 and V_2, which will result in from simple trigonometry,

$$\frac{C_1 C_2}{2}\left[\ \cos\{2\pi(f_1 - f_2)t\} - \cos\ \{2\pi(f_1 + f_2)t\}\ \ \right]$$

Thus, one of the outputs of the mixer is $\cos[2\pi(f_1 - f_2)t]$, which is a $50\ \mathrm{kHz}$ signal as per the above specifications of f_1 and f_2 which is sent to the oscilloscope. The other component from the output of the mixer $\cos[2\pi(f_1 + f_2)t]$, is unused in this present experiment.

[***Acknowledgement***: ***Brief information regarding frequency mixers gathered from Wikipedia***]



Close of CU in your experiment and what is inside it.

$\boldsymbol{Fig.d}$

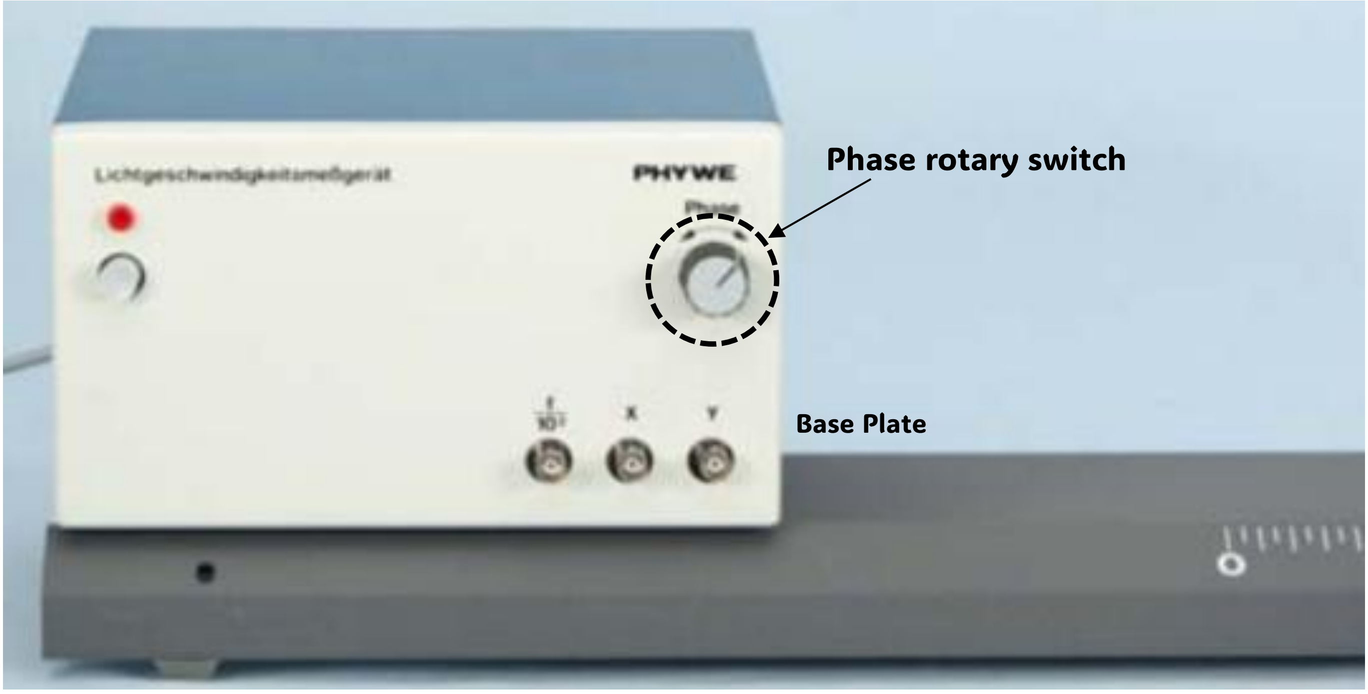
PROCEDURE

(Follow the procedure outline below to perform the experiment. See the appendix for a short review on the Cathode Ray Oscilloscope (CRO) which you will use in this experiment)

Setup and preparation:

Set up and prepare the experiment according to the following instructions and pictures:

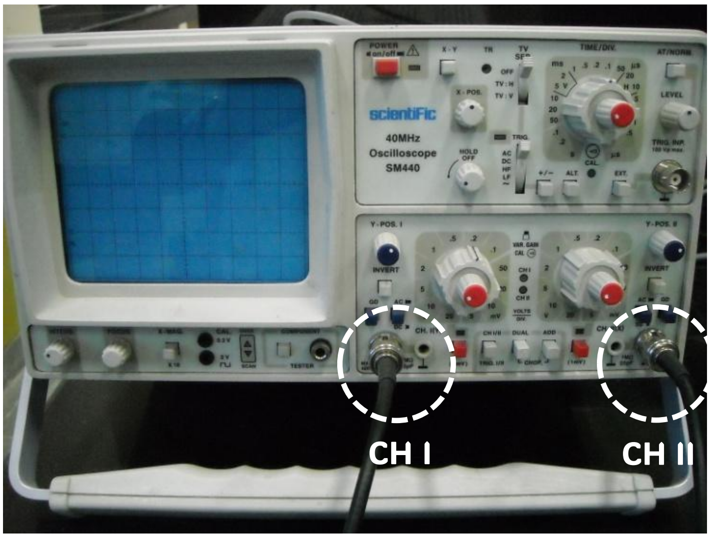
→ The control unit is shown in $fig.1$ and already discussed above. The control unit is already placed in the proper position (left side of the base plate next to the scaling) as shown in the figure.



$\boldsymbol{Fig.1}$

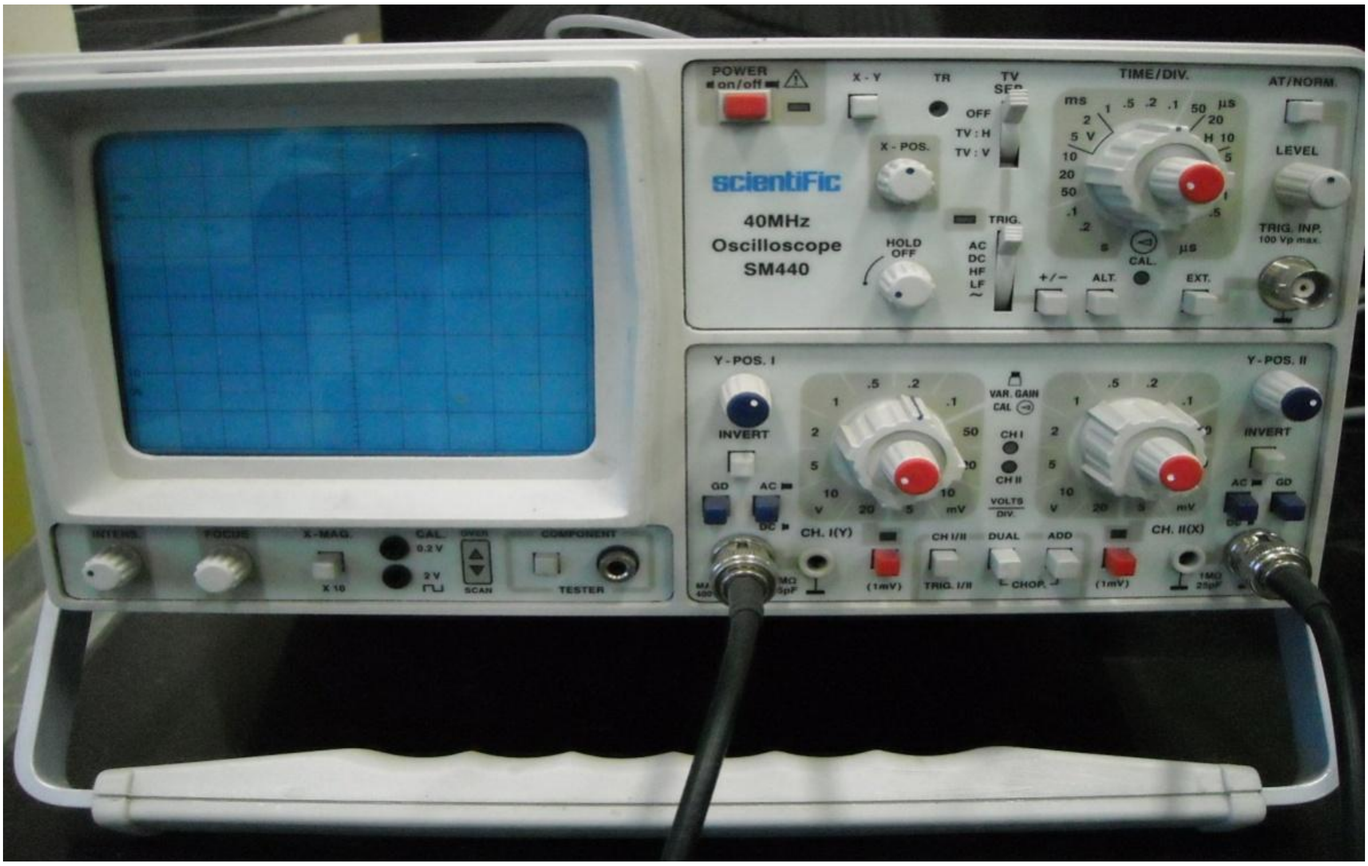
→ Connect the operating unit to the oscilloscope with the two screened cable as shown in $Fig.2$ and 3 [connect the X- and Y- outputs of the operating unit ($Fig.2$) to the respective channel I (CHI, which is X signal) and Channel II (CH II which is the Y signal) inputs of the oscilloscope ($Fig.3$)]

$\boldsymbol{Fig.2}$



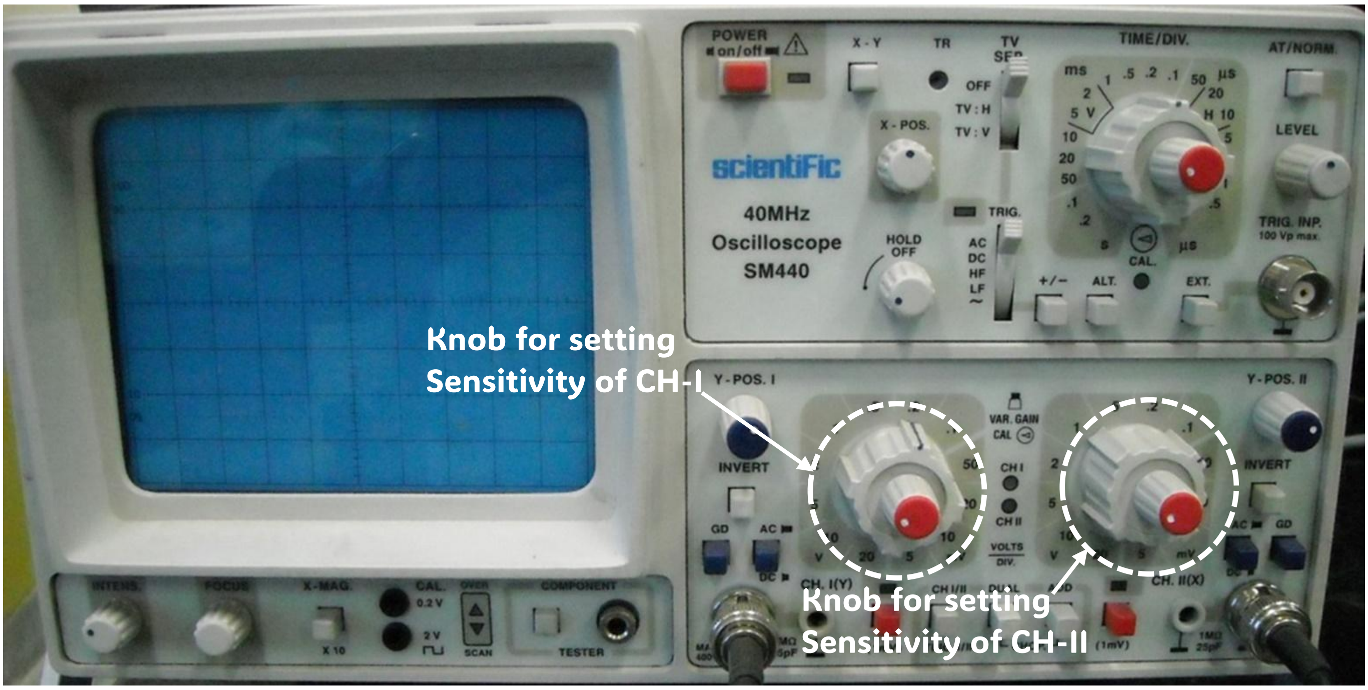
$\boldsymbol{Fig.3}$

→ Put the oscilloscope in the X-Y mode by pressing the button of the X-Y mode of the oscilloscope $(Fig. 4)$ *(the button may already be pressed in for your setup)*.



$\boldsymbol{Fig. 4}$

→ For the sensitivity of the Channel I (CH I) choose $200\ \mathrm{mV/cm}$ (on the oscilloscope it is labeled with "$.2$") and for the Channel II (CH II) set $20\ \mathrm{mV/cm}$ $(Fig. 5)$.



$\boldsymbol{Fig. 5}$

→ The condenser lens should have a distance about $3.5 - 4\ \mathrm{cm}$ from the transmitter diode of the operating unit ($Fig. 6$). Make sure that the plane side of the lens points to the operating unit.

→ Place the mirror onto the right end of the base plate ($Fig. 7$).

→ Switch on the operating unit and the oscilloscope.

→ Use a white piece of paper to follow the emerging light beam and hold it in front of the mirror.

→ Now, adjust the lens vertically, horizontally and in its height in such a way, that you can see a sharp image of the diode on the piece of paper ($Fig. 8$).

→ After removing the piece of paper, the red circle should illuminate the (left) mirror centrally.

$\boldsymbol{Fig. 6}$

$\boldsymbol{Fig. 7}$

$(an\ example\ location\ of\ mirror)$

$\boldsymbol{Fig. 8}$

→ Place the second lens with the plane side pointing to the operating unit about $5\ \mathrm{cm}$ in front of the receiver diode ($Fig. 9$).

→ Move the lens sideways ($Fig. 10$) until you see the image of the transmitter diode (little red circle) on the white ring around the receiver diode.

$\boldsymbol{Fig. 9}$

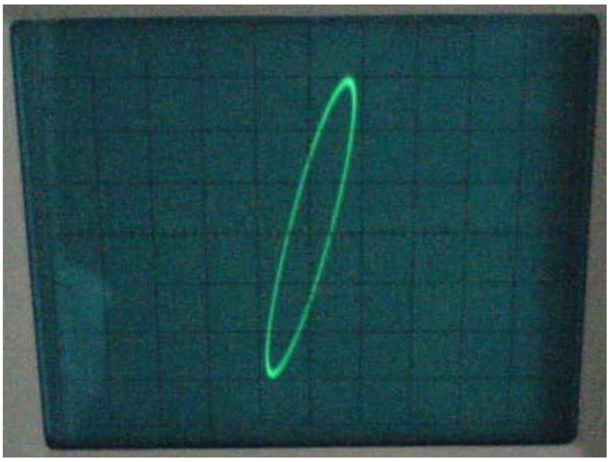
$\boldsymbol{Fig. 10}$

→ Now move the lens in the direction of the light beam until the image on the white ring appears as a little disc, which diameter is a little smaller than the opening of the diode ($Fig.11$).

→ Again, move the lens sideways and adjust it, so that the image of the transmitter diode hits the receiver diode in the best way.

→ If you see the oscilloscope screen you will see an ellipse ($Fig.12$).

$Fig.11$



$Fig.12$

Exp Part I - Determination of the velocity of light in air

$Fig.13$

→ Place the mirror near the operating unit, that the slit of the mirror's base points to the zero point of the base plate's scale ($Fig.13$).

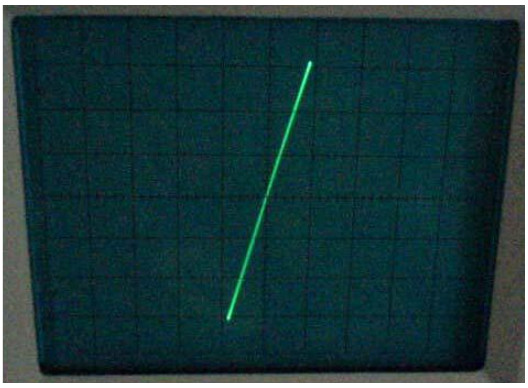
→ Change the sensitivity of the channel II ($\mathrm{CH\ II}$) (see $Fig.3$ above) to 100 or $200\,\mathrm{mV/cm}$ to see a Lissajous-figure (ellipse) on the oscilloscope ($Fig.12$).

→ Turn the "Phase" rotary switch ($Fig.1$) of the operating unit to the right until you see a straight line ($Fig.14$). Note this position of the mirror as x_1.

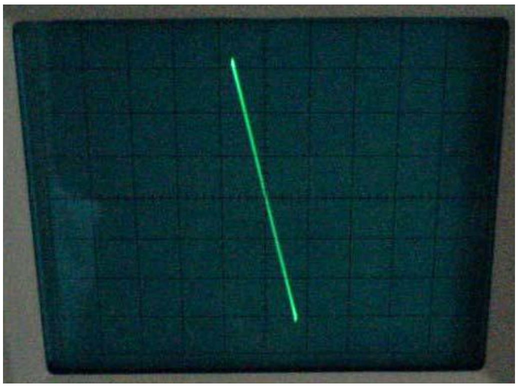
→ Now, move the mirror away from the operating unit as long as you see a straight-line sloping into the opposite direction ($Fig.15$). Note this position of the mirror as x_2.

→ Measure the distance Δx $(= x_2 - x_1)$ that you had to move the mirror and note this value in *table* 1.

→ Repeat this measurement a few times.



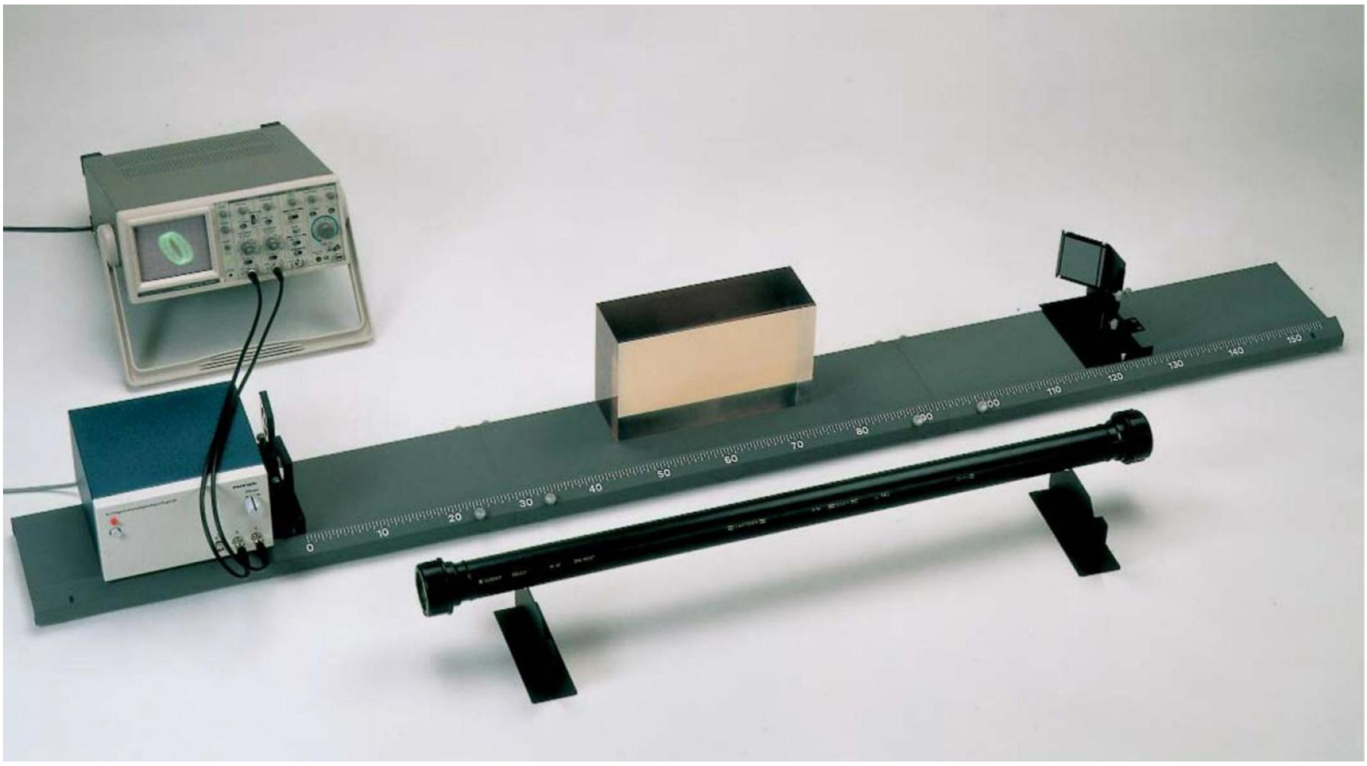
$Fig.14$



$Fig.15$

Exp Part II - Determination of the velocity of light in synthetic resin

Place the synthetic resin block on the base plate on the reflected path of the light, namely, on the light beam path which is obtained after reflecting from the mirror and returning to the receiver ($Fig. 16$).



$\boldsymbol{Fig. 16}$

- → Place the mirror directly behind the resin block.
- → Note this position of the mirror as x_1' as in Part I of the experiment.
- → On the display of the oscilloscope, there will appear a Lissajous-figure again.
- → Use the "Phase" rotating switch of the operating unit again to display a straight line on the oscilloscope.
- → Then take the resin block out of the light beam and move the mirror away from the operating unit until you see a straight-line sloping in the same direction again.
- → Record the new position of the mirror as x_2'.
- → Evaluate the distance $\Delta x'$ $(= x_2' - x_1')$ that you had to move the mirror and note this value in $table$ 2.
- → Repeat this measurement 7 - 10 times.

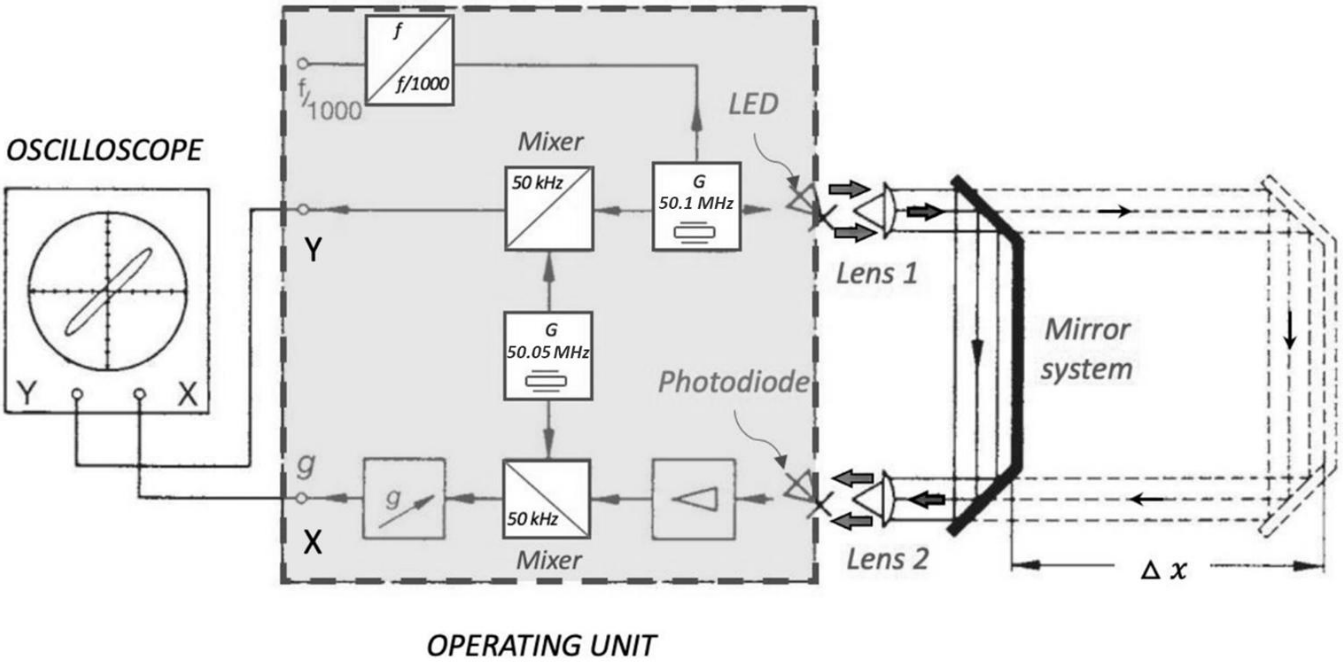
Note

Since this experiment requires no graph plotting, therefore take as many reading as possible. The data analysis should not only involve making proper estimates of the velocity of light in air and in the synthetic resin and the refractive index, but also making proper error estimates. Error estimation should involve calculating standard deviations as well as making error estimates using propagation of errors.

EVALUATION

Determination of the velocity of light in air:

In order to determine the velocity of light in air with this experiment, we consider that the light beam, which emerges from the transmitter diode, hits the receiver diode after a measurable path. On hitting the receiver diode (photo diode), it causes an alternating voltage with the same frequency but with a phase shift to the initial signal. This phase difference is displayed with the help of the oscilloscope (appears as an ellipse). Therefore, the modulation frequency $f = 50.10$ MHz of the transmitter diode is transformed within the operating unit to a frequency of about 50 kHz ($Fig.$ 17).



$\boldsymbol{Fig.}$ **17**

When the oscilloscope displays a straight line with a positive slope, the phase difference is 0. In our experiment, this happens when the mirror is placed to the zero point of the base plate and you turn the "Phase" rotating switch to the right. Then, when you move the mirror away from the operating unit until you see a straight line with a negative slope on the oscilloscope, the phase difference is 180° ($= \pi$). The mirror displacement Δx is measured and thus, the length of the path that the light beam has to cover until it reaches the receiver diode is $\Delta l = 2 \cdot \Delta x$

where, $\Delta x = x_2 - x_1$ .... (1)

(We have to multiply the measured mirror displacement with 2, since the light beam has to cover this distance on its way to the mirror and on its way back to the receiver diode.)

To cover this path Δl the light beam needs the time

$$\Delta \mathrm{t} = \frac{1}{2f} \quad \text{........................ (2)}$$

where f is the modulation frequency (50.10 MHz).

$$c = \frac{\Delta l}{\Delta t} = \frac{2\Delta x}{\left({}^{1}/_{2f}\right)} = 4f \cdot \Delta x \quad \text{........................ (3)}$$

Use the mean value for Δx from $Table$ 1 in the above formula to calculate the velocity of light in air.

Determination of the velocity of light in synthetic resin and refractive index:

In the following picture you can see a sketch of the experimental setup for this part of the experiment ($Fig. 18$).

In this part of the experiment, you obtain a straight line on the oscilloscope, which has the same slope at the mirror positions $x_1{}'$ and $x_2{}'$. This means, that the phase difference is the same in both cases. Thus, the light covers the two different paths at the same time t_1. The path that the light covered in the measurement with the medium is l_1. Therefore, the path of the light in the measurement without the medium is $l_1 + 2 \cdot \Delta x'$.

where,

$$\Delta x' = x_2{}' - x_1{}'$$

For the two measurements, one gets the following equations for the time t_1 -

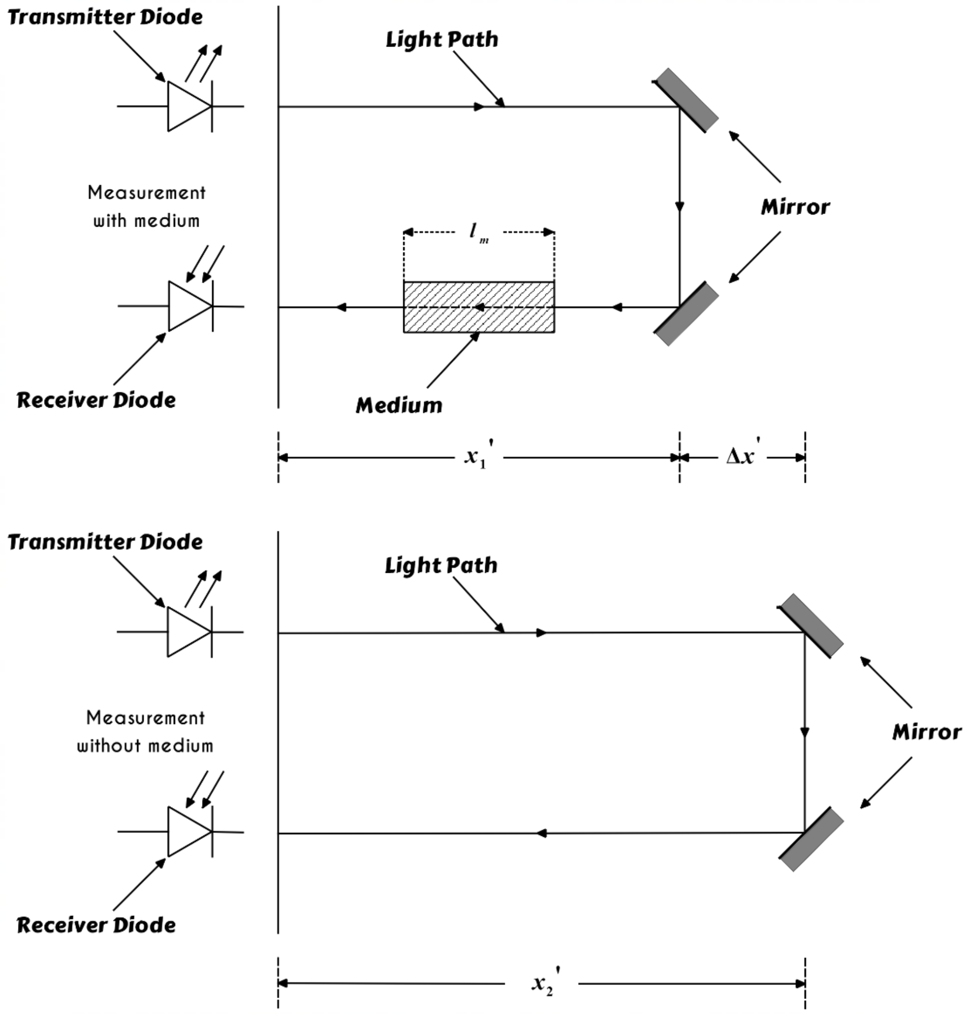


***Fig.* 18**

(Measurement with medium)

$$t_1 = \frac{l_1}{c} - \frac{l_m}{c} + \frac{l_m}{v} \qquad \text{........................} (4)$$

(Measurement without medium)

$$t_1 = \frac{l_1}{c} + \frac{2\Delta x'}{c} \qquad \text{........................} (5)$$

where $\boldsymbol{l_m}$ is the length of the light path through the medium, $\boldsymbol{c}$ be the velocity of light in air and $\boldsymbol{v}$ be the velocity of light in the medium.

From the equations (4) and (5) we get,

$$\frac{c}{v} = \frac{2\Delta x'}{l_m} + 1 \qquad \text{........................} (6)$$

$$\Rightarrow v = \frac{c \cdot l_m}{2\Delta x' + l_m} \qquad \text{........................} (7)$$

Insert your respective mean values for $\Delta x'$ from $Table$ 2 and your result for the velocity of light in air c in $eq.$ (7) to calculate the velocity of light in synthetic resin.

Then calculate the refractive index of the medium $n = {}^{c}/_{v}$ using $eq.$ (6).

REPORT SHEET FORMAT

Velocity of Light

Aim:

Working Formulae:

$$c = 4f \cdot \Delta x \qquad v = \frac{c \cdot l_m}{2\Delta x' + l_m} \qquad n = \frac{c}{v} = \frac{2\Delta x'}{l_m} + 1$$

where, $c = velocity\ of\ light\ in\ air$

$f = modulation\ frequency\ (50.10\ \mathrm{MHz})$

$\Delta x = mirror\ displacement\ (as\ mentioned\ in\ Exp\ part\ I)$

$l_m = the\ length\ of\ the\ light\ path\ through\ the\ medium$

$\Delta x' = mirror\ displacement\ (as\ mentioned\ in\ Exp\ part\ II)$

Observation/Table:

Length of resin block l_m = ____________ cm = ____________ m

Least Count of the base table = ____________ cm = ____________ m

Table I: Determination of the velocity of light in air (Take at least 7 readings)

Sl. No.	Initial position (x_1) in cm	Final position (x_2) in cm	Mirror displacement $\Delta x = x_2 - x_1$ in cm
1			
2			
3			
4			
5			
6			
7			

Δx_{avg} = _________ cm = _________ m

Table II: Determination of the velocity of light in synthetic resin (Take at least 7 readings)

Sl. No.	Initial position (x_1') in cm	Final position (x_2') in cm	Mirror displacement $\Delta x' = x_2' - x_1'$ in cm
1			
2			
3			
4			
5			
6			
7			

$\Delta x'_{avg}$ = _________ cm = _________ m

Calculations:

1. Calculate the velocity of light in air (c).
2. Calculate the velocity of light in the given medium (v).
3. Calculate refractive index (n) of the given medium.

Error Analysis:

1. Calculate standard deviation $(S.D.)$ in Δx and $\Delta x'$.
2. Determine the errors in c, v and n.

Final Result: Write your final results in this format - $Result \pm Error$. [All units should be mentioned]

Experiment 03

Helmholtz Coils

Aim:

- *To measure the magnetic field B along the axis of the flat coils when the distance between these two coils (S) is equals to the radius of a coil (R).*
- *To obtain the B components for each coil separately in this configuration. $(Optional)$*
- *To determine e/m ratio of electron using Narrow beam electron tube.*

PRINCIPLE

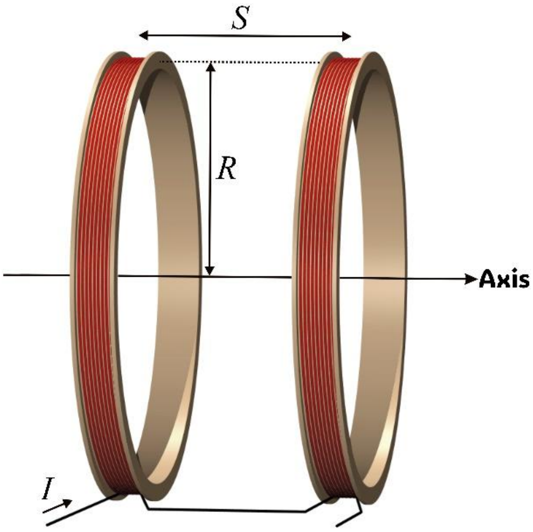
Magnetic Field in Helmholtz Coil:

The magnetic field of a circular coil of radius R, carrying a current I, at a distance z from the center of the loop along the axis is given by

$$\vec{B} = \frac{\mu_0 I}{2} \cdot \frac{R^2}{(R^2 + z^2)^{3/2}} \hat{k}$$

If there are two such parallel coils at a distance s in a such a way that the magnetic field adds in the space between them, then the magnetic field in between the coils is given by

$$B = \frac{\mu_0 I R^2}{2} \left[\frac{1}{\left\{R^2 + \left(\frac{S}{2} + z\right)^2\right\}^{3/2}} + \frac{1}{\left\{R^2 + \left(\frac{S}{2} - z\right)^2\right\}^{3/2}} \right]$$



Using this formula, we can show that

$$(i)\ At\ the\ midpoint\ \frac{\partial B}{\partial z}\ is\ zero.$$

$$(ii)\ \frac{\partial^2 B}{\partial z^2}\ is\ zero\ if\ S = R.$$

Because of these properties, the axial magnetic field is fairly constant over a certain region in the middle of the pair of coils. This arrangement is very popular in producing uniform axial fields in regions easily accessible to experimental situations needing such uniformity.

In this experiment we will investigate the magnetic field variations in space in such a pair of Helmholtz coils. The magnetic field is measured using a Hall probe connected to the Teslameter.

Related Concepts: Magnetic Field due to current carrying Coils, Motion of an electron in crossed E and B Fields.

References: *Introduction to Electrodynamics by D.J. Griffiths*, 2^{nd} Ed. $Ch.$ 5.

Specific Charge ($^{e}/_{m}$) of electron:

Electrons moving in a magnetic field are acted upon by a force normal to the field direction and normal to the direction of movement. The magnitude of the force is proportional to the charge e and the velocity v of the electrons as well as the magnetic flux density B. In this experiment we use a "narrow beam tube" which consists of a glass chamber containing Argon gas at low pressure and in which an electron beam can be generated from an electron gun. The path of beam becomes visible when appropriate potentials are applied. If the narrow beam tube is arranged in the magnetic field of the Helmholtz coils so that the beam leaves the electron gun normal to the field direction, the force F acting on the electrons is given by

$$F = evB$$

Under the influence of this force the beam is deformed into an arc of a circle and bent into a complete circle of radius r when the magnetic field is sufficiently strong. Using equation of motion and conservation of energy, it can be shown that

$$\frac{e}{m} = \frac{2V}{r^2B^2}$$

where, V is the accelerating voltage which gives a kinetic energy eV to each electron.

Since all of the quantities on the right-hand side of the equation can be determined by measurement, it is possible to calculate the specific charge of the electron.

APPARATUS

Construction of Helmholtz Coil:

The two coils given to you are wound from copper wire in 14 layers, each of 11 turns, giving the number of turns $n = 154$. The sockets of the coil winding are cast into the plastic foot of the coil and the connecting leads can be used to connect the coils in parallel or series as required.

SPECIFICATIONS OF THE COILS	*The Coil Diameter*	400 mm
	No of turns per Coils	154
	Coil Resistance	2.1 Ω

In the Helmholtz arrangement, the coils are positioned by three spacer rails so that their axial spacing is equal to the average coil radius. The rails can be removed after undoing knurled screws, allowing coils to be used individually or with variable spacing.

Principle of Teslameter:

The Teslameter uses a Hall Probe as a sensor to measure the magnetic field. The Hall probe is made of a semiconductor and operates on the principle of Hall's effect, which can be briefly described as follows. A semiconductor carrying current develops an $e.m.f.$, when placed in a magnetic field, in a direction perpendicular to the direction of both electric current and magnetic field. The magnitude of this $e.m.f.$ is proportional to the field intensity if the current is kept constant. This $e.m.f.$ is called Hall voltage. This small Hall voltage is amplified so that a millivoltmeter connected at the output of the amplifier can be calibrated directly in magnetic field units.

PROCEDURE

- Familiarize yourself with the coil (its construction and terminals), the Hall probe and the Teslameter provided to you.
- Connect the coils with the power supply in such a way that both coils have the same current in proper direction. Think carefully before you do this. ***In no case the current should exceed*** $\mathbf{2\ A}$.

Operating Instructions for Teslameter:

→ Turn the stepping switch (3) to left most position to select the measuring range $0-20\ \mathrm{mT}$.

→ Keep the changeover switch (4) in the "Direct Field" measurement mode (the lower position).

→ Switch on the Teslameter $10\ \mathrm{min}$ before starting to take measurements.

→ With no current in the Helmholtz coil, the Teslameter should show zero reading. If not, then using the zero-adjustment knob $(2\ and\ 6)$, set the reading to zero in the following manner - First turn the fine adjustment knob (6) to the middle "0" position. The display value is then minimized by turning the course zero-adjustment screw (2). Fine zeroing is then done with the fine adjustment knob (6).

→ Insert the Hall Probe into the magnetic field to be measured. The flat part of the probe should be perpendicular to the field.

→ Note down the readings. (The reading displayed is directly in milli-Tesla).

Precaution: The Hall probe is very ***delicate*** and should be handled with extreme care. ***If you have any doubts with respect to handling the Teslameter and Hall Probe, then please contact your TA/Tutor.***

The student may take the characteristic of magnetic field at the midpoint of the two coils and choose a current from that curve to be used in the entire experiment.

Exp Part I – Measuring Magnetic Field B of Helmholtz Coil:

(a) Adjust the spacing between the coils to $S=R$. Measure $B(z)$ as a function of distance from the midpoint along the axis taking measurements for every 10 millimeters when currents in the coils are in the same sense. Make sure that the Hall probe flat surface is parallel to the plane of the coil.

(b) Similarly obtain contributions of each coil separately and plot all the three cases in the same graph paper. $[\boldsymbol{Optional\ part}]$

B vs z graph

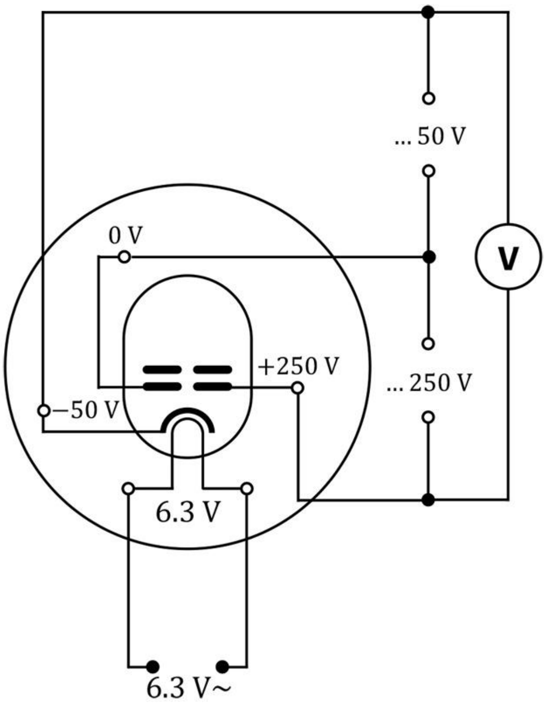
→ Calculate each value of $|\vec{B}|$ corresponding to each z keeping the current fixed at the experimental value and draw an analytical plot from the given formula.

→ On the same graph you will draw the experimental B vs z plot and compare with the analytical plot.

Exp Part II – Determining specific charge of electron using narrow beam tube:

Before the tube is turned on it must be ensured that the two potentiometers $0-50\text{ V}$ and $0-300\text{ V}$ of the mains adapter are on zero. This avoids voltage being present at the grid or anode of the electron gun when the filament voltage is switched on. This makes absolutely sure that the cathode layer cannot be damaged during heating. Only after a heating time of about one minute are the two potentiometers turned up so that the presence of the narrow beam can be observed in a well darkened room. The level of the anode voltage is chosen with the $0-300\text{ V}$ potentiometer, whereas with the aid of the $0-50\text{ V}$ potentiometer the grid voltage and hence the focus and brightness of the narrow beam can be suitably adjusted. The full intensity of the narrow beam is not generally achieved until heating has been continued for 2 to 3 minutes.

When measurement is interrupted for quite some time it is advisable to turn the two potentiometers back to zero. This considerably extends the life of the narrow-beam tube.



$(Electrical\ circuit\ of\ tube)$

The following experimental procedure can be adopted to determine the specific charge of an electron:

1. After the heating period the narrow beam is adjusted appropriately and a certain acceleration voltage V selected.
2. The current through the Helmholtz coils is then switched on and the circuit path described by the narrow beam under the influence of the homogeneous magnetic field observed (maximum permissible continuous current $2\ \mathrm{A}$).
3. The narrow-beam tube is turned in its clips to ensure that the beam leaves the electron gun exactly normal to the direction of the magnetic field and describes a full circle when the strength of the magnetic field is sufficient.
4. The coil current is set so that the narrow beam impinges on one of the four measuring rings. A full circle with radii of $2\ ,3, 4, 5\ \mathrm{cm}$ can be set up in this manner.
5. The coil current I required to produce a full circle of radius r at the accelerating voltage V indicated by the voltmeter is now read off the ammeter and the specific charge e/m of the electron calculated. Obtain the accuracy with which the specific charge e/m of an electron is determined in your apparatus.

❓ QUESTIONS

1. What is the principle of operation for magnetic field probe used in your experiment?
2. List all precautions in doing this experiment?
3. What are advantages, if any, of using Helmholtz arrangement to obtain uniform field over that of a solenoid?
4. To what accuracy e/m ratio is known in the literature?
5. Name a few other experiments from which we can determine e/m ratio.

You may also do the following:

To measure the axial and radial components of magnetic flux density when distance between coils $S = R$ using the rotational symmetry of the set-up.

- i. axial $B(Z)$ at $r = 100\ \mathrm{mm}, r = 140\ \mathrm{mm}$
- ii. radial $B_r(Z)$ at $r = 100\ \mathrm{mm}, r = 140\ \mathrm{mm}$

Reference: Introduction to Electrodynamics by D.J. Griffiths, 2^{nd} Ed. $Ch.$ 5.

REPORT SHEET FORMAT

Helmholtz Coil

Aim:

Working Formulae:

Observation/Table:

Least count of Teslameter =

Reference Point =

Radius of coil =

No. of turns =

Least count of Potentiometer =

[Before taking readings check with Hall probe that the magnetic field at center of the coils is non-zero (which confirms that you have done right connections)]

Table I: Measurement of magnetic field 'B' due to both the coils (in between and outside) when the current ($\boldsymbol{set\ at}\ \mathbf{1.5\ A}$) in both the coils flows in the same direction.

In between coils			Outside coils (left side)			Outside coils (right side)		
Sl. No.	Distance (measured w.r.t reference point) (in mm)	Magnetic Field ($\boldsymbol{B}$) in mT	Sl. No.	Distance (measured w.r.t reference point) (in mm)	Magnetic Field ($\boldsymbol{B}$) in mT	Sl. No.	Distance (measured w.r.t reference point) (in mm)	Magnetic Field ($\boldsymbol{B}$) in mT
1			1			1		
2			2			2		
3			3			3		
4			4			4		
5			5			5		
6			6			6		
7			7			7		
8			8			8		
9			9			9		
10			10			10		

$$Avg\ magnetic\ field\ in\ between\ coils\ B_{avg} = \_\_\_\_\_\_\_\_\ \mathrm{mT}$$

Table II: Determination of e/m ratio for electron

Sl. No.	**Radius** (r) *in* cm	**Voltage** (V_1) *in* volt	**Voltage** (V_2) *in* volt	**Sq. of Radius** (r^2) *in* cm^2	**Potential Difference** $(V = V_2 - V_1)$ *in* volt
1					
2					
3					
4					
5					

Graph:

1. Plot $distance\ vs\ magnetic\ field(B)$ from $Table\ I$.
2. Plot $V\ vs\ r^2$ from $Table\ II$.

Calculations:

1. Calculate slope (m_s) of $V\ vs\ r^2$ graph.
2. Calculate e/m ratio of the electron from the following formula -

$$\frac{e}{m} = \frac{2V}{r^2B^2} = \frac{2m_s}{B_{mean}{}^2}$$

Error Analysis:

1. Calculate standard deviation (S.D.) of magnetic field in the constant region (between the coils).

$$\sigma_B = \sqrt{\frac{\sum_{i=0}^{n}(B_i - B_{mean})^2}{n-1}}$$

2. Determine the errors in m_s and $({}^{e}/_{m})$.

$$\Delta\left(\frac{e}{m}\right) = \left[\frac{\Delta m_s}{m_s} + \frac{2\Delta B}{B_{mean}}\right] \cdot \left(\frac{e}{m}\right) \qquad \Delta m = \frac{\Delta m_1 + \Delta m_2}{2\sqrt{N}}$$

Final Result: Write your final results in this format - $Result \pm Error$. [All units should be mentioned]

1. Magnetic Field $B =$
2. Specific charge of electron $({}^{e}/_{m}) =$

Experiment 04

Coupled Pendulum

Aim:

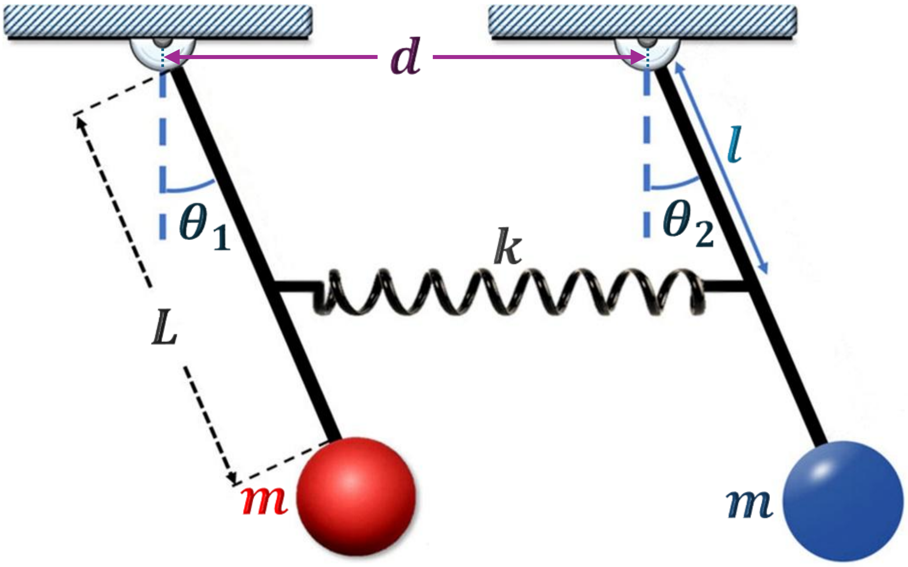
- *To determine normal mode frequencies of coupled pendulum.*
- *To establish a relation between characteristic frequencies and coupling lengths.*
- *To calculate the spring constant k.*
- *To see the comparison between ω_{spring} and ω_0.*

INTRODUCTION

In a periodic system, the number of vibration frequencies is generally equal to the number of degrees of freedom, which in turn is the minimum number of co-ordinates needed to completely describe its motion. For example, a single pendulum that is constrained to pivot in one plane can have its position specified by a single coordinate (usually angular displacement from the vertical) and has only one natural frequency of vibration. A spring that can pivot around its attachment point has at least two degrees of freedom and therefore two vibration frequencies. The most interesting (and useful) examples of this type are systems with several oscillators that are somehow coupled together. *In this experiment, you will study a system that has two gravity pendula coupled together by a spring.*

THEORY AND PRINCIPLE

You should be able to work out the theory behind the experiment either from energy considerations or from force-torque considerations. We outline it briefly using the former though it is equally easy by both methods.



$\boldsymbol{Fig.1}$ – *Schematic of coupled pendula with coupling spring attached at the end.*

Given two pendula with the same mass 'm' and length 'L' attached with a spring. When the pendula are displaced by angles θ_1 and θ_2, the masses are at positions $(L\sin\theta_1, -L\cos\theta_1)$ and $(d + L\sin\theta_2, -L\cos\theta_2)$, where 'd' is the horizontal distance between the pendula attachment points. Let $'l'$ be the coupling length of the pendula (*i.e.* the distance of the point, where spring is attached to the pendula, from the point of suspension). Assuming $\theta_1 \ll 1$ or by ignoring vertical motion of the masses so that the amount by which the spring is stretched is given by

$$\Delta x = l(\sin\theta_2 - \sin\theta_1) \approx l(\theta_2 - \theta_1) \qquad \text{..................} \ (1)$$

Combining the potential energy of the two masses with that of the springs gives:

$$U = mgh_1 + mgh_2 + \frac{1}{2}k(\Delta x)^2$$

$$\Rightarrow U = -mgL(\cos\theta_1 + \cos\theta_2) + \frac{1}{2}kl^2(\theta_2 - \theta_1)^2 \qquad \text{..................} \ (2)$$

$$\Rightarrow \dot{U} = mgL\left(\sin\theta_1\,\dot{\theta}_1 + \sin\theta_2\,\dot{\theta}_2\right) + kl^2(\theta_2 - \theta_1)\left(\dot{\theta}_2 - \dot{\theta}_1\right) \qquad \text{..................} \ (3)$$

Using the small angle approximation again in $eq.$ (3), we get

$$\Rightarrow \dot{U} = mgL\left(\theta_1\dot{\theta}_1 + \theta_2\dot{\theta}_2\right) + kl^2(\theta_2 - \theta_1)\left(\dot{\theta}_2 - \dot{\theta}_1\right) \qquad \text{..................} \ (4)$$

The kinetic energy of the masses is

$$K = \frac{1}{2}mL^2\left(\dot{\theta}_1{}^2 + \dot{\theta}_2{}^2\right) \qquad \text{..................} \ (5)$$

$$\Rightarrow \dot{K} = mL^2\left(\dot{\theta}_1\ddot{\theta}_1 + \dot{\theta}_2\ddot{\theta}_2\right) \qquad \text{..................} \ (6)$$

Conservation of energy $\dot{E} = \dot{U} + \dot{K} = 0$ then gives

$$\dot{E} = \dot{\theta}_1\left[mL^2\ddot{\theta}_1 + mgL\theta_1 - kl^2(\theta_2 - \theta_1)\right] + \dot{\theta}_2\left[mL^2\ddot{\theta}_2 + mgL\theta_2 + kl^2(\theta_2 - \theta_1)\right] = 0 \qquad \text{..................} \ (7)$$

Separating this equation gives

$$mL^2\ddot{\theta}_1 = \theta_1(-mgL - kl^2) + \theta_2(kl^2) \qquad \text{..................} \ (8)$$

$$mL^2\ddot{\theta}_2 = \theta_1(kl^2) + \theta_2(-mgL - kl^2) \qquad \text{..................} \ (9)$$

Solving for $\ddot{\theta}_1$ gives

$$\ddot{\theta}_1 = -\theta_1\left(\frac{g}{L} + \frac{kl^2}{mL^2}\right) + \theta_2\left(\frac{kl^2}{mL^2}\right) \qquad \text{................} \ (10)$$

Similarly,

$$\ddot{\theta}_2 = \theta_1\left(\frac{kl^2}{mL^2}\right) - \theta_2\left(\frac{g}{L} + \frac{kl^2}{mL^2}\right) \qquad \text{................} \ (11)$$

Now,

$$\omega_0{}^2 = \frac{g}{L} \ \ and \ \ \omega_c{}^2 = \frac{k}{m}\left(\frac{l}{L}\right)^2$$

Using these, the two equations of motion $[eq. (10) \& (11)]$ reduce to

$$\ddot{\theta}_1 = -({\omega_0}^2 + {\omega_c}^2)\theta_1 + {\omega_c}^2\theta_2 \qquad \text{................. (12)}$$

$$\ddot{\theta}_2 = {\omega_c}^2\theta_1 - ({\omega_0}^2 + {\omega_c}^2)\theta_2 \qquad \text{................. (13)}$$

Here ω_c is called the ***coupling frequency***.

Adding and subtracting the above two differential equations, we get two ***normal mode equations***

$$\left(\ddot{\theta}_1 + \ddot{\theta}_2\right) = -{\omega_0}^2(\theta_1 + \theta_2)$$

$$\left(\ddot{\theta}_1 - \ddot{\theta}_2\right) = -({\omega_0}^2 + 2{\omega_c}^2)(\theta_1 - \theta_2)$$

having ***normal-mode frequencies***

$$\omega_I = \omega_0 \;\; and \;\; \omega_{II} = \sqrt{{\omega_0}^2 + 2{\omega_c}^2}$$

Special Cases (*To obtain normal mode frequencies*) -

$$(In\ all\ cases\ the\ pendula\ are\ released\ from\ rest)$$

Asymmetric Stretch (in phase)	
Initial angular displacement	$\theta_1 = \theta_2 = \theta_0$
Angular displacement of each pendulum as function of time	$\theta_1(t) = \theta_2(t) = \theta_0 \cos\omega_0 t$ $\Rightarrow \theta_1(t) = \theta_2(t) = \theta_0 \cos\omega_I t$

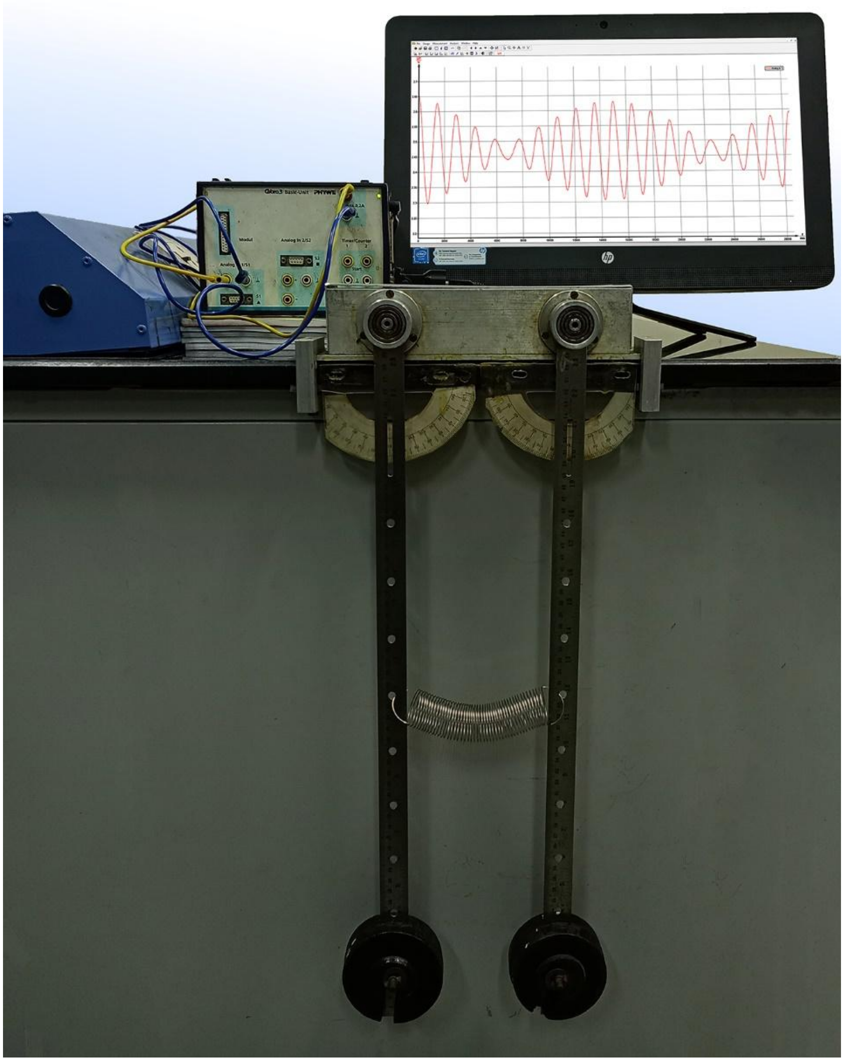
Symmetric Stretch (out of phase)	
Initial angular displacement	$\theta_1 = -\theta_2 = \theta_0$
Angular displacement of each pendulum as function of time	$\theta_1(t) = \theta_0 \cos\sqrt{{\omega_0}^2 + 2{\omega_c}^2}t = \theta_0 \cos\omega_{II} t$ $\theta_2(t) = -\theta_0 \cos\sqrt{{\omega_0}^2 + 2{\omega_c}^2}t = -\theta_0 \cos\omega_{II} t$

Single pendulum Stretch (beat case)	
Initial angular displacement	$\theta_1 = \theta_0,\ \theta_2 = 0$
Angular displacement of each pendulum as function of time	$\theta_1(t) = \theta_0 \cos\omega_B t \times \cos\omega_{III} t$ $\theta_2(t) = -\theta_0 \sin\omega_B t \times \sin\omega_{III} t$ where, the beat frequency ω_B and the oscillation frequency ω_{III} are given by $\omega_B = \left(\frac{\sqrt{{\omega_0}^2 + 2{\omega_c}^2} - \omega_0}{2}\right), \omega_{III} = \left(\frac{\sqrt{{\omega_0}^2 + 2{\omega_c}^2} + \omega_0}{2}\right)$

SETUP

The experimental set up consists of two gravity pendula mounted on a rigid platform. The angular position of one of them can be recorded using rotary potentiometer. The voltage drop across the variable slider on the potentiometer is proportional to the angle by which the pendulum is displaced. The computer interface helps you record this voltage variation as a function of time and displays it on the screen. The masses must be placed symmetrically about the strip for each pendulum.

The instructor in charge of this experiment will show you how this data is obtained and how to read values of particular points of interest from the display. Do not change any settings or electrical connections without express permission of the instructor. However, you can choose software parameters according to your needs.



PROCEDURE

For Single uncoupled Pendulum:

- Find the time period T_0 of a single pendulum without any coupling from a record of displacement as a function of time.
- Average over several oscillations and do this many times. Familiarize yourself with the data acquisition process at this stage.

For Asymmetric mode:

- Make sure that the pendulum is always oscillating in the vertical plane as much as possible. Force the pendulum to the required amplitude by using your fingertip at about one third of the length from the suspension.

- Attach the coupling spring at the holes closest to the bob. Displace both pendula by the same amount in the same direction (this needs care) to obtain initial conditions corresponding to *asymmetric stretch*. The effect of coupling spring would be seen to be negligible. Though you would notice damping as in the case of single pendulum.

- Obtain the time period T_I for this oscillation from several measurements. This is one of the two normal modes.

⋙ For Symmetric mode:

- Now create initial conditions for *symmetric stretch* by displacing the pendula equal amounts in opposite direction (out of phase).

- Record enough data points to obtain the time period T_{II} and the frequency of oscillations.

⋙ For Beat Case:

- Repeat the previous step, with the initial conditions such that only one of them is displaced (beat case). For the *beat case*, measure the oscillation time period T_{III} and the beat semi time period ${}^{T_{beat}}/_{2}$ corresponding to the envelope oscillation (time interval between n_{th} and $(n+1)_{th}$ envelope minimum or maximum).

- Repeat the beat case at least for 5 coupling lengths (l). Measure T_{III} and ${}^{T_{beat}}/_{2}$ in each case.

The screen displays of displacement as a function of time must be qualitatively sketched in your report for each case.

Caution: ***When we are making small oscillations, we need to use a first order type lever (like any uniform solid or pen for making small oscillations).***

QUESTIONS

1. Give at least two real life examples of coupled oscillators.
2. How close is your pendulum to a simple pendulum? What differences do you expect in characteristics from a simple pendulum and why?
3. What other dominant modes of vibration, other than the modes you have tried to measure, are present in the system. Suggest ways of reducing their effect.
4. How can the system you are using for this experiment be improved?

REPORT SHEET FORMAT

Coupled Pendulum

Aim:

Working Formulae:

$$\omega = \frac{2\pi}{T}$$ $[\omega \rightarrow frequency\ of\ oscillation]$
$[T \rightarrow time\ period\ of\ oscillation]$

$$\omega_0{}^2 = \frac{g}{L} \quad ; \quad \omega_{spring}^2 = \frac{k}{m}$$ $[\omega_0 \rightarrow natural\ frequency]$
$[\omega_{spring} \rightarrow spring\ frequency]$

$$\omega_c{}^2 = \frac{k}{m}\left(\frac{l}{L}\right)^2$$ $[\omega_c \rightarrow coupling\ frequency]$

$$\omega_B = \left(\frac{\sqrt{\omega_0{}^2 + 2\omega_c{}^2} - \omega_0}{2}\right)$$ $[\omega_B \rightarrow beat\ frequency]$

$$\omega_{III} = \left(\frac{\sqrt{\omega_0{}^2 + 2\omega_c{}^2} + \omega_0}{2}\right)$$ $[\omega_{III} \rightarrow oscillation\ frequency]$

where,

$L = length\ of\ the\ pendulum$

$l = coupling\ length\ of\ the\ pendulums$

$k = spring\ constant$

$m = mass\ of\ pendulums$

Observation/Table:

Least count of computer timer =

Least count of measurement scale =

Table I: Table for natural frequency (Take at least 3 readings)

Sl. No.	No. of oscillations (N)	Initial Time (T_i) *in* ms	Final Time (T_f) *in* ms	$\Delta T = T_f - T_i$ *in* ms	Time Period $T_0 = \frac{\Delta T}{N}$ *in* ms
1					
2					
3					

Average Time Period $\boldsymbol{T_{avg}}$ = __________ ms *Natural Frequency* $\boldsymbol{\omega_0}$ = __________ rad/sec

Table II: Table for Asymmetric Stretch (Take at least 3 readings)

Coupling length = __________ cm

Sl. No.	No. of oscillations (N)	Initial Time (T_i) *in* ms	Final Time (T_f) *in* ms	$\Delta T = T_f - T_i$ *in* ms	Time Period $T_I = \frac{\Delta T}{N}$ *in* ms
1					
2					
3					

Average Time Period $\boldsymbol{T_{I_{avg}}}$ = __________ ms *Normal-mode Frequency* $\boldsymbol{\omega_I}$ = __________ rad/sec

Table III: Table for Symmetric Stretch (Take at least 3 readings)

$Coupling\ length =$ __________ cm

Sl. No.	No. of oscillations (N)	Initial Time (T_i) *in* ms	Final Time (T_f) *in* ms	$\Delta T = T_f - T_i$ *in* ms	Time Period $T_{II} = \frac{\Delta T}{N}$ *in* ms
1					
2					
3					

$Average\ Time\ Period\ \boldsymbol{T_{II_{avg}}} =$ __________ ms $Normal\text{-}mode\ Frequency\ \boldsymbol{\omega_{II}} =$ __________ rad/sec

Table IV: Table for beat frequency (Take readings for 5 coupling lengths and take 2 readings of time period for each coupling length)

Sl. No.	Coupling length (l) *in* cm	Initial Time (T_i) *in* ms	Final Time (T_f) *in* ms	ΔT ($T_f - T_i$) *in* ms	$T_B = 2\Delta T$ *in* ms	Mean T_B *in* ms	Beat frequency (ω_B) *in* rad/sec
1							
2							
3							
4							
5							

Table V: Table for oscillation frequency (Take readings for 5 coupling lengths and take 2 readings of time period for each coupling length)

Sl. No.	Coupling length (l) *in* cm	No of osc. (N)	Initial Time (T_i) *in* ms	Final Time (T_f) *in* ms	ΔT ($T_f - T_i$) *in* ms	$T_{III} = \frac{\Delta T}{N}$ *in* ms	Mean T_{III} *in* ms	Oscillation frequency (ω_{III}) *in* rad/sec
1								
2								
3								
4								
5								

Calculations:

1. Calculate the various oscillation frequencies $\omega_0, \omega_I, \omega_{II}, \omega_{III}$ *and* ω_B.
2. Compare
   a) ω_I with ω_0 and comment.
   b) $\omega_{III} + \omega_B$ with ω_{II} only for one particular coupling length for which you have calculated T_{II}.
   c) $\omega_{III} - \omega_B$ with ω_I for all coupling lengths.
3. Plot $\sqrt{2\omega_B/\omega_0}$ $vs\ l$. This should be a straight line if the coupling is weak [Prove this]. Calculate spring constant (k) from the slope.

Sl. No.	Coupling length (l) *in* cm	ω_B *in* rad/sec	ω_0 *in* rad/sec	$\sqrt{2\omega_B/\omega_0}$
1				
2				
3				
4				
5				

4. Calculate ω_{spring} and compare it with ω_0.

Error Analysis:

1. Calculate the error in slop Δm , using this calculate the error in spring constantΔk.
2. Calculate the error in various oscillations frequencies $\Delta\omega_0$, $\Delta\omega_B$ and $\Delta\omega_{III}$ for a particular coupling length only.

Final Result: Write your final results in this format - $Result \pm Error$. [All units should be mentioned]

All results should be in significant figures.

Experiment 05

Linear Air Track

Aim:

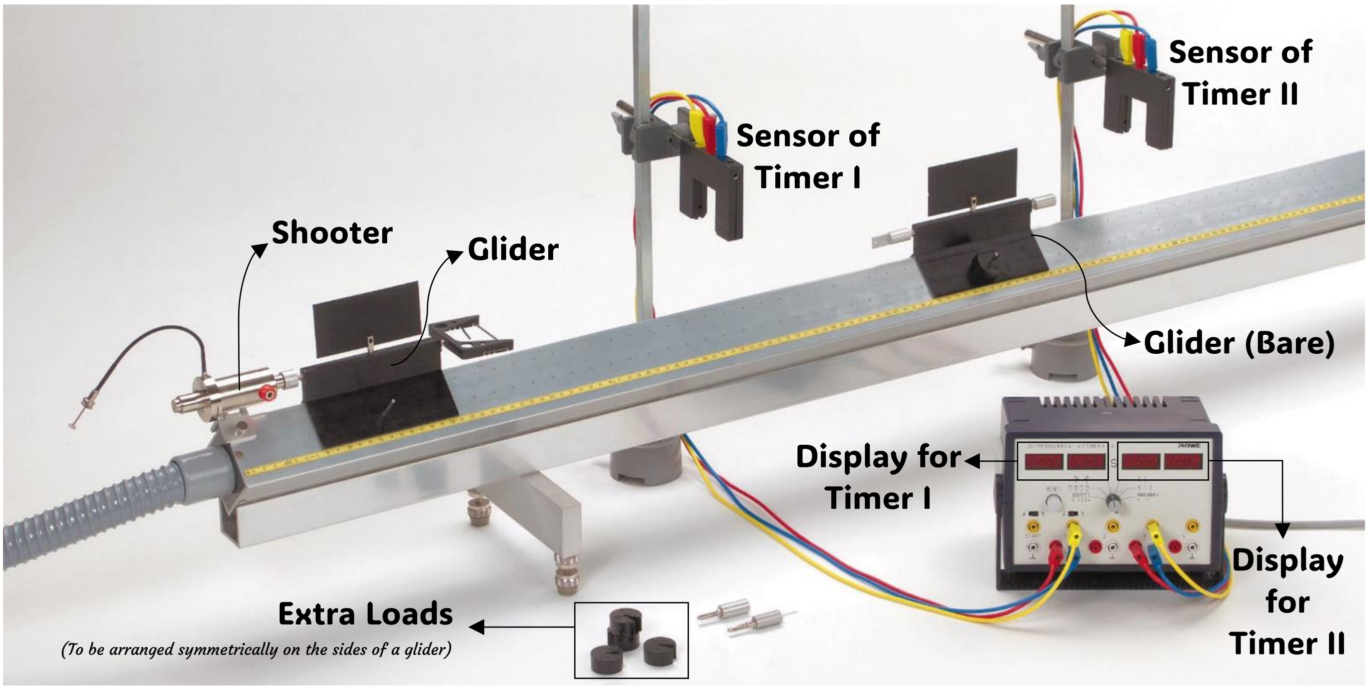
- *To verify law of momentum conservation for elastic collision.*
- *To calculate coefficient of restitution for elastic collision.*

INTRODUCTION

We shall study experimentally collisions in one-dimension using a linear air track. A linear air track is an apparatus which enables us to produce motion of masses (gliders) in a straight line with very little friction. The apparatus consists of a long tube, rectangular in cross section, with small holes spread uniformly over the two surfaces. Air at high pressure is forced through these holes, using a blower. Gliders move on the track. These gliders barely touch the track, because they are lifted up by the air flowing through the holes. This enables the gliders to move over the track with very little friction.

Preliminary experiments about the quality of the air track:

To be able to appreciate the validity of the conclusions from the experiment it will be good to have some preliminary experiments to get familiar with the air track and other components. Note that velocity is measured using timers which get the signal from the light barrier sensors. Gliders carrying masses will be mounted with screens which block the sensor for a specified period which is read by the timers. You will also be given a shooter which is basically a mechanical system for launching gliders with specified energy each time. The starting device has three reproducible initial energy settings. Observe that once launched the gliders move back and forth several tens of times since the friction is very low.



Learn how to use the sensor-timer to measure the velocity of the glider going through the sensor. There are two sensors and four displays on the timer. Two of the displays measure the time of the first pass and the second pass of an object through the first sensor.
Similarly, the other two displays measure the time of the first pass and the second pass of an object through the second sensor.

Look at the glider and the attachments. You can put the masses (cylindrical pellets) on the glider to increase its mass. A 10 cm screen can be put in vertical position on the glider, which passes through sensors during the motion of the glider.

A fork with a rubber band can be attached to the glider and the plate with a plug into the other glider. These face each other and if the two gliders with these attachments move towards each other. They push each other to make elastic collision. Another type of attachment is for inelastic collision. A rod with a sharp needle can be attached to one glider and a tube with plasticine on the other glider. When the gliders collide, the pin gets fixed in the tube and the two gliders move together.

PROCEDURE

Levelling and preliminary experiments:

a) Start the Timer/Counter and set the measuring parameters for the velocity measurement as told by your instructor.

b) Start the air blower and set the appropriate pressure ($\sim 3\ units$).

c) Place the light barrier that is connected to the Timer 1 jack to the left at the 60-cm mark. Position the light barrier that is connected to the Timer 2 jack to the right at the 140-cm mark. You may need to adjust these positions according to the mass used, so that the projectile sees both the light barriers.

d) Mount the 10 cm screen on a glider.

e) Adjust the air track until it is level. Allow the glider to move through the two light barriers at a constant velocity several times in order to determine whether there is a velocity gradient between the two light barriers. If necessary, readjust the air track.

Elastic Collision:

Keep the blower speed such that there is no turbulence and minimal friction. Weigh the target and projectile masses (m_t & m_p respectively) with their elastic appendages. Using three firing speeds, [max gun, min gun, & by hand] for four sets of different mass combination [bare-bare; ($m_t + 300gm$)-bare; bare-($m_p + 300gm$); (as you wish)-(as you wish)] of target and projectile, perform the experiment with 12 sets of data.

- ***Remember to write the mass of each extra loads separately.***
- ***<u>Caution:</u> Extra loads should be arranged symmetrically on the sides of the gliders.***
- ***Initially keep the target glider at rest.***

$t_p^i(\mathrm{s})$ = time taken by the flag of the projectile to pass through the sensor before collision.

$t_p^f(\mathrm{s})$ = time taken by the flag of the projectile to pass through the sensor after collision.

$t_t^f(\mathrm{s})$ = time taken by the flag of the target to pass through the sensor after collision.

As mentioned above the initial velocity (v_t^i) of the target is always kept at zero.

Table for elastic collisions -

Firing Speed	Mass Combination	t_p^i (s) before collision	Projectile velocity before collision v_p^i (m/s)	t_p^f (s) after collision	Projectile velocity after collision v_p^f (m/s)	t_t^f (s) after collision	Target velocity after collision v_t^f (m/s)
Max	1] m_t, m_p 2] $m_t + 300^*, m_p$ 3] $m_t, m_p + 300^*$ 4] $As\ You\ Wish$						
Min	---*DO*---						
Hand	---*DO*---						

$^*300 = [(50+50+50)_L + (50+50+50)_R]$

Calculate (at home) -

a) Calculate and plot {with all of these 12 data sets} $P_f\ vs.P_i$ with error bars. Check that the 45° straight line comes within the scope of error bars of the straight line you plot.

b) Calculate and plot {with all of these 12 data sets} $E_k^f\ vs\ E_k^i$ with error bars. Check that the 45°straight line comes within the scope of error bars of the straight line you plot.

c) Calculate and plot the modulus of "velocity of separation" vs. modulus of "velocity of approach". Hence find the coefficient of restitution, being the slope of the graph.

d) Calculate the coefficient of restitution for each set of reading (12 values) and find the mean and standard deviation. As the number of data points are large, take this S.D. as the error in the coefficient of restitution.

Inelastic Collision (Optional):

Keep the blower speed as it was in the elastic case. Repeat the experiment with the target and projectile with the correct appendages. This time use only maximum and minimum firing speed. Weigh the two gliders bare mass. *Here also the target is at rest initially.*

Use the target masses of $m_t, m_t + 20\ \mathrm{g}, m_t + 40\ \mathrm{g}, m_t + 60\ \mathrm{g}, m_t + 80\ \mathrm{g}\ and\ m_t + 100\ \mathrm{g}$, keeping $m_t + m_p + 100\ \mathrm{g} = constant\ (M)$.

Here you are going to measure only two times. One is projectile initial "t_p^i", and the other is combined final "t_c^f". Since in inelastic collision, the two gliders will stick to each other after collision.

Table for inelastic collisions -

Firing Speed	Mass of the Target	Mass of the Projectile	t_p^i in s	t_c^f in s	v_p^i in m/s	v_c^f in m/s
Max	$Bare$ $Bare + 20\text{g}$ $and\ so\ forth$ $up\ to\ Bare\ + 100\text{g}$	$Bare\ +\ 100\text{g}$ $Bare\ +\ 80\text{g}$ $and\ so\ forth$ $up\ to\ Bare$				
Min	---*DO*---					

Necessary Formula -

Momentum conservation gives

$$m_p' v_p^i = [(m_p + \cdots) + (m_t + \cdots)] v_c^f$$

where $m_p' = m_p\ , m_p + 20\ \text{g}\ , m_p + 40\ \text{g}\ , m_p + 60\ \text{g}\ , m_p + 80\ \text{g}\ and\ m_p + 100\ \text{g}.$

Analysis (at home) -

a) Calculate and plot the final vs. initial momentum graph as you did for elastic collisions. Graphs should be with error bars.

b) Plot $\frac{v_c^f}{v_p^i}$ vs m_p' and find $(m_p + m_t + 100)$ gm from the graph and compare with the measured values of $m_p + m_t + 100 = constant\ (M)$.

c) Comment on why we are not interested in the kinetic energy and the coefficient of restitution in this part.

Precautions

1. Set the air blower to appropriate pressure (approx. 3 units) so that there is neither friction nor disturbance due to air.
2. Clean the setup before starting the experiment so that there is no undue friction due to dust particles.
3. Ensure that two light barrier sensors are parallel to each other.
4. Ensure that the settings in Timer/Counter program are correct.
5. ***Always put the same number of slotted weights on both sides of gliders as there could be toppling otherwise.***
6. Always ensure that impact takes place at a very slow speed so that the instrument is not harmed.
7. The table should not be disturbed during the experiment.
8. ***Always note velocities with the correct sign according to the convention set by you. Momentum should also be noted with correct sign.***

Reference:
1. Kleppner and Kolenkow, An Introduction to Mechanics (McGraw Hill, 1978).
2. Resnick & Halliday, Physics part 1 (John Wiley).

REPORT SHEET FORMAT

Linear Air Track

Aim:

Working Formulae:

$$Speed \ = \frac{l}{t}$$

where,

l = length of flag

t = time recorded by the timer

$$Momentum\ before\ collision\ (\vec{P_i}) = m_p\vec{v_p^i} \quad \left[\left|\vec{v_t^i}\right| = 0.0\ \mathrm{m/s}\right]$$

$$Momentum\ after\ collision\ (\vec{P_f}) = m_p\vec{v_p^f} + m_t\vec{v_t^f}$$

$$Modulus\ of\ velocity\ of\ approach = \left|\vec{v_p^i} - \vec{v_t^i}\right| = \left|\vec{v_p^i}\right| \quad \left[\left|\vec{v_t^i}\right| = 0.0\ \mathrm{m/s}\right]$$

$$Modulus\ of\ velocity\ of\ separation = \left|\vec{v_t^f} - \vec{v_p^f}\right|$$

$$Coefficient\ of\ restitution\ (e) = \frac{Modulus\ of\ velocity\ of\ separation}{Modulus\ of\ velocity\ of\ approach} = \frac{\left|\vec{v_t^f} - \vec{v_p^f}\right|}{\left|\vec{v_p^i}\right|}$$

Observation/Table:

Least count of weighing machine =

Least count of timer =

Mass of target (m_t) =

Mass of projectile (m_p) =

Length of flag = 10 cm

Target velocity before collision = 0.0 m/s

Mass Combinations (gm)	Firing Speed	Before Collision			After Collision					Modulus of Velocity of Separation (m/s)	Modulus of velocity of approach (m/s)
		t_p^i in (s)	$\vec{v_p^i}$ in (m/s)	$\vec{P_i}$ in (kg · m/s)	t_p^f in (s)	t_t^f in (s)	$\vec{v_p^f}$ in (m/s)	$\vec{v_t^f}$ in (m/s)	$\vec{P_f}$ in (kg · m/s)		
m_t m_p	Max										
	Min										
$m_t + 100^*$ m_p	Max										
	Min										
m_t $m_p + 100^*$	Max										
	Min										
$m_t + 200^{**}$ m_p	Max										
	Min										
m_t $m_p + 200^{**}$	Max										
	Min										

$*100 = [(50)_L + (50)_R]$

$**200 = [(50+50)_L + (50+50)_R]$

Important: *Other than the time (for speed, a scalar), remember to include **the direction of motion** of each glider, before and after collision. Directed arrows (→, ←) **or** plus and minus signs (+, −) can be used for this purpose. **Clearly mention**, what direction (AWAY/TOWARDS the shooter) each symbol signifies.*

Graph:

1. Plot P_f vs P_i (for calculation, use back page of graph paper).
2. Plot modulus of velocity of separation vs modulus of velocity of approach.

Calculations:

1. Calculate slope $\left({}^{P_f}/_{P_i}\right)$ of P_f vs P_i graph.
2. Calculate slope of modulus of velocity of separation vs modulus of velocity of approach graph which gives the coefficient of restitution.

Error Analysis:

Calculate error in slope for both graphs.

Final Result: Write your final results in this format - $Result \pm Error$. [All units should be mentioned]

1. The law of momentum conservation is verified with $\left({}^{P_f}/_{P_i}\right) =$
2. The coefficient of restitution $(e) =$

Experiment 06

Pohl's Pendulum

Aim:

- *To find natural frequency of the hair spring.*
- *To find the resonance curve (angular amplitude of the system vs. driving angular frequency).*
- *To find damping constant of the system.*

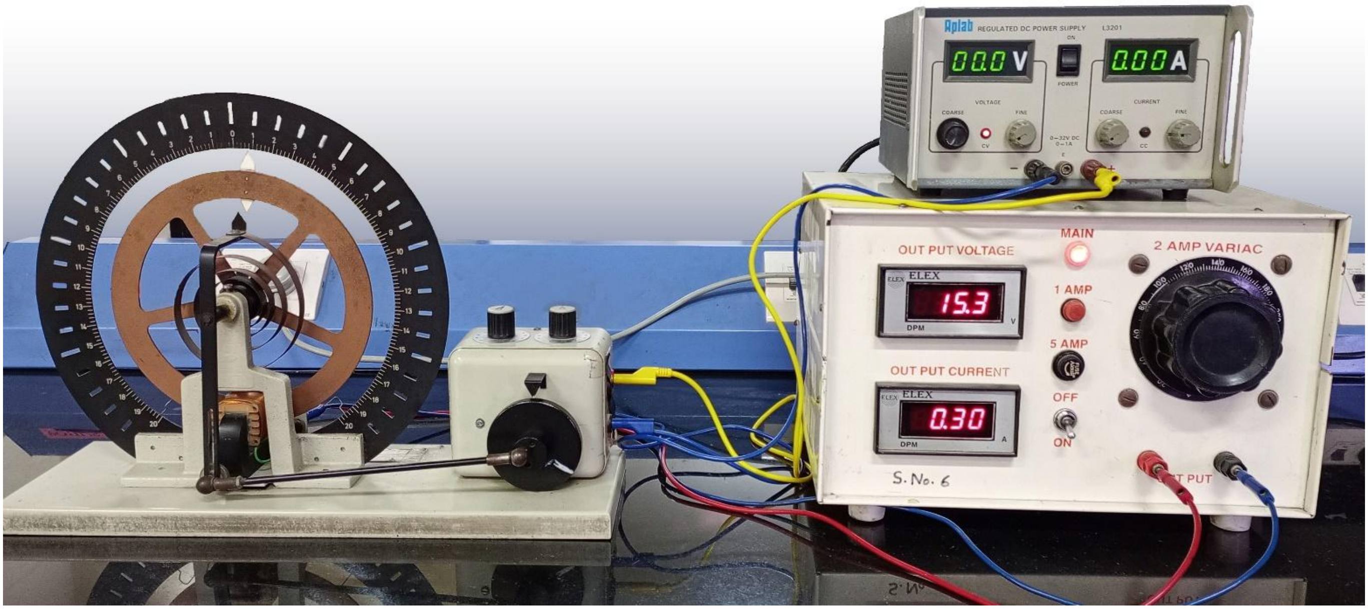
INTRODUCTION

The phenomenon of resonance is encountered in almost all branches of science and engineering in some form or the other. In this experiment, we will study a mechanical system consisting of a coil spring and a motor to drive oscillation in the spring. Damping in the system is provided by inducing eddy currents.

THEORY

In a forced damped oscillatory system, a sinusoidal force $F = F_0 \cos \omega t$ is applied on a system which can oscillate by itself with a natural angular frequency ω_0. The frequency ω is called driving frequency. The system also has a damping which is generally taken proportional to the velocity.

Pohl's torsional pendulum experiment is an excellent example of forced damped angular harmonic oscillations. The oscillations are angular and hence angular quantities are used.



The basic equation for an angular oscillator is

$$I\frac{d^2\theta}{dt^2} = \tau_{restoring} + \tau_{damping} + \tau_{driving} \qquad \text{..................} (1)$$

where, $I = Moment\ of\ inertia\ of\ the\ rotating\ system$

Each of the torques on the right are described below.

$$\tau_{restoring} = -k\theta \ (In\ this\ case)$$

where, $k = Torsional\ constant\ of\ the\ spring$
$\theta = instantaneous\ angular\ displacement\ of\ the\ oscillator$

This is the torque responsible for setting up the angular oscillations. It always acts in the direction opposite to the angular displacement of the body and thus brings it back to its equilibrium position.

$$\tau_{damping} = -b\frac{d\theta}{dt}$$

where, $b = proportionality\ constant\ that\ depends\ on\ the\ shape\ of\ the\ oscillator\ and\ the\ medium\ through\ which\ it\ moves$
$\frac{d\theta}{dt} = instantaneous\ angular\ velocity\ of\ the\ oscillator$

This torque acts in the direction opposite to the motion of the oscillator and has a retarding effect on the motion of the body. This torque can be applied by many factors, like the use of a dashpot, presence of friction etc.

$$\tau_{driving} = \tau_0 \cos \omega t$$

where, $\omega = angular\ frequency\ of\ the\ driving\ torque$
$\tau_0 = maximum\ torque\ applied$

This is the torque that is externally applied by the moving rod on the spring. This torque is independent of the motion of the oscillator.

Now that we know the torques involved, let's analyze the motion analytically. Rearranging $eq.$ (1) and putting in the expressions for the various torques, we get

$$\frac{d^2\theta}{dt^2} + \frac{b}{I}\frac{d\theta}{dt} + \frac{k}{I}\theta = \frac{\tau_0}{I}\cos \omega t$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \gamma\frac{d\theta}{dt} + \omega_0^2\theta = A_0 \cos \omega t$$

where, $\gamma = damping\ constant = \frac{b}{I}$

$A_0 = \frac{\tau_0}{I}$

and $\omega_0 = \sqrt{\frac{k}{I}} = angular\ frequency\ of\ oscillation$
$when\ no\ damping\ and\ no\ driving\ force\ is\ applied$

Writing the solution as $\theta = \theta_0 \cos(\omega t + \varphi)$ and comparing coefficients on both sides, we get

$$\theta_0 = \frac{A_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}} \text{ and } \varphi = \tan^{-1}\left(\frac{\omega\gamma}{\omega_0^2 - \omega^2}\right)$$

Damping constant γ is given by $\sqrt{2(\omega_0^2 - \omega_r^2)}$.

This equation of motion shows that the oscillator performs an angular simple harmonic motion with a constant amplitude. However, note that the amplitude and the phase shift depend on the relative magnitudes of ω and ω_0.

The most interesting situation arises when the driving frequency and frequency of oscillation of the oscillator match exactly. As can be seen from $eq. (2)$, the amplitude θ_0 reaches very nearly maximum of ${}^{\tau_0}/_{I\omega\gamma}$. This is known as the phenomenon of resonance. This experiment is all about analyzing the various aspects of resonance.

Another interesting aspect of resonance is the phase difference between the driving force and the angular motion of the oscillator. For frequency much smaller than the natural frequency, they are in phase and at frequencies beyond the resonance they get out of phase by π.

The damping in this experiment is provided by induced emf in the coil due to the motion of the oscillator through it, and the force thereof. A magnetic field is set up due to the copper coil by feeding in a current in it. When the oscillator moves through the field, there is a change in magnetic flux, which induces an emf and hence an induced current in the oscillator. This current carrying oscillator placed in the magnetic field of the coil experiences a retarding force and slows down. This is how the oscillatory motion of the oscillator gets damped. The damping is expected to be proportional to the current in the electromagnet.

PROCEDURE

Using the "grob" (rough) and "fein" (fine) controls on the power supply box the frequency of the driving force can be changed. A knob on the power supply controls a damping magnet. Driven oscillations, under-damped, over-damped, and critically damped motion can be shown, and the maximum amplitude in the presence of various damping forces can be ascertained.

Do not exceed the allowable damping current $(0.5\ \mathrm{A})$ for more than a few seconds at a time, or the coil may burn out.

Step 1: Make Proper Connection

a) Two chords connect the regulated power dc supply to the power supply box of the torsion pendulum. Request the instructor to check the right sockets.

b) Another pair of chords connect the regulated ac power supply to the damping magnet.

c) Note that the speed of the motor of the power supply box can be controlled both by changing the value of voltage in the power dc supply and also by using “grob” and "Fein" controls on the power supply box.

Step 2: Perform The Resonance Experiment

a) Find out the natural frequency of the system from free vibrations. Just oscillate it without switching on any of the power supplies and measure the frequency of oscillation. This will give you an idea of the range of frequencies (speed of motor) to be swept to obtain resonance. Write your observation. Take at least 5 observations.

b) Now you start taking readings. In this step you will draw and observe the graph between the amplitude of the oscillating pendulum at steady state and the angular frequency of the motor. Choose a particular value of damping current between 0.25 A and 0.45 A by turning the knobs on the AC power supply. Now adjust the speed of the motor by changing the value of voltage through the regulated dc supply. This should be done in such a way that you can achieve more readings near the resonance. Start taking values from minimum angular amplitude.

[*Note*: *This angular amplitude on apparatus has no units*. *Take at least* 20 *observations*.]

c) Note the time period of revolutions of the motor. This gives the driving frequency ω . Measure the amplitude of the oscillating pendulum when steady state is achieved. Note that when the steady state is reached the value of amplitude will not change. Get the steady state amplitude as a function of the driving frequency. Alter the number of revolutions used to calculate the time period of motor revolution depending upon the speed of the motor. More number of revolutions should be used when the speed is higher to minimize the error. Hence for a particular value of the damping current plot a graph between the angular displacement of the oscillating pendulum and the angular frequency of the rotating motor.

d) Find the value of the driving frequency ω for which the angular amplitude of oscillation is maximum. Call it ω_r. Find the two values of ω for which the angular amplitude becomes half the maximum amplitude. The difference $\Delta\omega = \omega_2 - \omega_1$ shows the sharpness of resonance.

e) [***Optional***] Repeat parts $(b), (c)$ *and* (d) for at least two more damping currents and compare the sharpness of resonance.

Step 3: Optional Part

Observe the phase difference between the driver and the pendulum for frequencies lower and also for frequencies higher than the resonance frequency. State your qualitative conclusions. Give a schematic sketch of phase difference as a function of ω.

Precautions

1. Check that all the connections are made properly.
2. There should be minimum disturbance during the experiment. Turning off the fans is recommended.
3. Level of the rod should not be changed during the experiment.
4. Wait until steady state is reached. Have patience, sometimes it may take time.
5. Don't feed more than a maximum of 1 ampere current into electromagnet otherwise it may burn out.
6. Near resonance, take as many readings as possible so that the resonance frequency can be located more precisely.

QUESTIONS

1. Why is $\omega_r < \omega_0$? Find out a relation between the two for the condition of resonance.

2. How does γ depend on damping current? Does ω_r depend on γ?

Reference: 1. Kleppner and Kolenkow, An Introduction to Mechanics (McGraw Hill, 1978).

REPORT SHEET FORMAT

Pohl's Pendulum

Aim:

Theory:

Working Formulae:

$$Natural\ frequency\ \ \omega_0 = \frac{2\pi}{T}$$

$$Damping\ constant\ \ \gamma = \sqrt{2(\omega_0^2 - \omega_r^2)}$$

where,

$$T = time\ period\ of\ oscillation$$

and $\omega_r = Resonance\ frequency.$

Observation/Table:

Least count of stopwatch =

Least count of amplitude scale =

Table I: Table for natural frequency (Take at least 4 readings)

Sl. No.	No. of oscillations (N)	Total Time (t) in s	Time Period $\left(T = \frac{t}{N}\right)$ in s	Frequency (ω) $in\ rad/sec$
1				
2				
3				
4				

$Average\ Natural\ Frequency\ \boldsymbol{\omega_0} =$ _________ rad/sec

Table II: Table for resonance frequency (Take at least 16-18 readings, max near the resonance)

$$Damping\ current = 0.3\ \mathrm{A}$$

Sl. No.	Voltage (V) $in\ volt$	Total time for 10 oscillations (t) in sec	Time Period $\left(T = \frac{t}{10}\right)$ in s	Driving Frequency (ω) $in\ rad/sec$	Amplitude $A = \frac{A_1 + A_2}{2}$ unit
1					
2					
..					
18					

Graph: Plot graph of amplitude (A) vs driving frequency (ω).

→ Draw graph with proper label, title & max. point near resonance.

→ Encircle all data points and draw a smooth graph.

→ Indicate resonance frequency and band width.

Calculations:

1. Natural frequency and resonance frequency.
2. Band width and damping constant.

Error Analysis: Calculate errors in -

1. Natural frequency
2. Resonance frequency
3. Band width
4. Damping constant

Final Result: Write your final results in this format - $Result \pm Error$. [All units should be mentioned]

1. Natural frequency $(\omega_0) =$
2. Resonance frequency $(\omega_r) =$
3. Band width $(\omega_b) =$
4. Damping constant $(\gamma) =$

Experiment 07

M.I. of Bicycle Wheel

Aim:

- *To study the angular motion and find the moment of inertia of a bicycle wheel.*

THEORY AND PRINCIPLE

A bicycle wheel is mounted in a bracket fixed to the wall. A brass cylinder (called collar) with a pin P protruding out from it is attached to the axle of the wheel. A small mass m, is attached to a string the other end of which has a small loop which is wound around the pin P $(Fig. 1)$, thus enabling the string to be wound uniformly around the collar. As the mass descends under the action of gravity it imparts an angular acceleration to the wheel. As long as the string is wrapped on the collar the velocity of the mass v, and the angular speed of the wheel ω, are related to each other as $v = \omega r$, where r is the radius of the collar. Suppose that the loop in the pin falls off when the mass has fallen through a height h below the point from where the mass started descending. After this the wheel goes on rotating by virtue of its rotational inertia but comes to rest after some time because of frictional losses of energy. At the instant the thread leaves the pin, we have

$$Loss\ in\ potential\ energy\ of\ m = Gain\ in\ kinetic\ energy\ of\ m + Gain\ in\ kinetic\ energy\ of\ wheel$$
$$+ Energy\ lost\ in\ overcoming\ friction\ at\ bearing$$

$$\Rightarrow mgh = \int_0^{v_0} mv\, dv + \int_0^{\omega_0} I\omega\, d\omega + n_1 f$$

$$\Rightarrow mgh = \frac{mv_0^2}{2} + \frac{I\omega_0^2}{2} + n_1 f = \frac{mr^2\omega_0^2}{2} + \frac{I\omega_0^2}{2} + n_1 f \qquad \text{..................} \ (1)$$

where I is the moment of inertia of the wheel with its axle etc., f is the (unknown) amount of energy lost per revolution, n_1 is the number of revolutions made by the wheel while m travelled through h, v_0 is the maximum velocity of mass and ω_0 is the maximum angular speed of the cycle wheel when the loop comes off the pin.

After this, the angular speed goes on diminishing and the wheel comes to rest when the energy ${I\omega_0^2}/{2}$ has been used up in overcoming friction. If the wheel has, by then, made n_2 further turns we can write

$$\frac{I\omega_0^2}{2} = n_2 f \quad \Rightarrow f = \frac{I\omega_0^2}{2n_2}$$

Substituting this in $eq. (1)$, we get

$$mgh = \frac{mr^2\omega_0^2}{2} + \frac{I\omega_0^2}{2} + \frac{I\omega_0^2 n_1}{2n_2}$$

$$\Rightarrow I = \frac{2mgh - \boxed{m\omega_0^2 r^2}}{\omega_0^2\left(1 + \frac{n_1}{n_2}\right)} \qquad \text{..................} \ (2)$$

This part may be neglected

A measurement of the various quantities enables I to be determined. You will probably find that the first term in $eq.(1)$ or (2) is negligible. In that case (but only if your estimate of other errors is greater than this term) you may neglect it.

PROCEDURE

→ Get familiar with the instruments. Locate the collar and the pin. Estimate the height available for the mass to fall and decide how many full turns n_1 you wish to wrap with this height. The height fallen $h = 2\pi r n_1$ will be easy to determine.

→ Study the action of the stopwatch given to you. If there are three events A, B and C in this order, the stopwatch can measure time intervals between the event A and B, and also between the event B and C in one go. Learn how to do it.

→ Also learn how to use light barrier detector to measure time of one revolution of the wheel. The instructor may explain this to students collectively.

→ Before beginning to take observations check if the wheel rotates freely. Make a loop at the free end of the string, put it around the pin and wrap n_1 turns of the string on the collar. n_1 will be an exact integer. Leave the mass from here, the wheel will accelerate till it completes n_1 turns (the string slips off the pin at this instant) and decelerates till it completes further n turns (it comes to rest now). You already know n_1. You have to measure n as well as time t_1 during acceleration and time t_2 during deceleration.

n_1 was arranged to be a whole number but it may not be possible to make n a whole number. So, you must devise some way of measuring fractional revolutions.

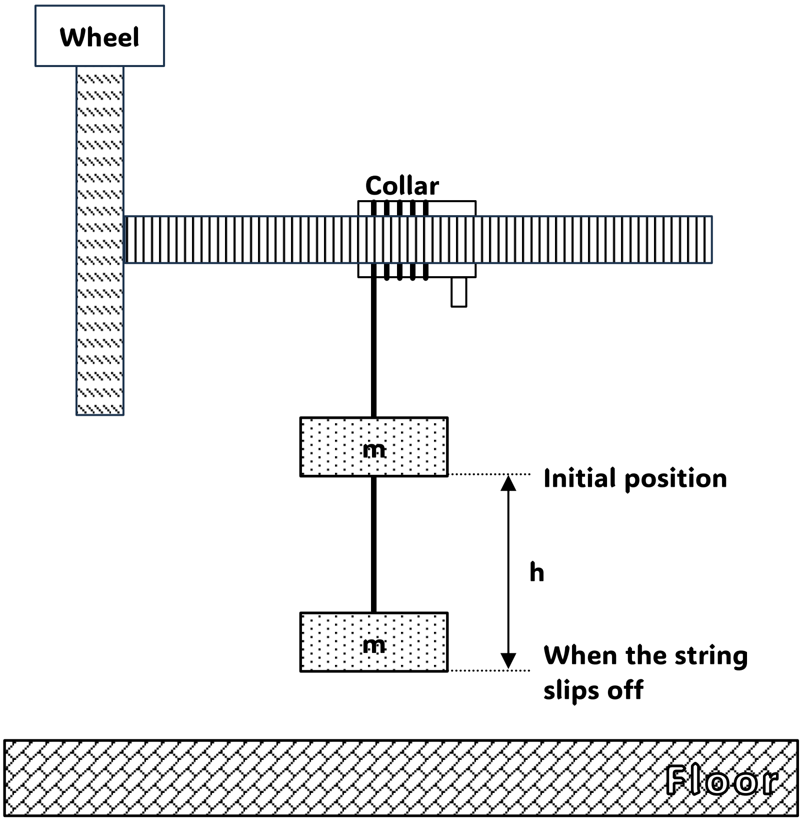


Fig. 1

→ Make observations at least three times in order to improve the accuracy of data. The velocity of the mass at the end of the descent is $v = {}^{2h}/_{t_1}$ and the angular speed of the wheel at this instant is

$$\omega = \frac{v}{r} = \frac{2h}{rt_1} \qquad \text{.................. } (3)$$

You can also get maximum ω from both acceleration (ω_1) and deceleration (ω_2) part. It is

$$\omega_{1,2} = \frac{4\pi n_{1,2}}{t_{1,2}} \qquad \text{.................. } (4)$$

Theoretically the ω_1 and ω_2 is equal.

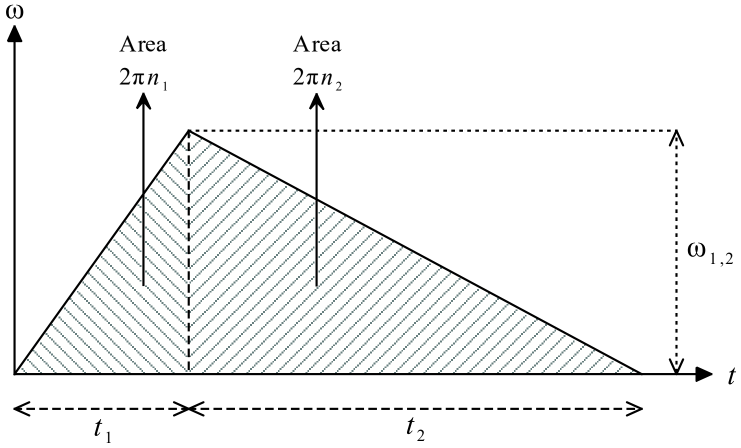
Make all measurements for at least three values of m.

→ The graph of ω vs t is a straight line with $+ve$ slope during acceleration and during deceleration it will be a straight line with $-ve$ slope.

The relation between ω and t during acceleration is given by

$$\omega = \left(\frac{\omega_1}{t_1}\right) t$$

The relation between ω and t during deceleration is given by



$$\omega = -\left(\frac{\omega_2}{t_2}\right) t + \omega_2 \left(1 + \frac{t_1}{t_2}\right)$$

→ You now have all the quantities you need to compute I from $eq.(3)$. You will find that the values of ω (as also those of v) obtained by measurement embodied in $eqns.(4)$ & (5). Choose the one you think to be the more accurate one and explain why you think so.

→ $(Optional)$ In order to estimate the effect of friction compute I also from

$$mgh = \frac{mr^2\omega^2}{2} + \frac{I\omega^2}{2} \qquad \text{..................} (5)$$

which is $eq.(3)$ except for the term in f. Calculate the fractional difference.

You may also do the following:

- ➔ Fasten two clamps of equal masses m_1 at the diametrically opposite positions of the rim of the wheel and get the moment of inertia I_1.
- ➔ Check if you get $I_1 - I_0 = 2m_1R^2$ as expected from the theory.

QUESTIONS

1. When m strikes the floor sound and heat energy are produced. Does it affect your result?
2. How would it affect your results if the string is not evenly wound on the axle?
3. Suggest a more accurate method of measuring ω.

Reference: Resnick and Halliday, Part 1, pp. 251-252.

REPORT SHEET FORMAT

M.I. of Bicycle Wheel

Aim:

Working Formulae:

$$I = \frac{2mgh - m\omega_0^2 r^2}{\omega_0^2\left(1+\frac{n_1}{n_2}\right)}$$

I = *moment of inertia of bicycle wheel*
ω = *angular velocity when string gets detached*
n_1 = *no. of rotation during acceleration*
n_2 = *no. of rotation during deceleration*

where $h = 2\pi r n_1$

$$\therefore I = \frac{(2mg \times 2\pi r n_1) - m\omega_0^2 r^2}{\omega_0^2\left(1+\frac{n_1}{n_2}\right)}$$

$$\Rightarrow I = \frac{\left(\frac{4\pi n_1 mgr}{\omega_0^2} - mr^2\right)}{\left(1+\frac{n_1}{n_2}\right)}$$

Angular velocity, $\omega = \frac{4\pi n_1}{t_1} = \frac{4\pi n_2}{t_2}$

Standard deviation $(\sigma) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$

Observation/Table:

Vernier constant $(v.c)$ of vernier calipers = [*Show Calculation*]

Least count of stopwatch =

Least count of weighing machine =

Least count of protractor =

Table I: Table for radius of cylinder

Sl. No.	Main scale reading (M.S.R) *in* mm	Vernier coincidence (x)	Vernier scale reading $(\text{V.S.R} = x \times v.c)$ *in* mm	Total Diameter $(d = \text{M.S.R} + \text{V.S.R})$ *in* mm
1				
2				
3				

Average diameter $\boldsymbol{d}$ = __________ m, *Average radius* $\boldsymbol{r} = \frac{\boldsymbol{d}}{\mathbf{2}}$ = __________ m

Table II: Table for moment of inertia

(for three different values of mass and three different readings for each mass)

» *No. of rotation during acceleration should be greater than or equal to 5* $(n_1 \geq 5)$.

Mass (m) *in* gm	No. of rotation during acceleration (n_1)	Time taken during acceleration (t_1) *in* sec	No. of rotation during deceleration (n_2)	Time taken during deceleration (t_2) *in* sec	Angular velocity obtained from acceleration data (ω_1) *in* rad/sec	Angular velocity obtained form deceleration data (ω_2) *in* rad/sec	Average Angular velocity (ω) *in* rad/sec	Moment of Inertia (I) *in* $\mathrm{kg \cdot m^2}$
$\boldsymbol{m_1}$								
$\boldsymbol{m_2}$								
$\boldsymbol{m_3}$								

$$Average\ moment\ of\ inertia\ \boldsymbol{I_{av}} = \underline{\hspace{3cm}}\ \mathrm{kg \cdot m^2}$$

Calculations: [*Each student in a group has to choose different set of readings to show calculation*]

1. Angular speed of cycle wheel (ω).
2. Moment of inertia (I).

Error Analysis:

1. Calculate standard deviation (σ_I) in moment of inertia.
2. Determine propagation error $(for\ any\ one\ measurement)$ of moment of inertia.

Final Result:

Write your final results in this format - $Result \pm Error$. [All units should be mentioned $(in\ S.I.)$]

Moment of inertia $(I) = I_{av} \pm \Delta I$.

Experiment 08

Electromagnetic Induction

Aim:

- *To study the relation between maximum emf and the velocity of the magnet.*
- *To study the flux in a solenoid coil as a function of time when a magnet passes through it.*

PRINCIPLE

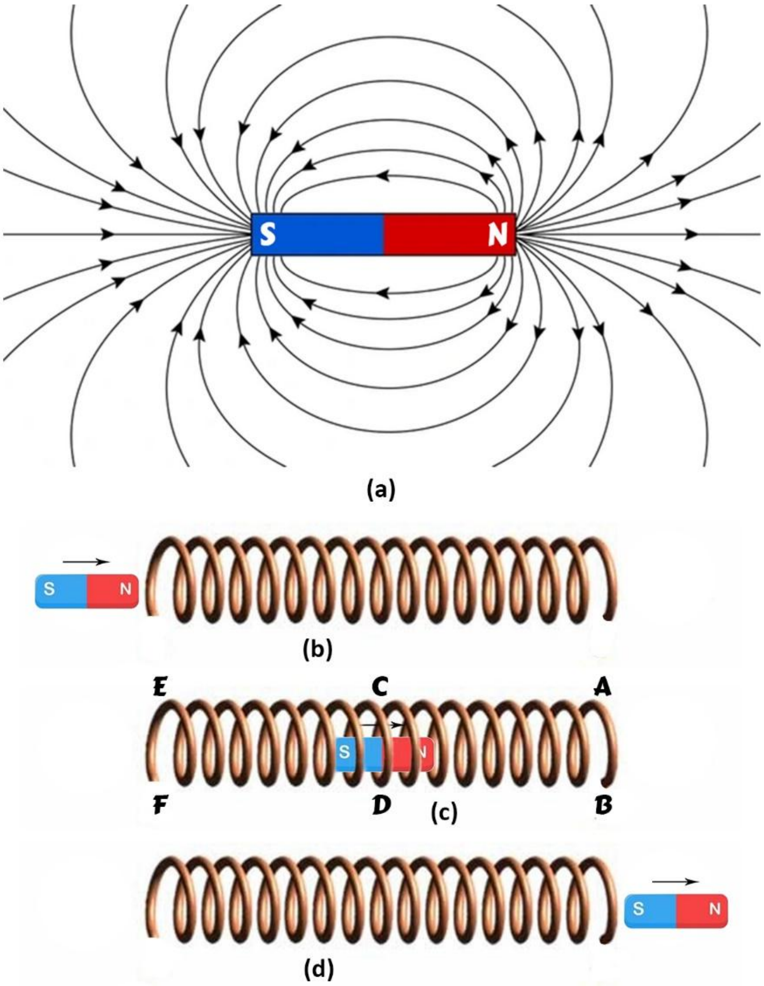
When the magnetic flux through a coil changes, an emf is produced in the coil which is given by

$$\varepsilon = -\frac{d\varphi}{dt} \qquad \text{..................} \; (1)$$

This is Faraday's law of induction. The objective of this experiment is to measure the induced emf ε as a function of time when a bar magnet moves through a solenoid with a velocity v.

Consider a fixed solenoid of finite length placed with its axis along the x-axis. A bar magnet moves along the x-axis with a speed v. Let x denotes the coordinate of the center of the magnet *(Can you guess how will the flux change as a function of x?)*. The flux through the i-th turn of the solenoid is $\varphi_i = \int B_x dA$, where dA is an area element in the plane of the turn. The total flux through the solenoid $\varphi = \sum_i \varphi_i$ where summation is made over all the turns. $Fig. 1(a)$ shows the magnetic field lines due to a bar magnet. Consider the situation in $Fig. 1(b)$. As the magnet comes closer, φ_i through each turn is positive and increases. When the magnet goes in [$Fig. 1(c)$], the flux through turns like EF will decrease as the magnet moves ahead and the flux through turns like AB will increase. Till the magnet reaches the middle, the net effect is that the total flux φ increases.

As the magnet moves in the second half of the solenoid, more number of turns are left behind from where contribution to φ decreases. Net result is that φ starts decreasing. This continues even after the magnet comes out. Convince yourself that the flux changes as a function of x as shown in $Fig. 2(a)$.



$Fig. 1$

The emf induced will be

$$\varepsilon = -\frac{d\varphi}{dt} = -\frac{d\varphi}{dx}\cdot\frac{dx}{dt} = -v\frac{d\varphi}{dx}$$

You can work out the shape of $\frac{d\varphi}{dx}$ *from* $Fig. 2\ (a)$*, it will be like that shown in* $Fig. 2\ (b)$.

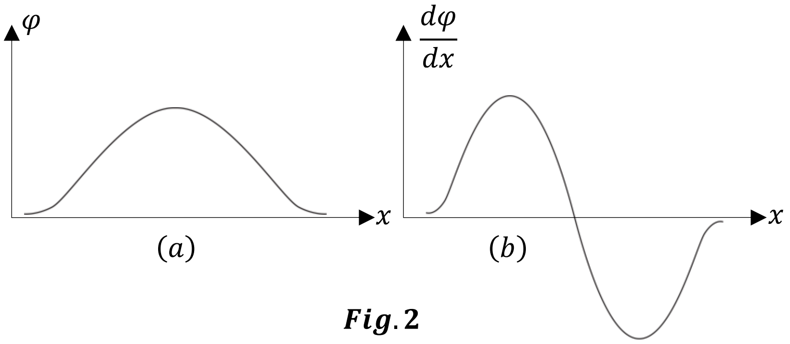


***Fig*.2**

APPARATUS

Oscillating Frame:

A simple apparatus has been designed to study the effect of flux change when a magnet goes at different rates through a coil of suitable area of cross section. A rigid frame of aluminum having the lower portion in the shape of a circular arc is pivoted at one point. The whole frame can oscillate freely in its own plane, about a horizontal axis passing through the pivot ($Fig. 3$). A bar magnet is mounted at the center of the arc and the arc passes through coil C. The angular amplitude can be read by means of a scale and a pointer.

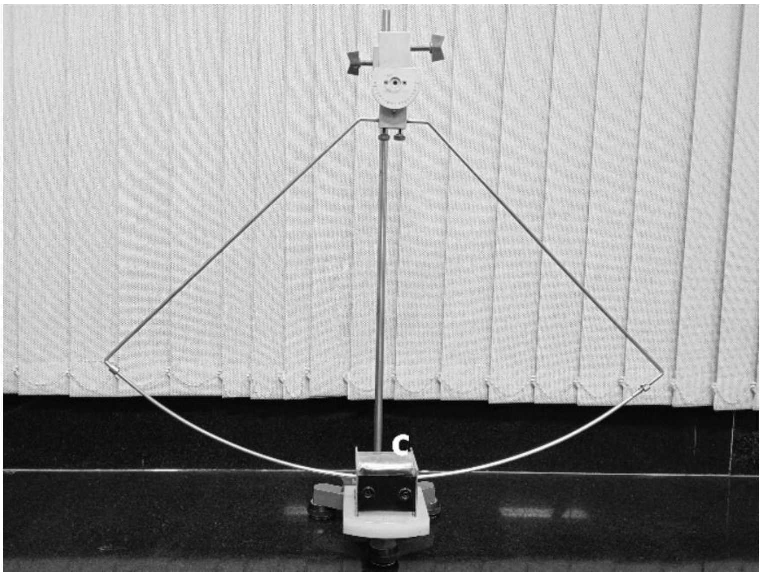


***Fig*.3**

If we leave the frame from an initial angular position θ_0, it will oscillate with a time period

$$T = 2\pi\sqrt{\frac{I}{Mgd}}$$

where d is the distance of the point of suspension (center of circle) from the center of mass.

You can check that the angular speed of the frame, when it passes through the equilibrium position, is

$$\omega_{max} = \frac{4\pi}{T}\sin\left(\frac{\theta_0}{2}\right)$$

and hence the speed of the magnet when it is at the center of the solenoid is

$$V_{max} = \frac{4\pi R}{T}\sin\left(\frac{\theta_0}{2}\right) \qquad \text{..................} (2)$$

where R is the radius of the circular arc of the frame. There will be small variation in the speed of the magnet as it goes through the solenoid, but for qualitative understanding, you can assume that it moves in the solenoid with roughly the same speed as given in $eq. (2)$.

Computer Interface:

The emf induced between the ends of the coil is directly read by the PC. The software 'Measure' installed in the PC gives you a graph of ε versus t.

Useful Features of The Software 'Measure'

It is easy to use the software given to you. Take a few minutes to familiarize yourself with it before going on to do the detailed experiment. All you have to do is to click the required icon given at the top. Some of the important ones are as follows -

Arrow	In this mode simply point the cursor at the required point to obtain values of the co-ordinates.
Mark	Use this to mark a portion of the curve. The x-coordinates of the marked portion are shown on the bottom. The marked portion is highlighted in a different color.
Survey	You can adjust the left bottom and right top co-ordinates of the cursor box to obtain coordinates and their differences in this mode. You can use this to calculate slopes around a point.
Show Integral	Mark the portion of the curve for which you need to calculate the integral and then click this icon to obtain the value. If you need to start from the origin each time, take the cursor out of the plotting area and drag it across the origin to ensure that the starting point is the same.
Slope	Mark the required portion of the curve for which you need the slope and then click this icon to get slope. However, we recommend the use of survey mode to get slope more accurately.

PROCEDURE

Ensure that the support for the apparatus is vertical by adjusting leveling screws. As far as possible, try to make zero of the scale as the mean position. Center the magnet inside the coil.

Play with the system

Connect the ends of the solenoid to the cobra interface with a bar magnet at its place, release the frame from the angular position θ_0. Stop it as it completes half oscillation. Look at the $\varepsilon\ vs\ t$ curve on the PC screen. Is it as you expect from *Faraday's law*? Let the frame oscillate for 2-3 rounds and record $\varepsilon\ vs\ t$. Can you locate the point on the curve, when the magnet entered the coil. Practice on finding slope and integrating the curve under desired limits using the software *MEASURE*.

Measure the time period T

First find out the time period of small oscillation T for the apparatus from the ε versus t which gets recorded on the PC. Measure the time period from the plot on your computer screen.

Make sure you account for whole one cycle while measuring T from the PC screen.

Plot of $\varphi(t)$

Record one or two oscillations on the PC. Focus on only one of them by using magnification button. Use the integration feature in the software to obtain the flux φ as a function of time (You may integrate only half of the pulse since the pulse is highly symmetric, although it is more accurate to integrate the full pulse). Plot this flux on a graph paper for the complete pulse. Write how much is the maximum flux. Find the maximum flux for different initial angular displacement θ_0. What do you observe?

V_{max} dependence of ε_{max}

At what position of the magnet is the emf maximum? At what position of the magnet is the flux maximum? Change the speed of the magnet in the solenoid by releasing the frame from different initial angular position θ_0. As the speed is more, the flux change will take place faster and the induced emf will be more. Record the emf vs time for several values of θ_0, get V_{max} from $eq.(3)$ and ε_{max} from the $\varepsilon - t$ plot on the PC screen. Plot on a graph paper ε_{max} vs V_{max}.

$(d\varepsilon/dt)_{max}$ as a function of V_{max}^2

As ε_{max} is related to V_{max}, so is $(d\varepsilon/dt)_{max}$ with V_{max}^2. Use the *'SURVEY'* mode of the software to calculate the slope $(d\varepsilon/dt)$ from the $\varepsilon - t$ curve and find $(d\varepsilon/dt)_{max}$. Do it for several values of θ_0 and plot $(d\varepsilon/dt)_{max}$ vs V_{max}^2.

Is there a relation between ε_{max} and V_{max}? From your $\varphi(t)$ plot you will notice that the rate of change of φ is maximum for positions close to the mean position (specially for amplitudes so that the magnet is initially far from the coil). The rate of change of velocity near the mean position is small and hence can be considered close to V_{max}. Hence, we can expect under these assumptions

$$\varepsilon_{max} = -\left(\frac{d\varphi}{d\theta}\right)_{max} \omega_{max}$$

To test the above relations record one or two oscillations for several values of amplitudes as given in the table. For each case choose one pulse and obtain ε_{max}, slope at mean position and φ. For calculating slope use the 'survey' mode to obtain Δx and Δy around the point of interest. For calculating φ use 'Show integral' mode as before.

QUESTIONS

1. Estimate the distance from the mean position to the point at which the induced emf is maximum.
2. Is it necessary that the magnet passes through the central axis of the coil to obtain maximum emf?

REPORT SHEET FORMAT

Electromagnetic Induction

Aim:

Working Formulae:

Observation/Table:

Least count of scale =

Least count of angular scale =

Table I: Table for V_{max} and ε_{max}

$Radius\ of\ the\ circular\ arc\ of\ the\ frame\ (R) =$ ________ cm = ________ m

No. of obs.	Angle (θ_0) $(in\ degree)$	T_1 $(in\ ms)$	T_2 $(in\ ms)$	Time Period $[T = T_2 - T_1]$ $(in\ ms)$	V_{max} $(in\ m/s)$	E_{max} $(in$ volt)	E_{min} $(in$ volt)	$\lvert\varepsilon_{max}\rvert = \dfrac{E_{max} - E_{min}}{2}$ $(in$ volt)
1	5°							
2	10°							
3	15°							
4	20°							
5	25°							
6	30°							
7	35°							

Table II: Table for $flux\ (\varphi)$ and $time\ (t)$

No. of observations	Time (t) in ms	Flux (φ) in V · ms
1		
2		
3		
.		
.		
20		

Calculations: Show calculations of V_{max} for all amplitudes.

Graph: Write down the title, unit, and smallest division in each graph and use the full graph paper.

1. Plot graph of ε_{max} vs V_{max} [*Fit the graph with straight line*].
2. Plot flux (φ) vs time (t)

Error Analysis: You have used the following relation to calculate error V_{max}.

$$\frac{\Delta V_{max}}{V_{max}} = \frac{\Delta R}{R} + \frac{\Delta T}{T} + \frac{\Delta\theta}{2}\cot\left(\frac{\theta}{2}\right)$$

Experiment 09

Current Balance

Aim:

- *To learn about the magnetic fields produced by current carrying wires.*
- *To obtain the value of free space permeability (μ_0) using a current balance.*

PRINCIPLE

If a current I is sent through an infinitely long straight wire in z-direction a magnetic field is set up around it, which is given by

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\varphi} \qquad \text{.................. (1)}$$

where r is the distance from the central axis of the wire and μ_0 is the permeability of free space ($4\pi \times 10^{-7}\ \mathrm{N/A^2}$). If another wire carrying the same current I is placed parallel to the first wire at a distance r from it, it will experience a force of magnitude given by

$$F = BIL = \frac{\mu_0 I}{2\pi r} \cdot IL$$

$$\Rightarrow F = \frac{\mu_0 L}{2\pi r} I^2 \qquad \text{.................. (2)}$$

where L is the length of the second wire. If the two currents are in opposite directions, they repel each other and if they are in the same direction, they attract each other. The expression for the force is, strictly speaking, valid only for infinitely long first wire, but we will assume it to be sufficiently accurate for this experiment.

The above expression for the force is used to define the ampere in SI. The ampere is defined as follows: “One ampere is the current which, if present in each of two parallel conductors of infinite length and one meter apart in a vacuum, causes each conductor to experience a force of exactly 2×10^7 newton per meter of length”.

In this experiment a current I is passed in opposite directions through two parallel horizontal bars which are connected in series. The lower bar AB is fixed; the upper one CD is part of a rectangular frame which can rotate about a horizontal axis PQ. It can be balanced a few millimeters above the fixed bar AB by adjusting a counterweight W. The upper bar supports a small pan onto which weights are placed, thereby causing the upper bar to drop down towards the lower one.

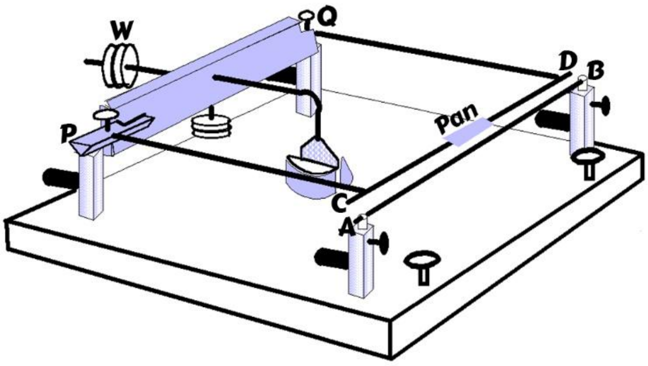
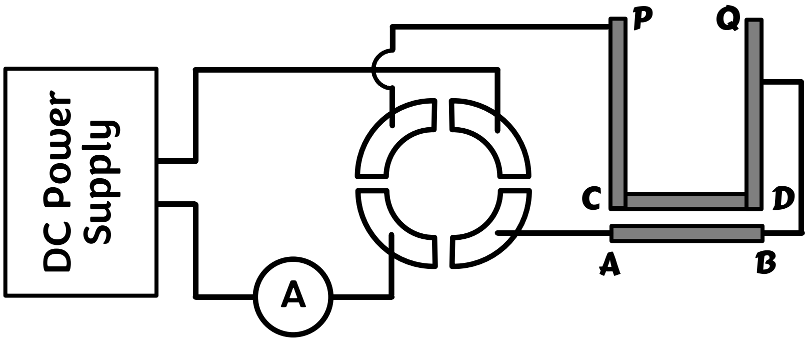


***Fig.* 1**

When the current is turned on and increased sufficiently, repulsion between the two bars causes the frame to rotate so that CD goes up. The frame stays at some angle where the torque due to weights and the magnetic repulsion force balance. By putting appropriate weights on the pan, the frame can be brought back to its initial equilibrium position. The position of the bar is observed by means of a mirror, a laser and a scale. With this experimental set up one can then determine the relationship between the force on either conductor and the current passing through the conductors.

Current is passed from the DC power supply to the bars through a commutator. It contains four holes and two keys are inserted in one set of opposite holes to pass the current in the bars in one sense. When the keys are put in the other set of opposite holes, the direction of current in bars gets reversed.



$\boldsymbol{Fig.2} - Wiring\ Diagram$

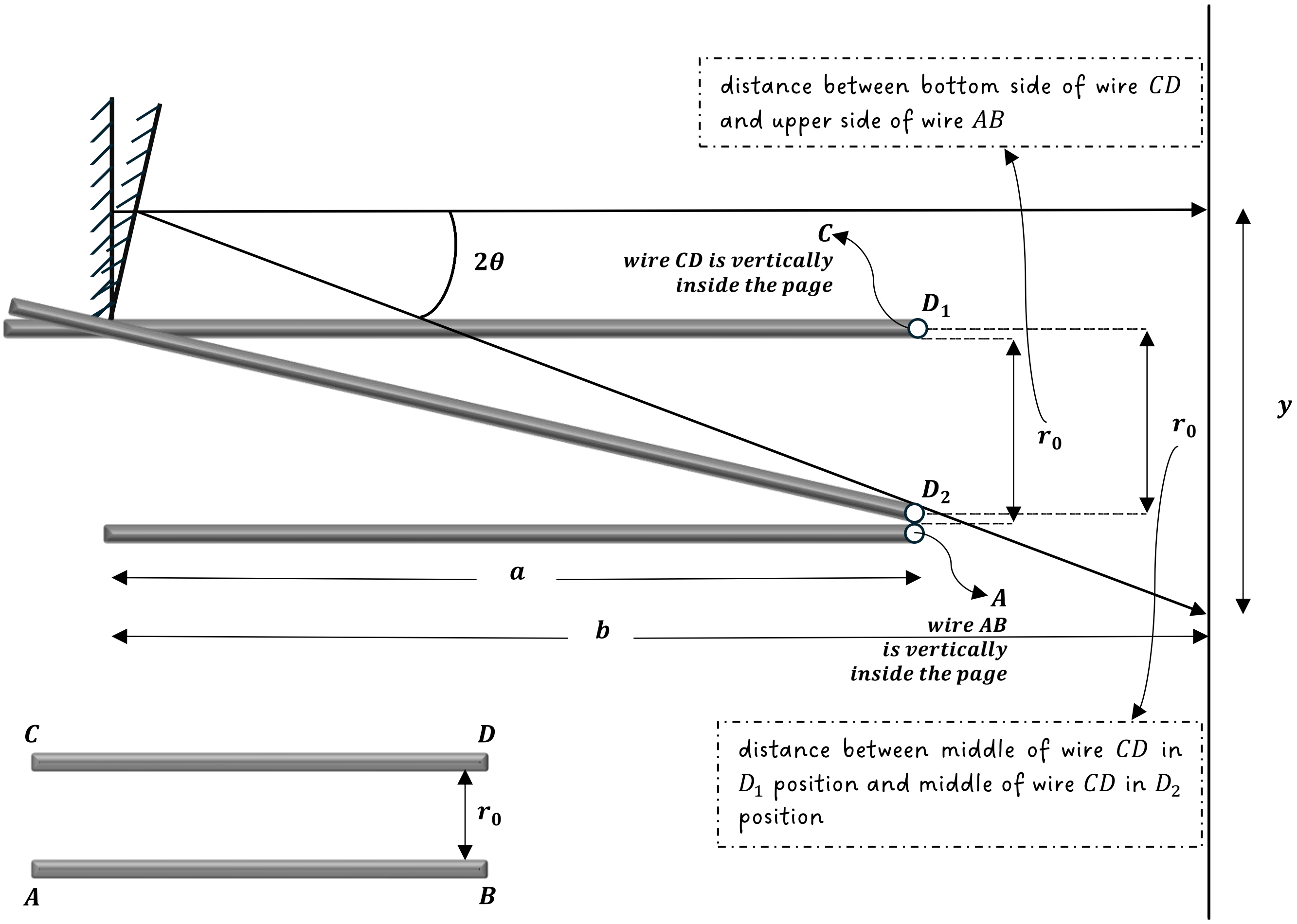
PROCEDURE AND ANALYSIS

In this experiment you will measure the force F exerted by the fixed bar AB on the upper bar CD as a function of the current I passed through them for a fixed distance r between them. From $eq.(2)$ you can determine μ_0.

- → Look at the apparatus and identify various parts of the set up. Check that the frame rotates freely about the axis PQ. Check that AB and CD are both horizontal and parallel to each other. If needed, you can adjust the inclination of AB with the help of the screws given on the binding posts art A and B. See how the frame rotates if you push the bar CD slightly in upward direction. Study how the current gets reversed when the positions of the keys in the commutator are changed.
- → Level the base of the current balance using a spirit level and adjusting screws at the bottom of the balance. *Do not turn the screws if the balance is already leveled.*
- → Connect the apparatus as indicated in the wiring diagram. Don't put keys in the commutator. Remember that the current carrying wires produce magnetic fields. Care has to be taken to avoid these fields exerting forces on the movable bar. Wire should leave binding posts at right angles.
- → Adjust the counterweight until the upper bar is a few millimeters above the lower bar.
- → Set up the laser and a scale at 1 to 3 meters from the mirror fixed with the frame. You are given a small laser for the measurement of small deviations of the bar. Adjust the laser mount until you can see the reflected laser beam spot on the scale clearly. Record the equilibrium point indicated by the reflected laser spot on the scale. Take care to choose a reference point on the reflected spot as the size of the spot (beam waist) could be fairly large. You may have to put off the fans in order to avoid oscillations of the frame.
- → In increments of $5\ \mathrm{mg}$, place weights in the pan. Adjust the current until the scale reading returns to its equilibrium value. Record the current. Reverse the current and repeat. Find the average current.
- → Measure the length of the upper bar carrying the current.

→ The center-to-center distance between the bars at equilibrium can be determined as follows. Measure the distance a from the knife-edge to the center of the front bar at each side (PC and QD) and take the average. Record the scale reading at equilibrium. Depress the upper bar CD by placing a small coin on the scale pan so that it touches the lower. Note the new scale reading. The distance r_0, between the facing surface of the two bars is given by

$$r_0 = \frac{ya}{2b}$$



where y is the difference in scale readings,

a is the mean distance from the knife edge to the bar, and

b is the distance from the mirror to the scale.

The center-to-center distance (r) is obtained by adding the diameter (d_0) of either wire to (r_0).

$$r = r_0 + d_0$$

Using the data obtained above plot the force F as a function of I^2 and determine the slope of the resultant curve. From the slope determine the value of the permeability of free space μ_0.

? QUESTIONS

1. Why do you not have to worry about the weight of the upper bar when you calculate the force F between the two bars?
2. Could the force between the conducting bars be determined in this manner if an alternating current were used?

Reference: **Introduction to Electrodynamics by D. J. Griffiths, 2nd Ed.**
Physics Part II by Halliday and Resnick.

REPORT SHEET FORMAT

Current Balance

Aim:

Working Formulae:

Observation/Table:

Least count of current display =

Least count of voltage display =

Least count of weighing machine =

Least count of vertical scale =

Least count of measuring tape =

Least count of screw gauge =

Table I: Table for force F and I^2 (Take 5 readings)

Sl. No.	Mass (m) in mg	I_1 in amp	I_2 in amp	I_{avg} in amp	$I^2 = (I_{avg})^2$ in amp	$F = mg$ in N
1						
2						
3						
4						
5						

Table II: Table for diameter of wire (CD) (make 3 data only at three different places of wire)

Sl. No.	Main scale reading (M.S.R) in mm	Circular scale coincidence with base line (x)	Circular scale reading $(C.S.R = x \times L.C)$ in mm	Total Diameter $(d = M.S.R + C.S.R)$ in mm
1				
2				
3				

Calculations:

1. Calculate r_0 and r.
2. Calculate slope of the graph of $F\ vs\ I^2$ and free space permeability μ_0.

Error Analysis:

Calculate errors in -

1. r_0
2. r
3. Slope of F vs I^2 graph
4. μ_0

Final Result:

Write your final results in this format - $Result \pm Error$. [All units should be mentioned]

1. r_0
2. r
3. Slope of F vs I^2 graph
4. μ_0

Experiment 10

Gyroscope

Aim:

- *To study the precession motion of gyroscope.*
- *To determine moment of inertia of gyroscope disc.*

INTRODUCTION

Gyroscopic motion plays an important role in many nuclear and celestial phenomena. Gyroscopes are used in compasses, in the steering mechanism of torpedoes and in inertial guidance systems. In this experiment you will study the *precession of gyroscope*. The objective is to find the moment of inertia of the gyroscope by measuring the precession frequency, as a function of the spin frequency of the gyroscope.

THEORY

The gyroscope is a uniform heavy disk G mounted on a rod as its axis. The disk can be spun about the rod and the rod can also be rotated freely in all directions. The gyroscope which is free to rotate about all the three axes, is balanced in horizontal position with the help of a counterweight C and is set to rotate with frequency ω about the x-axis [$see\ fig. 1$ & 2].

$Fig. 1$

If I is the moment of inertia of the gyroscope about its symmetric axis, the angular momentum L is given by

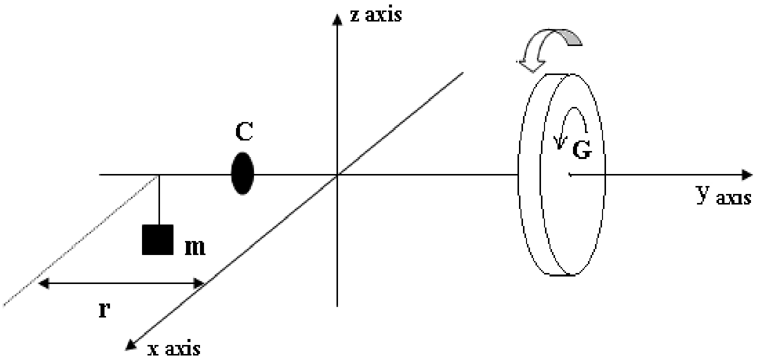
$$\vec{L} = I\omega\hat{j} \qquad \text{.................. (1)}$$

Now, the addition of an additional weight m, at a distance r from the support point O, introduces a supplementary torque τ.

$$\vec{\tau} = (-r\hat{j}) \times (-mg\hat{k}) = +mgr\hat{\imath}$$

$$\Rightarrow \frac{d\vec{L}}{dt} = mgr\hat{\imath} \qquad \text{.................. (2)}$$

$$\Rightarrow d\vec{L} = mgr \cdot dt\ \hat{\imath}$$



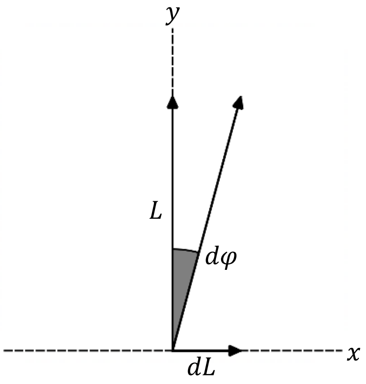
$\boldsymbol{Fig.2}$

From (1) and (2), you see that the angular momentum vector turns by an angle $d\varphi$ in the x-y plane ($see\ fig.3$). As the spin angular velocity is large, the angular momentum remains almost along the axis of the wheel, and hence to rotate the angular momentum, the wheel itself rotates.

Thus, the gyroscope starts precessing (rotates slowly about the vertical axis) with a frequency ω_p under the influence of τ.

The angle rotated in time dt is $d\varphi$, from $fig.3$,

$$d\varphi = \frac{|d\vec{L}|}{L} \qquad \text{.................. (3)}$$



$\boldsymbol{Fig.3}$

Using $eq.1, 2\ and\ 3$, we get the precession angular velocity as

$$\omega_p = \frac{d\varphi}{dt} = \frac{1}{L}\left|\frac{d\vec{L}}{dt}\right| = \frac{mgr}{I\omega} \qquad \text{.................. (4)}$$

If t_p is the time for one complete precessional revolution and t is the time taken by the gyroscope to spin about its axis (one rotation), then

$$\omega = \frac{2\pi}{t}\ \ \&\ \ \omega_p = \frac{2\pi}{t_p}$$

Putting these in $eq.(4)$, we get

$$\frac{1}{t} = \frac{mgr}{4\pi^2 I}t_p \qquad \text{.................. (5)}$$

Thus, a plot of $\left({}^{1}/_{t}\right)\ vs\ t_p$ should yield a straight line for a fixed m, from which the moment of inertia I of the gyroscope disc can be obtained.

PROCEDURE

Balance the gyroscope G horizontally, using the counterweight C $(see\ Fig. 1)$, without any hanging weight m.

- → Give a spin to the horizontally balanced gyroscope by pulling a string wrapped over its stem and measure the time (t) required to complete one revolution using the given light barrier counter. [For this, attach a soft opaque strip to the rim of the gyroscope].
- → Immediately after this, hang a mass m on the groove at the longer end of the gyroscope. This is at a distance $r = 27\text{cm}$. The gyroscope will precess $(can\ you\ guess\ the\ direction?)$. Using the stopwatch, measure the duration of half the rotation $\left({}^{t_p}/_2\right)$.
- → Without any delay, remove the mass m, so that gyroscope stops precessing and measure t again, using the light barrier counter.
- → The average of t measured in steps (a) and (c) above is to be used in $eq. (4)$.
- → Repeat for several different initial spins of the gyroscope for a particular m and plot $\left({}^{1}/_{t}\right)\ vs\ t_p$ and find the slope. Find I using $eq. (4)$.
- → Find I for two more values of m.
- → Estimate the uncertainty in your result.

QUESTIONS

1. What are the directions of L and τ in the frame of axes shown in Fig. 2?
2. Why are you asked to measure ${}^{t_p}/_2$ and double it, instead of measuring t_p directly?

You may also do the following:

- ➔ Rotate the wheel in opposite sense and see that the sense of precession also gets reversed.
- ➔ Spin the wheel and give a gentle tap on the axle rod from above. See if the rod rotating. Now give a tap at the same position from below. What do you see?
- ➔ Put the mass m on the groove while the gyroscope is not spinning. Analyze the motion by working out the direction of torque of mg and of the angular momentum produced during the motion.

Reference: Kleppner and Kolenkow, An Introduction to Mechanics (McGraw Hill, 1978), Chapter 7.

REPORT SHEET FORMAT

Gyroscope

Aim:

Working Formulae:

Observation/Table:

Least count of light barrier counter =

Least count of stopwatch =

Mass of hanger =

Mass of block =

Total mass =

Sl. No.	Initial time period of wheel (t_1) in sec	Final time period of wheel (t_2) in sec	Half precession time period $\left({}^{t_p}/_{2}\right)$ in sec	Average time period of wheel (t_{avg}) in sec	Inverse of Average time period $\left({}^{1}/_{t_{avg}}\right)$ in sec^{-1}	Complete precession time (t_p) in sec
1						
2						
3						
4						
5						
6						

Graph:

Plot $\left({}^{1}/_{t_{avg}}\right)$ vs t_p (for calculation, use back page of graph paper).

Calculations:

1. Slope of best fit line (m).
2. Average moment of inertia (I) of the wheel (use m to calculate this).

Error Analysis:

1. Calculate error in the slope (Δm).
2. Calculate error in moment of inertia (ΔI).

Final Result: Write your final results in this format – $Result \pm Error$. [All units should be mentioned]

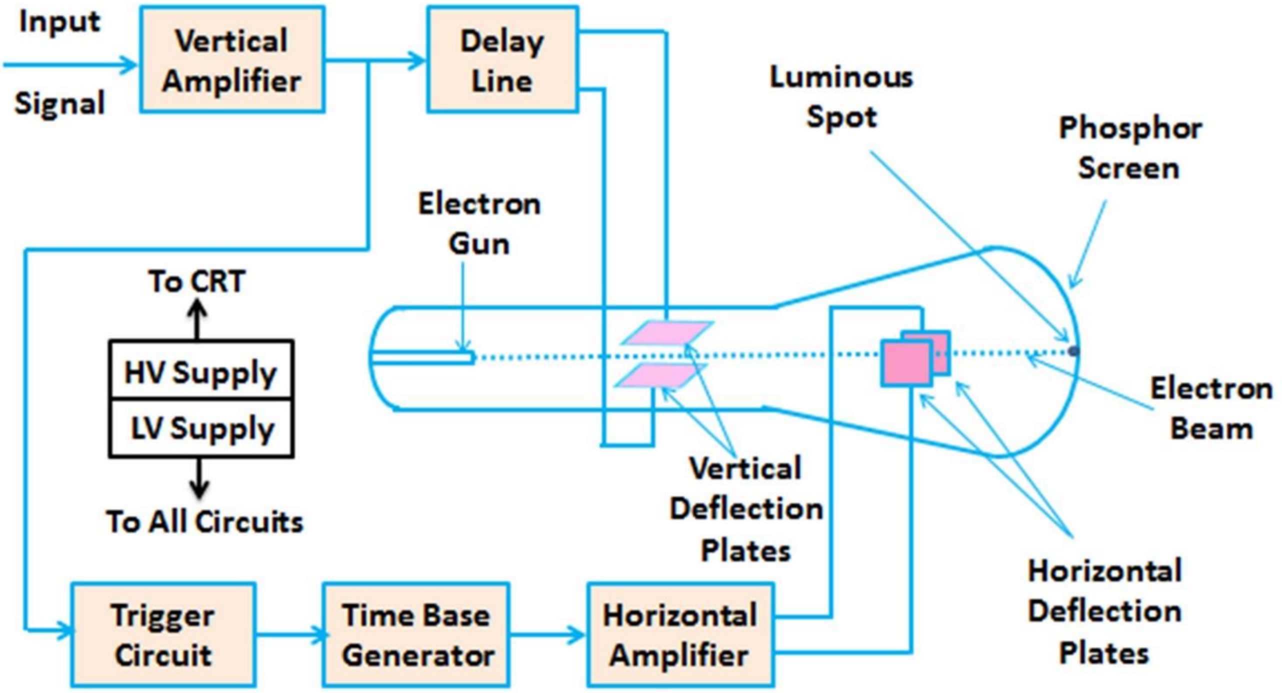
Appendix A

Cathode Ray Oscilloscope

(Velocity of Light)

BASIC PRINCIPLE OF CATHODE RAY OSCILLOSCOPE (CRO)

CRO is employed to study waveforms, transient phenomena, and other time varying quantities from very low to high frequencies. The major component of CRO is cathode ray tube (CRT). In the diagram below the schematic of the cathode ray tube (CRT) and a general purpose CRO is shown.



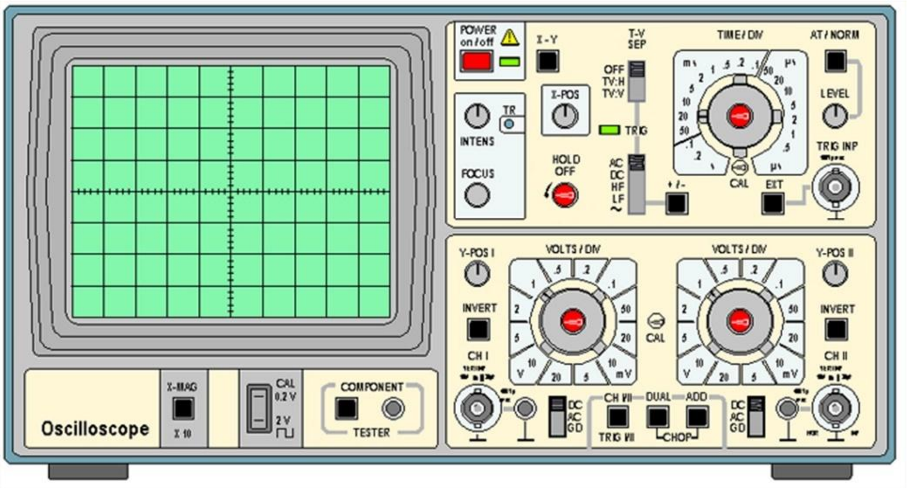
Block Diagram of a General Purpose CRO

The CRT consists of a highly evacuated funnel-shaped glass tube. The electrons are emitted from thermionic cathode. A number of electrodes transform the emitted electrons into a high velocity electron beam, known as cathode ray. The electron beam travels through the evacuated space of the tube towards a fluorescent screen. When the beam strikes the screen, the kinetic energy of the electrons is converted into light emission. A small light spot is thus produced on the CRT screen at a place where the electrons hit it. On its way towards the screen the electron beam can be deflected by suitable voltages. Usually, the signal under test deflects the spots vertically on the screen. Another voltage proportional to time is employed to deflect the spot horizontally. Thus, the time variation of the signal is displayed on the screen. If the external voltage is DC, a horizontal straight line would be traced out whose vertical position depends on the voltage applied. For sine or square wave voltages, the corresponding shape is traced on the screen, and the amplitude and period of the signal can be measured.

Precaution

It is instructive to operate the CRT with a low brightness of the spot. If a very bright spot is allowed to stay at rest on the screen for long time, burnout problems may appear.

To measure time, the horizontal displacement on the CRT screen is calibrated in time. The horizontal axis is referred to as the time base. The calibration of the horizontal axis is read from the front panel control marked TIME/DIV. The vertical axis is calibrated in volt to measure the signal voltage. The calibration is read from the front panel control marked VOLT/DIV. The sweep and the signal voltages are amplified before application to the deflecting plates. The corresponding amplifiers are known as horizontal amplifier and vertical amplifier, respectively.



In the dual beam CRO two electron beams are obtained in the CRT. The beams produce two spots of light on the CRT screen and facilitate simultaneous observation of two signal waveforms.

The steps described in the following subsections below are designed to introduce the different features of the scope one after another.

OPERATION

Do not turn the CRO on until you have studied the controls on the control panel of the scope. Disconnect any signal cables from either of the two input terminals on the control panel. Locate the intensity knob and turn it completely counter-clockwise. This will ensure that the beam does not damage the screen when the scope is later turned on. Turn the CRO on by pushing the power button in. Turn the intensity knob clockwise until the beam appears as a stationary spot. The spot should be near the center of the screen. Adjust the focus knob to make the spot sharp and in focus.

Locate the large knob marked SEC/DIV. This knob controls the speed of the beam sweeping across the screen horizontally. The actual setting of this knob will be displayed on the oscilloscope screen. For example, if SEC/DIV is set at 5 ms, the beam will move to the right by one division in five milliseconds.

Now find two similar knobs marked VOLTS/DIV. These knobs control the deflection of the electron beam in the vertical direction for a given input voltage. As with the SEC/DIV knob, the actual setting of the VOLTS/DIV knob will be displayed on the screen. For example, if a signal with strength of one volt is applied, then with the knob at 1 VOLT/DIV, would cause a vertical deflection of 1 division. With the knob turned clockwise to the 0.2 VOLT/DIV position, the same signal would cause a deflection of 5 divisions.

In addition to these two controls described above, there are a number of other knobs and buttons on the control panel. We will not describe the function of all these knobs here. Notice that many of the buttons have both an “in” and an “out” position. When the button is in the “in” position, that feature is enabled. Find the GND button on the control panel. When the GND button is in the “in” position, the vertical input of the scope is connected to ground potential (*i.e.*, a potential of 0 volts).

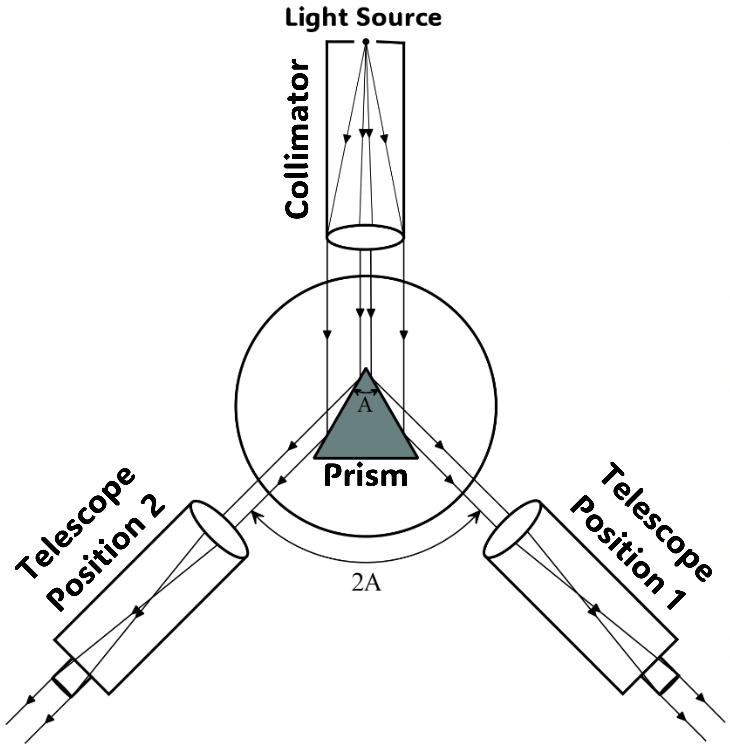
Appendix B

Angle of Prism

(Prism Spectrometer)

MEASUREMENT OF THE REFRACTING ANGLE ‘A’ OF PRISM

- Place the prism on its triangular base so that its refracting edge is at the center of the prism table and points towards the collimator.
- Turn the prism table such that about half of the light falls on each refracting face. Lock the prism table. You should be able to see the image of the slit with naked eye on reflection from either face of the prism.
- Now rotate the telescope to receive the reflected light on one side of the prism. Do the same on the other side. If the instrument is correctly leveled the images from both sides fall at the center of the telescope cross wires. If necessary, adjust the entrance slit width of the collimator to sharpen the image.
- Bring one edge of the slit image into coincidence with the intersection of the crosswires and lock the telescope. Record the reading using the vernier. Do the same on the other side. Use the same vernier each time.



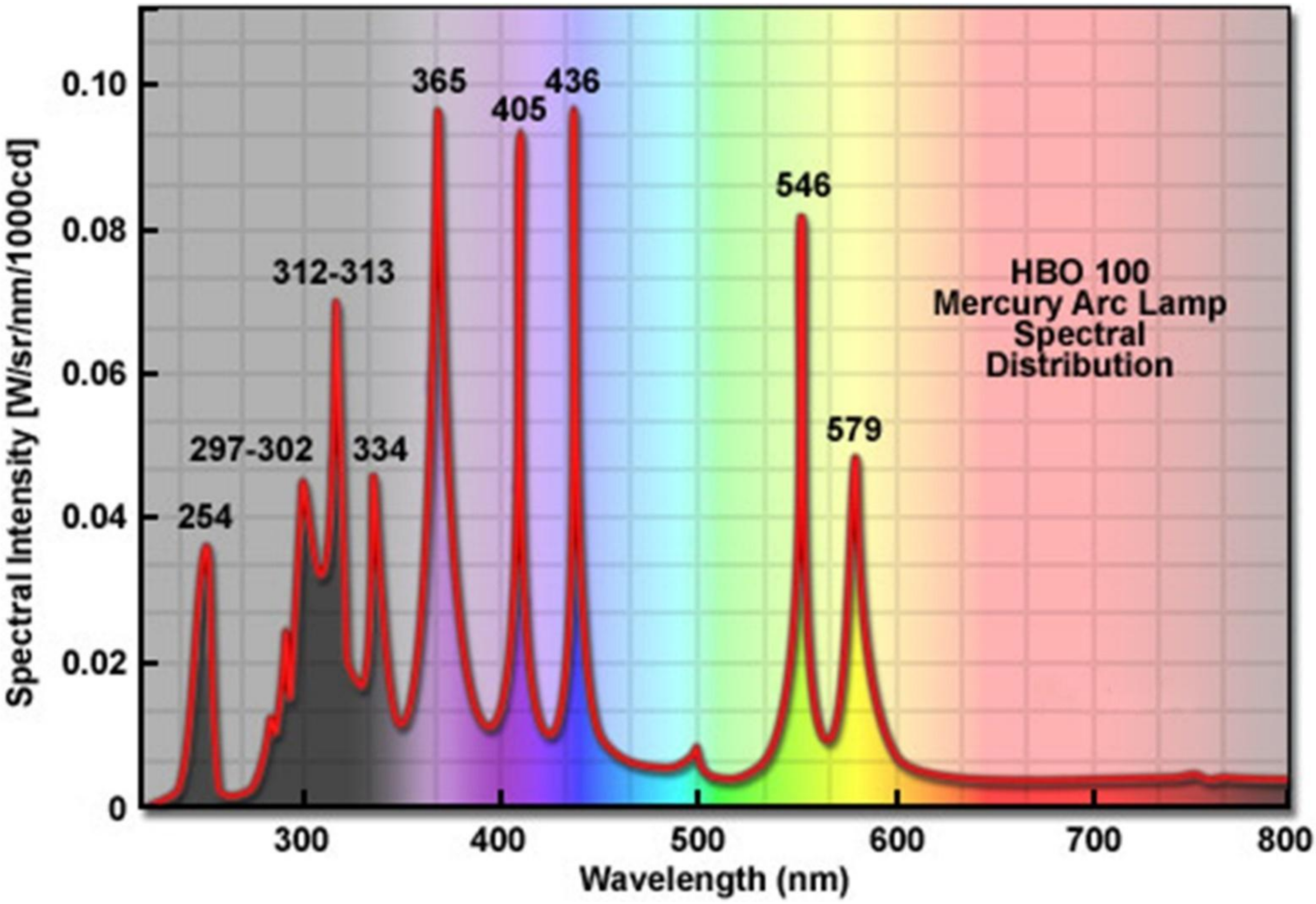
Measurement of prism angle

From these measurements, calculate the refracting angle A of the prism.

Appendix C

Mercury Spectrum

(Prism Spectrometer)



Note Your Marks

Sl. No.	Experiment	Marks		
		LP	LR	Total
01.	*Prism Spectrometer*			
02.	*Velocity of Light*			
03.	*Helmholtz Coils*			
04.	*Coupled Pendulum*			
05.	*Linear Air Track*			
06.	*Pohl's Pendulum*			
07.	*M.I. of bicycle Wheel*			
08.	*Electromagnetic Induction*			
09.	*Current Balance*			
10.	*Gyroscope*			